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On the Joint Evolution of Culture and Institutions*

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Abstract

Explanations of economic growth and prosperity commonly identify a unique causal effect, e.g., institutions, culture, human capital, geography. In this paper we provide instead a theoretical modeling of the interaction between culture and institutions and their effects on economic activity. We characterize conditions on the socio-economic environment such that culture and institutions complement (resp. substitute) each other, giving rise to a *multiplier* effect which amplifies (resp. dampens) their combined ability to spur economic activity. We show how the joint dynamics of culture and institutions may display interesting non-ergodic behavior, hysteresis, oscillations, and comparative dynamics. Finally, in specific example societies, we study how culture and institutions interact to determine the sustainability of extractive societies as well as the formation of civic capital and of legal systems protecting property rights.

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1 Introduction

”Era questo un ordine buono, quando i cittadini erano buoni [...] ma diventati i cittadini cattivi, divento’ tale ordine pessimo.”¹; Niccolò Machiavelli, *Discorsi*, I. 16, 1531

[...] ”among a people generally corrupt, liberty cannot long exist.” Edmund Burke, Letter to the Sheriffs of Bristol (1777-04-03).

”If there be no virtue among us, no form of government can render us secure. To suppose that any form of government will secure liberty or happiness without any virtue in the people is an illusion.” James Madison, 20 June 1788, Papers 11:163

The distribution of income across countries in the world is very unequal: according to World Bank data 2015, U.S. GDP per capita in international dollars is 71 times that of the Democratic Republic of Congo, 58 times that of Niger, 9 times that of India and 3 times that of Brazil, for instance. But accounts for economic growth and prosperity? What stands at its *origin*?

The question of *origin* is typically translated, in the economic literature, into one of causation in the language statistics and econometrics. Furthermore, often a single univariate cause is searched for and different possible causes are run one against each other.²

Identifying the main cause of economic growth, even if in different specific contexts, is complicated.³ The argument for institutions, for instance, essentially requires historical natural experiments where institutions are varied in geographical units with common geographical characteristics, culture, and other possible socio-economic determinants of future prosperity. Perhaps the most successful example is Acemoglu, Johnson and Robinson (2001)’s study of the institutional design of colonial empires, the more extractive the higher settlers’ mortality rates.⁴ Even in this

¹This was a good institutional order when citizens were good [...] but when citizen became bad, it turned into an horrible order; our translation

²Acemoglu and Robinson (2012), for instance, argue explicitly against each one of several potential alternative causes (geography, culture, ignorance; Ch. 2, *Theories that don’t work*) before laying their argument in favor of institutions in the rest of the volume.

³Besides the arguments following, there are also methodological reasons to be skeptical about the concept of causation when facing slow-moving non-stationary processes (as, arguably is long-run history). For instance, the origin of the Mafia in Sicily has been reduced with good arguments to a price shock on sulfur and lemon in the 1850’s (Buonanno, Durante, Prarolo, and Vanin, 2012); to the lack of city states in the XIV’t century - in turn a consequence of Norman domination (Guiso, Sapienza, and Zingales, 2016); to the Paleolithic split into nomadic pastoralism in 7th millennium B.C. (Alinei, 2007).

We might even suggest ironically that a single origin of economic growth and prosperity is a myth, like the one about the birth of all languages after the Christian God’s destruction of the Babel’s Tower, an “event” which was indeed “accurately” dated, allegedly on May 5th, 1491 B.C. by James Ussher, in 1650.

⁴Other examples include: the Spanish colonial policy regarding the forced mining labor system in Peru’ (Dell, 2010); the U.S.-Mexico border separating the city of Nogales (Acemoglu and Robinson, 2010); the border separating the island of Hispaniola into two distinct political and institutional systems, Haiti and the Dominican Republic (Diamond, 2005).

case, however, the assumption that the distinct institutions originated in the natural experiment arise in otherwise common cultural, geographical, environments is disputable. Indeed, settlers' mortality rates could be correlated with natives mortality rates and hence pre-colonial development; see e.g., Alsan, 2012, on the habitat for the Tse-Tse fly in Africa.⁵ Naturally, similar issues arise when identifying culture as the cause of prosperity, by isolating cultural variation in historical environments with a common institutional set-up.⁶

Even when not problematic, these causal analyses disregard the interactions between various determinant of economic growth: for instance, the same institutional change may have differential effects according to different cultural environments. Instances where this is the case have indeed been extensively documented.⁷ The main reference in this respect of course is the work of Putnam on social capital, following the differential effects in the North and in the South of Italy of the institutional decentralization of the 60's and 70's (Putnam, 1993).⁸ More generally, instances where institutions and cultural traits have manifestly jointly contributed to the development or the disruption of economic activity are common. This is the case for instance of Italian independent city states in the Renaissance (Guiso, Sapienza, and Zingales, 2008, 2016), industrialization and social capital in Indonesia (Miguel, Gertler and Levine, 2006), the technology of plough, patriarchal institutions and gender attitudes (Alesina, Giuliano and Nunn, 2013), the authoritarian culture of the sugar plantation regions of Cuba operated with slave labor as opposed to the liberal culture of the tobacco farms (Ortiz, 1963).⁹

⁵Furthermore, even the identification of the historical natural experiment as a change in institution is often debatable, as institutions generally reflect the cultural attitudes of the institution builders. Fischer (1989), for instance, studies institution formation during the early immigration waves in North America, showing how the cultural origins of the different groups of migrants (Puritans, Cavaliers, Quakers, Scots-Irish) affected the institutions they set in place; see also the well-know analysis by Greif (1994) of the institutional set-up of the Genoese and Maghrebi traders and Ben-Ner and Putterman (1998). The institutional changes in Medieval England from the Magna Charta onwards, which arguably stand at the origin of British prosperity and the Industrial Revolution (Acemoglu and Robinson, 2010) could also be attributed to a general bourgeois culture as forcefully argued by McCloskey (2006, 2010). The same can be said for the formation of Italian independent city states in the Renaissance (Guiso, Sapienza, and Zingales, 2008 and 2016).

⁶While the first example of such kind of analysis is perhaps Weber's protestant ethic arguments (Weber, 1930), recent examples include the effects of the slave trade on trust within african tribes differently exposed to it but with similar institutional set-up (Nunn and Wantchekon, 2011); individual values about the scope of application of norms of good conduct in Europe (Tabellini, 2008a).

⁷See Alesina and Giuliano (2015) for a related argument, accompanied by a comprehensive survey of this literature; along similar lines, see also Nunn (2012).

⁸More recently, see e.g., Durante, Labartino, and Perotti, 2011, on university reform in Italy; Nannicini, Stella, Tabellini, and Troiano, 2010 on voting reform again in Italy; Mauro and Pigliaru (2012) on how culture has different effects when political institutions are centralized or decentralized; Grosjean (2011) on the traditonal (Scottish-Irish) pastoral society honor code in the U.S.; Minasyan (2014) on the effects of development aid institutions depending on donor-recipient cultural differences.

⁹Even the presumption that culture is fundamentally immutable in the relevant time-frame, that is, changing at a much more slower pace than institutions, seems unfounded. Attitudes towards redistribution after the

Motivated by (this reading of) this literature, therefore, we study socio-economic environments in which culture and institutions jointly evolve and interact.¹⁰ Our objective is to develop an abstract model of culture, institutions, and their joint dynamics. But while we aim at an abstract model, we are not after full generality. Rather we aim at a simple model which could help identify conditions under which the interaction of culture and institutions produces specific outcomes of interest. In these environments the *origin*, and hence the causation, question loses most of its interest: culture and institutions are jointly and endogenously determined and they jointly affect economic growth and prosperity, indeed all sorts of economic activity.¹¹ The focus is moved from the cause (both culture and/or institution can have causal effects) to the process as determined by the interaction.

We characterize conditions under which cultural and institutional dynamics reinforce a specific (e.g., desirable) socio-economic equilibrium pattern, and economies in which on the contrary the interaction between culture and institutions ends-up weakening this equilibrium outcome. In this context, we can define the *cultural multiplier*, as the ratio of the total effect of institutional change divided and its direct effect, that is, the *counterfactual* effect which would have occurred had the distribution of cultural traits in the population remained constant after the institutional change. We also show how the joint dynamics of culture and institutions may display non-ergodic behavior, in which initial conditions determine important qualitative properties of their evolution, including the stationary state the process converges to. Finally, we study how interesting examples of hysteresis, oscillations, and comparative dynamics can emerge from the interaction of culture and institutions.

In specific example societies, we then study the sustainability of extractive institutions as well as the formation of civic capital and of institutions protecting property rights. In a society in which the elite is cultivating a differentiated value for leisure and is exercising its power by

institution of welfare states in Europe, for instance, (Alesina and Angeletos, 2005; Alesina and Giuliano, 2010) and in East Germany after unification (Alesina and Fuchs Schuendeln, 2007) also changed very rapidly. So did in various instances the applications of the honor code studied by Appiah (2010). This is also arguably the case for social/civic/human capital after colonization (Glaeser, La Porta, Lopez-de-Silanes, Shleifer, 2004; Easterly and Levine, 2012; and Bisin and Kulkarni, 2012) and for various social preferences after the creation of the Kuba Kingdom in the early 17th century Africa (Lowe et al., 2015). An example of a rapid joint change of institutions and culture induced by pro-active policies is the case of the fight against corruption in Hong Kong in the last decades which was driven by institutional change but engendered a deep modification of norms and attitudes towards corruption in the population in just a few years (Clark, 1987 and 1989; see also Hauk and Saez-Marti', 2002).

¹⁰While the existing related theoretical literature is very thin, one such environment has been studied by Greif and Tabellini (2010, 2011), where norms of cooperation (local vs. global) interact with institutional set-ups (informal vs. formal, clan vs. cities) to determine distinct paths of economic activity (China vs. Europe). See also Birdner and François (2011) for an analysis of how the size of gains from trade opportunities matters for the co-evolution of institutions and honesty norms.

¹¹The quotation at the head of this section suggest this view has been shared by political scientists and social philosophers since the early times of these disciplines.

extracting resources from the mass workers via taxation, we study the conditions under which the cultural and institutional dynamics maintain or reverse *extractive institutions*. We show that, in such society, depending on initial conditions, the elite might or might not have an interest in establishing less-extractive institutions. When it does, it devolves part of the fiscal authority to workers, to indirectly commit institutions to a lower tax rate (in turn inducing workers to exert an higher labor effort and hence to contribute to fiscal authorities). We also show that, in this society, culture and institutions are complements: institutional change devolving some fiscal authority to the workers weakens the incentives of the elite to transmit its own leisure culture, while in turn a smaller size of the elite augments its incentives to devolve fiscal authority to workers.

Our next example analyzes the case of a society where a fraction of the mass population is endowed with a *civic culture* which induces actions that may have beneficial effects on the functioning of public governance structures and on other members of the society. In such a context, we study conditions under which the cultural and institutional dynamics favors or hinders the spread of the *civic culture*. We show that, in this society, culture and institutions may act as substitutes. Indeed, on the one hand, civic values are more likely to diffuse when the degree of political representation given to the mass population increases; on the other hand, the larger the diffusion of civic values in society, the smaller the need to design institutional changes devolving formal power in a way that prevents the misgovernance of public policies. As it turns out, an exogenous institutional change (democratization) enlarging political representation to a mass population, may ends up having its effects mitigated by the induced *civic culture* diffusion associated to this institutional shock.

Finally, in a society where socio-economic interactions consist of conflictual relationships between groups with different propensions to act violently, we study conditions under which cultural and institutional dynamics favors or hinders the development of a legal system for the protection of *property rights*. We show that, in this society, when the more conflict-prone agents are powerful and institutions are stuck in their control, the cultural dynamics might work to un-do this equilibrium, favoring first the spread of a conflict-prone culture until a threshold, after which endogenous institutional dynamics are triggered which lead to the devolution of power, more property rights protection and a long term disappearance of the conflict-prone culture.

We proceed, in turn, with an abstract model of the dynamics of of institutions (Section 2) and then with an abstract model of cultural evolution (Section 3). We then study the interaction of the two (Section 4). Finally, we construct three simple examples aimed at illustrating the analysis and the different forms of interactions (Section 5).

1.1 Culture and institutions: Conceptual frame

We conceptualize culture as preference traits, norms, and attitudes which can be transmitted across generations by means of various socialization practices or can be acquired through socio-economic interactions between peers. Models of the population dynamics of cultural traits along these lines have been extensively studied in economics, e.g., by Bisin and Verdier in a series of papers (1998, 2000a,b), 2001).¹² The model of culture adopted in this paper conveniently remains inside the confines of this literature.

Less straightforwardly, we conceptualize institutions as mechanisms through which social choices are delineated and implemented. This is in line with the recent work effort by Acemoglu, Johnson, and Robinson in various pathbreaking contributions,¹³ but it also diverges from it in several ways. More specifically, e.g., in Acemoglu (2003) and in Acemoglu and Robinson (2006), institutions coincide with the political pressure group exercising the power to control social choice; and institutional change represents an effective commitment mechanism on the part of one political group to limit the extraction of resources from the others. With political power and control embedded in one single group, institutional change is then the result of the voluntary transfer of power across groups, typically under threats of social conflict.¹⁴

In this paper instead, while we share the view of institutional change as a commitment mechanism, we depart from the notion of political power and control as embedded in one single group. Specifically, we model institutions as Pareto weights associated to the different groups in the social choice problem.¹⁵ This allows us to view institutional change as more incremental (formally, a continuous rather than a discrete change in political control) than just revolutions and regime changes.¹⁶ Most importantly, it also allows us to eschew relying necessarily on social conflict as an explanation of institutional change: institutional change can much more generally occur as a

¹²This literature builds on the work of Cavalli-Sforza and Feldman (1973, 1981) in evolutionary biology and of Boyd and Richerson (1985) in anthropology; see Bisin and Verdier (2010) for a recent survey. For empirical work using this framework, see Bisin and Topa (2003), Bisin, Topa and Verdier (2004), and more recently Giavazzi, Petkov, and Schiantarelli (2014).

¹³See Acemoglu, Johnson, and Robinson (2006) for a survey; see also Acemoglu, Egorov, and Sonin (2015).

¹⁴Similarly, Besley and Persson (2009a,b, 2010) study a society with pressure groups alternating in the power to control economic institutions regarding taxation and contractual enforcement. Along these lines, Angelucci and Meraglia (2013) study charters to city states in the early Renaissance in Europe as concessions from the king to citizens to check and control the extractive power of fiscal bureaucracies. Levine and Modica (2012) and Belloc and Bowles (2012) take instead an alternative, explicitly evolutionary, approach to the dynamics of institutions.

¹⁵See also Guimaraes and Sheedy (2016) who ground the study of institutions in the theory of coalition formation; and Lagunoff (2009) who provides a general study of the theoretical properties of political economy equilibria with dynamic endogenous institutions.

¹⁶This approach is consistent with the view expanded by Mahoney and Thelen (2010) whereby institutional change occurs through gradual and piecemeal changes that only ‘show up’ or ‘register’ as change if a somewhat long time frame is considered. Mahoney and Thelen (2010) distinguish between four modal types of such institutional change: displacement, layering, drift, and conversion.

mechanism to imperfectly and indirectly internalize the lack of commitment and the externalities which plague social choice problems; social conflict being only one of them and not necessarily the most prevalent in history.¹⁷

2 The society

Consider a society with a continuum of agents separated into distinct groups defined in terms of relevant characteristics, i.e., political power and cultural traits.¹⁸ We shall assume first that political and cultural groups are aligned and indexed by $i \in I$. In Section 4 and in several of the examples in Section 5 we allow for groups which are distinct in terms of political power and cultural traits (with no substantial effects on the general analysis). In this paper we also restrict for simplicity to dichotomous groups, that is $I = \{1, 2\}$.¹⁹

Let a^i denote the action of an agent belonging to group i and $\mathbf{a} = \{a^1, a^2\}$ the vector profile of actions, which we assume lies in some compact set. Let p denote economic policy in society, also in some compact set.²⁰

We characterize culture in this society in terms of the distribution of the population by cultural groups. Let q^i denote the fraction of agents of group i in the population, with $\sum_{i \in I} q^i = 1$. We adopt the shorthand $q^1 = q$, $q^2 = 1 - q$.

The preferences of the fraction of agents belonging to group (i) are represented by an indirect utility function:

$$u^i(a^i, p; \mathbf{a}, q). \tag{1}$$

The dependence of u^i on a captures indirectly any externality in the economy. The dependence of u^i on q captures instead indirectly the dependence of technologies and resources on the distribution of the population by groups. A natural example would have the externality being represented by the mean action in the population: $A = qa^1 + (1 - q)a^2$.

¹⁷Consistently with this view, e.g., Lizzeri and Persico (2004) challenge Acemoglu and Robinson (2000, 2001, 2006) and Conley and Temimi (2001)'s rationalization of the extension of the franchise in early nineteenth century England, as an effect of threats to the established order. They argue instead that such institutional change had been motivated by the necessary evolution of public spending which required a commitment to limit particularistic politics in favor of public programs.

¹⁸Groups can of course be defined also in terms of resources, technologies, and so on. But we shall abstract from these characteristics for simplicity in the paper.

¹⁹With more than two groups the issue of coalition formation in institutional set-up and change becomes central. We leave this for a subsequent paper. The dynamics of $n \geq 2$ cultural traits has been studied by Bisin, Topa and Verdier (2009) and Montgomery (2009).

²⁰Of course policies might be multi-dimensional, an extension we avoid for simplicity. Also, without loss of generality we could add a parametrization of the component of economic institutions which acts directly on the economic environment. We avoid clogging the notation when not necessary.

We conceptualize institutions as mechanisms through which social choices are delineated and implemented at equilibrium. Specifically, we model institutions as (Pareto) weights associated to the different groups in the social choice problem which determines policy making at equilibrium.²¹ Let $\beta^i \geq 0$, denote the weight associated to group i , with $\sum_{i \in I} \beta^i = 1$. Again, we adopt the shorthand $\beta^1 = \beta$, $\beta^2 = 1 - \beta$.

2.1 Societal optimum and equilibrium

”[...] gli assai uomini non si accordano mai ad una legge nuova che riguardi uno nuovo ordine nella citta’ se non e’ mostro loro da una necessita’ che bisogni farlo; e non potendo venire questa necessita’ senza pericolo, e’ facil cosa che quella republica rovini, avanti che la si sia condotta a una perfezione d’ordine.”²²; Niccolo’ Machiavelli, *Discorsi*, I. 2, 1531.

We introduce the concepts of social optimum and of equilibrium for our society, given institutions β and a distribution of the population by cultural group q .

Definition 1 *The societal optimum is a tuple $\{\mathbf{a}^{eff}, p^{eff}\}$ such that:*

$$\{\mathbf{a}^{eff}, p^{eff}\} \in \arg \max \beta u^1(a^1, p; \mathbf{a}, q) + (1 - \beta) u^2(a^2, p; \mathbf{a}, q). \quad (2)$$

The societal optimum is a normative concept. It will be generally unattainable at equilibrium in our society. Two distinct equilibrium concept which will play a fundamental role in our analysis.

Definition 2 *The societal equilibrium is a tuple $\{\mathbf{a}, p\}$ such that:*

$$\begin{aligned} p &\in \arg \max_p \beta u^1(a^1, p; \mathbf{a}, q) + (1 - \beta) u^2(a^2, p; \mathbf{a}, q) \\ a^i &\in \arg \max_a u^i(a, p; \mathbf{a}, q) \quad i \in I = \{1, 2\} \end{aligned} \quad (3)$$

That is, the *societal equilibrium* is a Nash equilibrium of the game between agents of the two groups and the policy maker, in an institutional set-up characterized by weights β and distribution by cultural group q .

This simple formulation of the *societal equilibrium* captures lack of commitment on the part of the policy maker, who is not allowed to pick the policy p in advance of the choices of the economic agents.²³ The appropriate equilibrium notion, in a society in which the policy maker

²¹Importantly, we interpret the social choice problem not just normatively, but rather as the indirect choice problem solved by the political process.

²²[...] the majority of people will never agree to a new institutional order for the city unless necessary; and since necessity cannot come without danger, it is easily the case that institutions get into ruins before being perfected in a new order; our translation.

²³No issues other than notational ones are involved in modeling a policy maker choosing after the economic agents, thereby strengthening its lack of commitment.

has commitment is the *societal commitment equilibrium*. It is defined as the Stackelberg Nash equilibrium of the same game, in which the policy maker is the leader.

Definition 3 *The societal commitment equilibrium is a tuple $\{\mathbf{a}^{com}, p^{com}\}$ such that:*

$$\begin{aligned} \{\mathbf{a}^{com}, p^{com}\} &\in \arg \max \beta u^1(a^1, p; \mathbf{a}, q) + (1 - \beta) u^2(a^2, p; \mathbf{a}, q) \\ \text{s.t. } &a^i \in \arg \max_a u^i(a, p; \mathbf{a}, q), \quad i \in I = \{1, 2\} \end{aligned} \quad (4)$$

Under general conditions the *societal optimum*, the *societal equilibrium*, and the *societal commitment equilibrium* are distinct. More precisely,

Proposition 1 *Given institutions β and a distribution of the population by cultural group q , the societal equilibrium and the societal commitment equilibrium are both weakly inefficient, that is, they are weakly dominated by the societal optimum. On the other hand, the societal commitment equilibrium weakly dominates the societal equilibrium.*

Proof. The statement is a straightforward consequence of the fact that, for any (β, q) : i) problem (4), which defines a *societal commitment equilibrium*, is a constrained version of problem (2), which in turn defines a *societal optimum*; ii) any *societal equilibrium* satisfying (3) is always contained in the constrained feasible set of problem (4), which defines a *societal commitment equilibrium*.²⁴ ■

Societal equilibrium is generally not efficient in this society. Furthermore, while its institutional dynamics do drive society towards efficiency, it is not generally the case that institutions are efficient in a stationary state. By combining the results of Proposition 1 and Proposition 3 we obtain that a stationary *societal equilibrium* at best constitutes a *societal commitment equilibrium* for some institutions. In particular, the *societal commitment equilibrium* will not represent a *societal optimum* when the government policy p does not span the whole set of possible values of the vector profile \mathbf{a} . In other words, the institutional dynamics induces equilibria to tend towards efficiency, but i) not generally all the way towards a *societal optimal*, and ii) to a specific point of the attainable efficiency frontier (corresponding to a specific institutional set-up), that is, not necessarily towards a Pareto improvement. Several of the examples we study clearly demonstrate these points.

Making the dependence on (β, q) explicit, the *societal equilibrium*, the *societal commitment equilibrium*, and the *societal optimum* can be denoted, respectively:

$$[\mathbf{a}(\beta, q), p(\beta, q)]; \quad [\mathbf{a}^{com}(\beta, q), p^{com}(\beta, q)]; \quad [\mathbf{a}^{eff}(\beta, q), p^{eff}(\beta, q)].$$

²⁴Of course, under robust conditions - in particular in all examples we study - domination holds strictly.

A simpler formulation of our analysis of the dynamics of institutions is obtained imposing the following regularity assumptions.²⁵

Assumption 1 *Utility functions are sufficiently regular so that*

$$\mathbf{a}(\beta, q), p(\beta, q), \mathbf{a}^{com}(\beta, q), p^{com}(\beta, q) \text{ are continuous functions.}$$

Assumption 2 *Utility functions are sufficiently regular so that $p(\beta, q)$ is monotonic in β .*²⁶

2.2 Institutional dynamics

In our formulation, turning to an explicit notation for time, a given current set of institutions in period t , β_t , induces the social preference order internalized by the policy maker at t . Future political and economic institutions, β_{t+1} are designed, at the end of period t , to maximize the current social preference order by means of future policy choices, at $t + 1$. We assume that institutional design is myopic, that is, institutions are designed for the future as if they would never be designed anew in the forward future.²⁷

More precisely, institutions at time $t + 1$ are designed at time t as a solution to:

$$\begin{aligned} & \max_{\beta_{t+1}} \beta_t u^1 (a^1(\beta_{t+1}, q_{t+1}), p(\beta_{t+1}, q_{t+1}); a(\beta_{t+1}, q_{t+1})) \\ & + (1 - \beta_t) u^2 (a^2(\beta_{t+1}, q_{t+1}), p(\beta_{t+1}, q_{t+1}); a(\beta_{t+1}, q_{t+1})) ; \end{aligned}$$

That is, the societal equilibrium induced by institutions β_{t+1} at $t + 1$ is chosen to maximize the social welfare induced by institutions set β_t . The solution is characterized as follows:

Proposition 2 *Under Assumptions 1-2, and given (q_t, q_{t+1}) , the dynamics of institutions β_t is governed by the following implicit difference equation:*

$$\beta_{t+1} = \begin{cases} \beta \text{ such that } p^{com}(\beta_t, q_{t+1}) = p(\beta, q_{t+1}) & \text{if it exists} \\ \begin{cases} 1 & \text{if } p^{com}(\beta_t, q_{t+1}) > p(\beta, q_{t+1}), \forall 0 \leq \beta \leq 1 \\ 0 & \text{if } p^{com}(\beta_t, q_{t+1}) < p(\beta, q_{t+1}), \forall 0 \leq \beta \leq 1 \end{cases} & \text{else} \end{cases} . \quad (5)$$

²⁵ Assumptions 1-2 are translated into obvious but stringent restrictions on fundamentals; see Appendix C for details.

²⁶ See the Appendix A for a formal generalization of the analysis when Assumption 2 is not imposed.

²⁷ A greater degree of forward looking behavior when modeling institutional change is analytically intractable when joined with cultural dynamics. For an analysis of forward looking institutional change per se, see Lagunoff (2008), and Acemoglu, Egorov and Sonin (2015).

At any time t , current institutions β_t induce the choice $p(\beta_t, q_t)$ at equilibrium. But under the current social preference order, the choice $p^{com}(\beta_t, q_t)$ would be the most preferred. Therefore, institutions for time $t + 1$ are designed to induce a choice $p^{com}(\beta_t, q_{t+1})$ whenever possible at equilibrium; that is, β_{t+1} is such that $p(\beta_{t+1}, q_{t+1}) = p^{com}(\beta_t, q_{t+1})$. Whenever this is not possible, under our assumptions, institutions will be designed which induce a policy choice p as close as possible to $p^{com}(\beta_t, q_{t+1})$.

To characterize the stationary states of the dynamics of institutions and their stability properties, it is convenient to define $P(\beta, q) := p^{com}(\beta, q) - p(\beta, q)$.²⁸ For a given society characterized by institutions β and a distribution by cultural trait q , $P(\beta, q)$ is an indicator of the extent of the policy commitment problem faced by such society. Intuitively, the absolute value of $P(\beta, q)$ indicates the intensity of the commitment problem as it reflects the distance between what can best be achieved under commitment and what is actually achieved at equilibrium. The sign of $P(\beta, q)$ on the other hand indicates the direction of the institutional change in β needed to resolve the commitment problem.

Proposition 3 *Under Assumption 1-2, and given q , the dynamics of institutions governed by (5) have at least one stationary state. Any interior stationary state β^* obtains as a solution to $P(\beta, q) = 0$. The boundary stationary state $\beta = 1$ obtains when $P(\beta, q)|_{\beta=1} > 0$; while the boundary stationary state $\beta = 0$ obtains when $P(\beta, q)|_{\beta=0} < 0$.²⁹ In the continuous time limit, the dynamics governed by (5) satisfies the following properties:*

- if $P(\beta, q) > 0$ for any $\beta \in [0, 1]$, then $\beta = 1$ is a globally stable stationary state;
- if $P(\beta, q) < 0$ for any $\beta \in [0, 1]$, then $\beta = 0$ is a globally stable stationary state;
- any boundary stationary state is always locally stable;
- any interior stationary state β^* is locally stable if $\frac{\partial P(\beta^*, q)}{\partial \beta} < 0$.

2.3 Cultural dynamics

Cultural transmission is modeled as the result of *direct vertical* (parental) socialization and *horizontal/oblique socialization* in society at large. *Direct vertical* socialization to the parent's trait $i \in I = \{1, 2\}$ occurs with probability d^i . If a child from a family with trait i is not directly socialized, which occurs with probability $1 - d^i$, he/she is *horizontally/obliquely* socialized by

²⁸We collect here the properties of (5) which are most relevant in our subsequent analysis. A more complete global stability analysis is not particularly complex but tedious. We relegate it to Appendix B.

²⁹Note that we arbitrarily define $\beta = 1$ (resp. $\beta = 0$) as an *interior stationary state* if $P(\beta, q^i)|_{\beta=1} = 0$ (resp. $P(\beta, q^i)|_{\beta=0} = 0$).

picking the trait of a role model chosen randomly in the population inside the political group (i.e., he/she picks trait i with probability q^i and trait $i' \neq i$ with probability $q^{i'}$).

If we let $P^{ii'}$ denote the probability that a child, in (a family in) group $i \in I$ is socialized to trait i' , we obtain:

$$P^{ii'} = d^i + (1 - d^i)q^{i'}$$

Let $V^{ii'}(\beta, q)$ denote the utility to a cultural trait i parent of a type i' child. It depends on the institutional set-up and the cultural distribution the child will face when he/she will make his/her economic decision $a^{i'}$:

$$V^{ii'}(\beta, q) = u^i \left(a^{i'}(\beta, q), p(\beta, q); a(\beta, q), q \right) \quad (6)$$

Let $C(d^i)$ denote socialization costs. Direct socialization, for any $i \in I = \{1, 2\}$, is then the solution to the following parental socialization problem:

$$\begin{aligned} \max_{d^i \in [0,1]} & -C(d^i) + \sum_{i' \in I} P^{ii'} V^{ii'}(\beta, q) \\ \text{s.t.} & P^{ii'} = d^i + (1 - d^i)q^{i'} \end{aligned}$$

As usual in this literature, define $\Delta V^i(\beta, q) = V^{ii}(\beta, q) - V^{ii'}(\beta, q)$ as the *cultural intolerance* of group i . It follows that direct socialization, with some notational abuse, has the form:

$$d^i = d^i(q, \Delta V^i(\beta, q)) = d^i(\beta, q), \quad i \in I = \{1, 2\}. \quad (7)$$

The following assumption simplifies the analysis.³⁰

Assumption 3 *Utility and socialization cost functions are sufficiently regular so that $d^i = d^i(\beta, q)$ is continuous in (β, q) , for any $i \in I = \{1, 2\}$.*

Let $D(\beta, q) = d^1(\beta, q) - d^2(\beta, q)$. Turning again to the explicit notation for time t , the dynamics of q_t is straightforwardly determined.

Proposition 4 *Under Assumption 3, and given β_{t+1} , the dynamics of the distribution by cultural group q_t is governed by the following difference equation:*

$$q_{t+1} - q_t = q_t(1 - q_t)D(\beta_{t+1}, q_{t+1}). \quad (8)$$

The dynamics of q_t governed by (8) have at least the two boundaries as stationary states, $q = 0$ and $q = 1$. Any interior stationary state $0 < q^ < 1$ obtains as a solution to $D^i(\beta, q) = 0$. In the continuous time limit, the dynamics governed by (8) satisfies the following properties:*

³⁰See the Online Appendix for the associated conditions on fundamentals.

- if $D(\beta, q) > 0$ for any $q \in [0, 1]$, then q_t converges to $q = 1$ from any initial condition $q_0 > 0$;
- if $D(\beta, q) < 0$ for any $q \in [0, 1]$, then q_t converges to $q = 0$ from any initial condition $q_0 < 1$;
- if $D(\beta, 1) > 0$, then $q = 1$ is locally stable ;
- if $D(\beta, 0) < 0$, then $q = 0$ is locally stable;
- any interior stationary state q^* is locally stable if $\frac{\partial D(\beta, q^*)}{\partial q} < 0$.

It is often convenient to impose the following assumption (we do so in the examples as it simplifies the study of the dynamics of culture essentially without loss of generality).

Assumption 4 *Socialization costs are quadratic:*

$$C(d^i) = \frac{1}{2} (d^i)^2.$$

The following corollary characterizes the resulting simplification:

Corollary 1 *Under Assumption 4,*

$$D(\beta, q) = \Delta V^1(\beta, q)q - \Delta V^2(\beta, q)(1 - q).$$

Given β , any interior stationary state q^ is obtained as a solution to:*

$$\frac{\Delta V^1(\beta, q)}{\Delta V^2(\beta, q)} = \frac{q}{1 - q}. \quad (9)$$

With quadratic socialization costs an interior stationary state of the cultural dynamics has the property that the relative share of the two groups in the population is equal to their relative cultural intolerance; hence in particular, higher intolerance corresponds to a larger share at the stationary state.

3 Joint evolution of culture and institutions

Under Assumptions 1-3, the joint dynamics of institutions and culture is governed by the system (5,8), which we report here for convenience:

$$\begin{aligned} \beta_{t+1} &= \begin{cases} \beta \text{ such that } p^{com}(\beta_t, q_{t+1}) = p(\beta, q_{t+1}) & \text{if it exists,} \\ \begin{cases} 1 & \text{if } p^{com}(\beta_t, q_{t+1}) > p(\beta, q_{t+1}), \forall 0 \leq \beta \leq 1 \\ 0 & \text{if } p^{com}(\beta_t, q_{t+1}) < p(\beta, q_{t+1}), \forall 0 \leq \beta \leq 1 \end{cases} & \text{else} \end{cases} \\ q_{t+1} - q_t &= q(1 - q_t)D(\beta_{t+1}, q_{t+1}). \end{aligned}$$

Any *interior* stationary state of the system (5,8), (β^*, q^*) , solves:

$$P(\beta, q) = D(\beta, q) = 0$$

Let $\beta = \beta(q)$ be the steady state manifold associated with the first equation in (??) and $q(\beta)$ be the steady state interior cultural manifold associated with the second equation.

Very little can be proved in general about the non-linear dynamical system (5,8); as a consequence we will turn to phase-diagrams in specific examples. We can nonetheless show the following:³¹

Proposition 5 *Under Assumptions 1-3 the dynamical system (5,8) has at least one stationary state.*

Let (β^*, q^*) denote an interior stationary state of the dynamical system (5,8). To obtain a more detailed analysis of the dynamics of the non-linear system (5,8) it is very useful to distinguish environments where culture and institutions are complements from those where they are substitutes.³²

Definition 4 *Institutional and cultural dynamics are complementary, at (β^*, q^*) , when the steady state manifolds $\beta(q)$ and $q(\beta)$ have slopes of the same sign:*

$$\text{sign} \left(\frac{d\beta(q)}{dq} \right)_{(\beta^*, q^*)} = \text{sign} \left(\frac{dq(\beta)}{d\beta} \right)_{(\beta^*, q^*)} .$$

Conversely they are substitutes at (β^, q^*) when the slopes have opposite signs.*

As it turns out, in fact, the existence of cycles in a neighborhood of a stationary state requires some form of substitutability in the dynamics of culture and substitution. In other words, sufficient conditions to rule out oscillatory dynamics are characterized in the next two propositions, stated for simplicity under the following separability condition on preferences:³³

Assumption 5 *Agents' preferences satisfy*

$$u^i(a^i, p; a, q^i) = v^i(a^i, p) + H^i(p; a, q^i) .$$

Proposition 6 *Under Assumptions 1-5, at a locally stable interior steady state (β^*, q^*) , the local dynamics show no converging cycles (dampening oscillations) if institutional and cultural dynamics are complementary.*

³¹The proof is detailed in Appendix B.

³²To simplify the analysis we consider the continuous time limit of the system in what follows; see Appendix A for details and a discussion.

³³See Appendix A for technical details.

The existence of a stable spiral steady state requires that institutional and cultural dynamics be substitutes and relative rates of change of culture and institutions which balance each other.³⁴ A related complementarity condition rules out limit cycles (periodic orbits) in institutional and cultural dynamics as an implication of the Bendixon Negative Criterion.

Proposition 7 *Under Assumptions 1-5, there are no periodic orbits and limit cycles in the neighborhood of at a locally stable interior steady state (β^*, q^*) if: i) β^* is locally stable given $q = q^*$; and ii) $q = q^*$ is locally stable given $\beta = \beta^*$.*

3.1 The cultural multiplier

In this section we study more in detail the interaction between culture and institutions along their dynamics, to identify conditions under which it produces specific outcomes of interest. More specifically, we aim at conditions under which cultural and institutional dynamics reinforce or hinder a specific socio-economic equilibrium pattern, e.g., induced by an exogenous shock to the dynamical system. To this end, we introduce and study the concept of *cultural* (resp. *institutional*) *multiplier*, the ratio of the long run change in institutions (resp. culture) relatively to the counterfactual long run change that would have happened had the cultural composition (resp. institutional set-up) of society remained fixed. In fact, motivated by the literature discussed in the Introduction, which mostly stresses the economic effects of institutions for given cultural composition, we shall concentrate on the *cultural multiplier*, under the understanding that symmetric arguments and conditions hold for the *institutional multiplier*.

Consider the effects of a change in a parameter γ at a *stable interior* stationary state of the dynamics, $(\beta^*, q^*) \in (0, 1)^2$. Adding explicit reference to γ in the notation, we normalize the arbitrary components of the comparative dynamics environment we study so that:

γ increases, locally at the steady state, both the policy p as well as the extent of the commitment problem:

$$\frac{dp^{com}(\beta^*, q^*; \gamma)}{d\gamma} > \frac{dp(\beta^*, q^*; \gamma)}{d\gamma} > 0;$$

members of group 1 (with institutional power β) aim at a relatively larger policy level, p :

$$\frac{\partial p(\beta^*, q^*; \gamma)}{\partial \beta} > 0.$$

As a consequence, a positive change in γ induces a process of convergence to a new steady state characterized by a larger *societal equilibrium* policy p (through a larger β); that is, in the absence of cultural change, $\left(\frac{d\beta^*}{d\gamma}\right)_{q=q^*} > 0$.

The *cultural multiplier* measures the strength of the interaction between culture and institutions.

³⁴See Appendix A for precise conditions ensuring converging oscillations towards the steady state:

Definition 5 *The cultural multiplier on institutional change, at a locally stable interior steady state (β^*, q^*) , is*

$$m = \left(\frac{d\beta^*}{d\gamma} \right) / \left(\frac{d\beta^*}{d\gamma} \right)_{q=q^*} - 1. \quad (10)$$

A positive *cultural multiplier* reinforces the effect of a change in institutions at a socio-economic equilibrium. Whether the multiplier is indeed positive, in turn, crucially depends on culture and institutions being complements or substitutes:

Proposition 8 *Under Assumptions 1-4 at a locally stable interior steady state (β^*, q^*) , the cultural multiplier m is positive (resp. negative) if and only if the institutional and cultural dynamics are complementary (resp. substitute).*

When the slopes of $\beta(q)$ and $q(\beta)$ have the same sign, institutional and cultural dynamics are complementary and the cultural multiplier is positive. As an illustration, suppose that culture and institutions are complements in the sense that $\frac{d\beta(q)}{dq}$ and $\frac{dq(\beta)}{d\beta} > 0$. Then in our environment an increase in γ is set to induce an increase in β . Because of complementarity, this in turn induces an increase in q which feedbacks positively on the institutional weight β . Any exogenous institutional change, through an increase in γ , is amplified by the associated cultural dynamics that interact with institutions. Conversely, an institutional change would be hindered by cultural changes (i.e., the *cultural multiplier* is negative) when culture and institution are substitutes, that is, when the slopes of $\beta(q)$ and $q(p)$ have opposite signs.

A better intuition for the mechanisms driving complementarity, and hence the sign of the cultural multiplier, can be developed under Assumption 5. In this case, the complementarity condition on the slopes of $\beta(q)$ and $q(\beta)$, can be shown to require that³⁵

$$\frac{\partial P(\beta^*, q^*)}{\partial q}, \quad \frac{d \frac{\Delta V^1(p)}{\Delta V^2(p)}}{dp} \text{ have the same sign.}$$

The term $\frac{\partial P(\beta^*, q^*)}{\partial q}$ reflects how the institutional commitment problem is affected by a change of the distribution by cultural groups. Institutional change represents a mechanism to solve the commitment problem and therefore to induce an increase in the societal equilibrium policy p . This is obtained by giving more institutional weight to the group supporting relatively more the policy p , that is, by increasing β (given our normalization of the comparative dynamics environment).

Conversely, the term $\frac{d \frac{\Delta V^1(p)}{\Delta V^2(p)}}{dp}$ reflects how a change in the equilibrium policy p affects the dynamics of the distribution of the population by cultural group. A higher p is associated to a larger steady state frequency of the trait that is relatively more in favor of that policy, that is, an increase in q .

The *cultural multiplier* governs the effects of the interaction between culture and institutions on an aggregate economic variable of interest, e.g., per capita income, public good provision, or

³⁵See Appendix A for details.

any other measure of economic activity. Let $A(p, q, a^1(p), a^2(p))$ formally denote the economic aggregate. A *cultural multiplier on A* can then be defined as

$$m_A = \frac{dA}{d\gamma} / \left(\frac{dA}{d\gamma} \right)_{q=q^*} - 1.$$

In Appendix B we decompose m_A , and show that it is proportional to $1 + m$,

$$m_A = K \cdot (1 + m)$$

where the coefficient of proportionality K depends on three components: the compositional cultural effect of q on $A(\cdot)$, the full effect of the policy p on $A(\cdot)$, and the sensitivity of the equilibrium policy $p(\beta, q)$ to cultural change.

4 Distinct political and cultural types

In this section we extend our analysis to consider a society in which political and cultural groups are distinct. Let $i \in I$ index the political groups and $j \in J$ the cultural groups. Let a^{ij} denote the action of agents of subgroup (i, j) and $a = \{a^{ij}\}_{i,j}$ the vector profile of actions. Let q^{ij} denote the distribution of the population by cultural group and by $q = \{q^{ij}\}_{i,j}$ the vector profile satisfying $\sum_{j \in J} q^{ij} = 1$, for $i \in I$. Let λ^i denote the fraction of agents in political group i . Utility functions are then written $u^{ij}(a^{ij}, p; a, q)$.

In this society, we continue to identify political institutions with the weights of the groups $i \in I$ in the social choice problem which determines economic policy, $\beta = \{\beta^i\}_i$ satisfying $\sum_{i \in I} \beta^i = 1$.

As for cultural transmission, we assume for simplicity that political groups are perfectly segregated, so that the reference population for an agent in subgroup (i, j) is the subgroup itself. Fixing a political group $i \in I$, direct vertical socialization to the parent's trait, say $j \in J$, occurs with probability d^{ij} ; $P_t^{i,jj}$ (resp. $P_t^{i,jj'}$) denote the probability that a child, in (a family in) political group $i \in I$ with trait j is socialized to trait j (resp. j') at t ; $V^{i,jj}(\beta_{t+1}, q_{t+1})$ (resp. $V^{i,jj'}(\beta_{t+1}, q_{t+1})$) denotes the utility to a cultural trait j parent in political group i of a type j (resp. j') child.

It is then straightforward to extend the analysis of the previous sections to this society, with distinct political and cultural groups, to obtain the following system for the joint dynamics of culture and institutions:

$$\beta_{t+1}^i = \begin{cases} \beta \text{ such that } p^{com}(\beta_t^i, q_{t+1}) - p(\beta, q_{t+1}) & \text{if it exists,} \\ \begin{cases} 1 & \text{if } p^{com}(\beta_t^i, q_{t+1}) > p(\beta, q_{t+1}), \forall 0 \leq \beta \leq 1 \\ 0 & \text{if } p^{com}(\beta_t^i, q_{t+1}) < p(\beta, q_{t+1}), \forall 0 \leq \beta \leq 1 \end{cases} & \text{else} \end{cases}$$

$$q_{t+1}^{ij} - q_t^{ij} = q^{ij}(1 - q_t^{ij}) \left(d^{ij} - d^{ij'} \right), \text{ with } d^{ij} = d^{ij}(q_t, \Delta V^{ij}(\beta_{t+1}, q_{t+1})).$$

5 Specific example societies

In this section we work out several specific example societies, simple but rich enough to display some interesting cultural and institutional dynamics.³⁶

5.1 Elites, workers, and extractive institutions

In this example we study *extractive institutions*, that is, a society where the power of a political group is exercised by extracting resources from the other group, e.g., via taxation. We study in particular conditions under which the cultural and institutional dynamics in this society maintain or reverse its extractive character.

Consider a society populated by two groups, workers and members of the elite, with distinct cultural traits and technologies. In particular, the preferences of the members of the elite are shaped by cultural norms which let them value leisure greatly, more than workers. Hence in equilibrium, while both the elite and workers are endowed with the same technology which transforms labor into private consumption goods, workers will work and the elite will generally eschew labor and constitute a *leisure class*. In this society, taxes on labor income finance public good consumption, which is valued by both groups. But since members of the elite do not work, (fiscal) institutions are extractive in that only workers bear the fiscal weight of the public good.

Institutions lack commitment; that is, fiscal authorities choose the tax rate without internalizing its effect on workers' labor effort. This gives institutions generally an incentive to tax labor excessively. As a consequence, in this society, the elite might have an interest in establishing less-extractive institutions, by devolving part of the fiscal authority to workers. This would indirectly commit institutions to a lower tax rate, in turn inducing workers to exert an higher labor effort and hence to contribute more to the public good. Indeed, we will show that, in this society, culture and institutions are complements: institutional change devolving some fiscal authority to the workers weakens the incentives of the elite to transmit its own culture and hence reduces the size of the leisure class, while in turn a smaller leisure class augments the incentives of the elite to devolve fiscal authority to workers.

Formally, let workers be group $i = 1$ and the elite be $i = 2$. Both groups can transform labor one-for-one into private consumption goods. Let a^i denote labor exerted by any member of group i . Let s denote the initial resources all agents are endowed with. Let p denote the (income) tax rate and G the public good provided by fiscal institutions. Preferences for group i are represented by the following utility function³⁷:

$$u^i(a^i, G, p) = u(a^i(1-p) + s) + \theta^i v(1 - a^i) + \Omega \cdot G.$$

³⁶In all the examples we impose and exploit various regularity conditions without explicit mentioning them. We discuss however all the details in the Online Appendix.

³⁷ $u(\cdot)$ and $v(\cdot)$ are concave increasing function with sufficient regularity. See the Online Appendix for details.

Our characterization of the distinction between workers and the elite just in terms of cultural values requires that:³⁸

the parameter θ^i , representing the preference for leisure, satisfies $\theta^1 < \theta^2$.

To better illustrate the dynamics of culture and institutions in this society we consider extreme preferences for leisure of the elite, $\theta^2 > \frac{u'(s)}{v'(1)} > 1 = \theta^1$. In this case, members of the elite never work, $a^2 = 0$, and consume their resources, s . Workers instead exert some effort level $a^1 > 0$ and consume in fact $a^1 + s$ units of the private consumption good. Both groups consume the public good G , in an amount equal the tax burden, to balance the budget of the fiscal institutions: $G = p [a^1 q + a^2(1 - q)]$ where q is again the fraction of workers type $i = 1$.

The *societal equilibrium* and the *societal commitment equilibrium* are then easily characterized, for any institutional set-up, β , and distribution of the society by cultural traits, q . Equilibrium policies, that is, tax rate p , are as in Figure 1. Consider first the *societal equilibrium*. Typically, for β small enough, $\beta \leq \underline{\beta}(q)$, all policies p inducing no labor effort are a *societal equilibrium*. In this case, workers have so little power that the natural ex post incentive is to tax them to the extent that they do not provide any labor supply, $p \geq p_0$. On the contrary, for $\beta \geq \bar{\beta}(q)$, workers have effectively control of the fiscal authority. In this case, labor income is either not taxed (if $\beta > \bar{\beta}(q)$); or else it is taxed only inasmuch as it is necessary to finance the amount of public good preferred by workers themselves, p^* (in this last case $\bar{\beta} = 1$). For intermediate values of $\beta \in (\underline{\beta}(q), \bar{\beta}(q))$, the *societal equilibrium policy* $p(\beta, q)$ takes interior values and is a decreasing function of β . The ex-post incentives to finance the public good through labor income taxes are lower when the workers' interest are better represented.

In the *societal equilibrium*, the policy maker does not internalize the negative distortion of taxation on the tax base. Hence taxes at a *societal equilibrium* are systematically (and inefficiently) higher than at the *societal commitment equilibrium* where such effect is internalized. The *societal commitment equilibrium*, $p^{com}(\beta, q)$, more specifically, is also a decreasing function of β , always smaller than the tax rate p^{\max} which maximizes tax revenue. Furthermore, $p^{com}(\beta, q) = 0$ when β is larger than the threshold $\bar{\beta}(q)$ (or $p^{com}(\beta, q) = p^*$ if $\bar{\beta}(q) = 1$). Most importantly,

$$p^{com}(\beta, q) < p(\beta, q), \quad \forall \beta < \bar{\beta}(q), \quad 0 < q < 1.$$

³⁸In the next subsection, we also some distinction in terms of initial resource endowment.

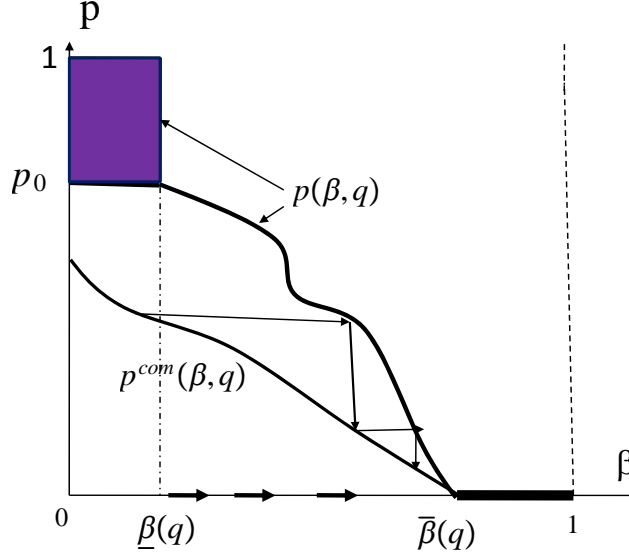


Figure 1: Elites, Workers, and Extractive Institutions
Equilibrium Policies and Institutional Dynamics

Institutional dynamics. The institutional dynamics tend to internalize the inefficiency of the *so-cietal equilibrium* which is due to lack of commitment, for any given cultural distribution in the population q . In this society, therefore, fixing $0 < q < 1$, for any $\beta < \bar{\beta}(q)$, the institutional dynamics tend towards increasing the fiscal authority of workers, that is towards increasing β . This leads to reducing the excessive (and inefficient) tax rate p until it is optimal for the workers to do so. At the stationary equilibrium, therefore, all fiscal authority ends up effectively in the hands of workers.³⁹ Importantly, the public good consumption G^* , at the stationary equilibrium, is efficient. It is the preferred level under the stationary institutional set-up β^* (which effectively does not account for the preferences of the elite), but it does not generally constitute a Pareto improvement with respect to the equilibrium level of public good associated to the initial institutional set-up (which does account for the preferences of the elite).

Cultural dynamics. For every value of β the cultural dynamics tend to an interior stationary state $q(\beta)$, whereby q increases when $q < q(\beta)$ and decreases instead when $q > q(\beta)$. In other words, given the institutional set-up, both group tend to engage in more intense cultural transmission when their trait is relatively minoritarian in society.⁴⁰ Furthermore, the relative incentives to

³⁹Even though $\beta < 1$, a higher β would have no effect on equilibrium policies.

⁴⁰This is the case when transmission is driven by *cultural substitution*; see Bisin and Verdier (2001) and (2010)

socialization $\Delta V^1(p)/\Delta V^2(p)$ are decreasing in p . Indeed, as taxation leads to increased rent extraction on labor, leisure class norms are more likely to be transmitted than those of the workers: the larger the rents of the elites, the larger their socialization advantage.

Joint evolution of culture and institutions. The joint cultural and institutional dynamics of this society are concisely represented in the phase diagram in Figures 2a) and 2b). The curve along which β is constant, $\bar{\beta}(q)$, is weakly increasing in q : to a larger fraction of the workers' trait in the distribution, (i.e., a larger q), is associated a (weakly) larger β in the long run dynamics of institutions. Indeed, the larger is q , the larger are the incentives of the elite to extract resources from workers by taxing labor income. Consequently a larger fiscal authority to workers is necessary to reach their most preferred tax rate. At the stationary state all fiscal authority is effectively devolved to the workers, $p = 0$, and no public good is consumed in the society, $G = 0$.

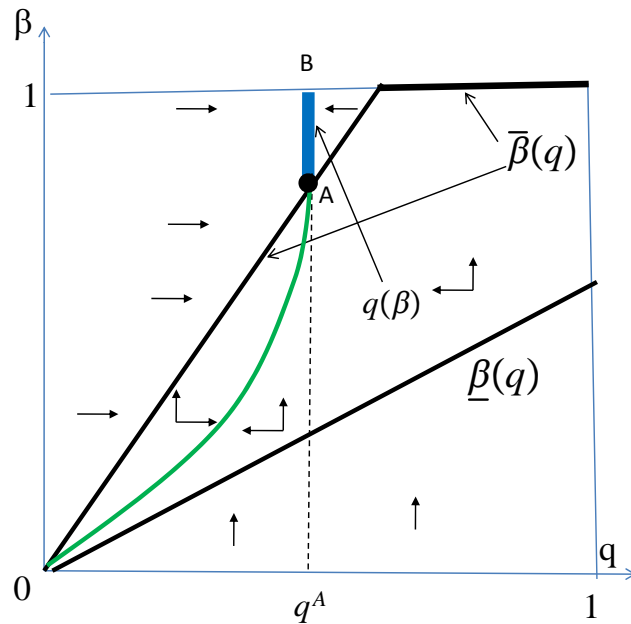


Figure 2a: Elites, workers, and Extractive Institutions
 Joint Dynamics (case a: $G^* = 0$)

for, respectively, the underlying conceptual analysis and for a survey of the empirical evidence with respect to several cultural traits.

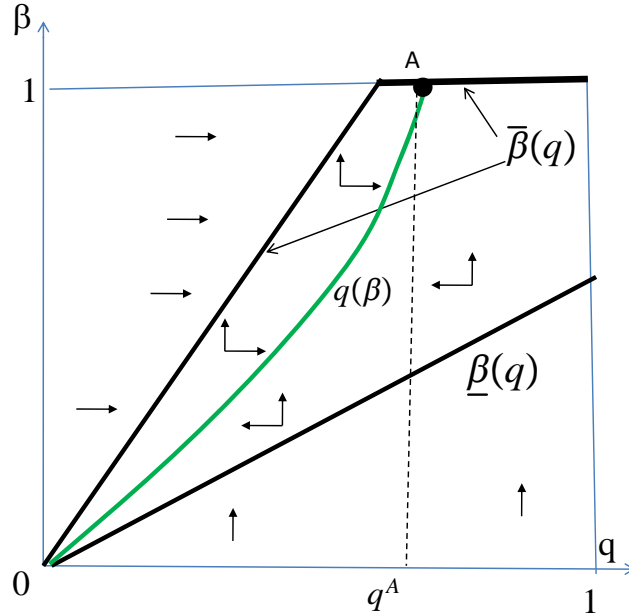


Figure 2b: Elites, workers, and Extractive Institutions
 Joint Dynamics (case b: $G^* > 0$)

The curve along which q is constant, $q(\beta)$, is also weakly increasing in β : more fiscal authority to workers leads to a lower equilibrium tax on labor p and therefore to an increase in the prevalence of the workers' cultural trait in the population in the long run dynamics of culture. Indeed, the lower is the tax rate, the higher are the relative gains of workers in the socialization process.

Since both $\beta(q)$ and $q(\beta)$ are increasing, culture and institutions are cultural complements in this society, reinforcing each other: the cultural multiplier is positive (Proposition 8). Furthermore, the joint evolution of culture and institutions displays a unique ergodic stationary state. The parameter configuration of the society determines whether any of the public good is provided in the long run, that is whether the society is in case a) or b) in shown in Figures 2a) and 2b).⁴¹ In either case, however, *extractive* institutions are undermined by their own inefficiency (due to the lack of commitment of the policy maker). The transition away from *extractive* institutions is inevitable, from any initial condition.⁴²

⁴¹Formally, case a) without long run provision of G holds when $\bar{\beta}(q^A) \leq 1$, while case b) with long run provision of G holds when $\bar{\beta}(q^A) > 1$.

⁴²The mechanism driving the dynamics of institutions is distinct from the one stressed by Acemoglu (2003), Acemoglu and Robinson (2006, 2010) and Acemoglu, Johnson, and Robinson (2006). In our society, the transition is triggered independently of any technology on the part of the workers to threaten, e.g., by means of a revolution, the power of the aristocrats. Furthermore, in this society, extractive institutions are not stable, independently of

5.1.1 The transition away from extractive institutions.

In the society we have just studied in the previous section, the elites' lack of commitment leads to very inefficient equilibrium outcomes when the elites themselves control the institutional set-up. As a consequence the joint dynamics of culture and institutions necessarily drive the society towards less extractive institutions where fiscal authority is devolved mostly (or even completely, in some parameters' configurations) to workers. In this section we study a simple extension of this society with the objective of providing a more articulate and interesting representation of the transition away from extractive institutions, one which delves more deeply into the cultural preferences and the incentives of the elites.

Consider a society alike to the one studied in the previous section except in that i) a fraction of the members of the elite hold a cultural trait which specifies work-ethic norms akin to those of the workers, rather than those of the leisure class;⁴³ ii) workers are not endowed with an initial endowment of resources, they can only consume off their production; iii) workers face a survival constraint, a minimum level of consumption necessary for survival. Furthermore, in the society we study in this section, taxes are not raised to finance the public good but are instead purely extractive, being redistributed pro-capita to the members of the elite.⁴⁴

As in the previous section, the elites, as a political group, have the power of taxing workers, but cannot commit ex-ante to a tax rate. In this society, however, their incentives and preferences are heterogeneous: the members of the elite who share work-ethic norms (the *bourgeois*) are more aligned with workers' interests than those who do not (the *aristocrats*). Depending on their distribution by cultural trait and the political control they exert on the fiscal authority in society, the elites might impose a tax rate such that workers are constrained to subsistence (an *extractive regime*).⁴⁵

The institutional dynamics of this economy will in general be *non-ergodic*, depending crucially on initial conditions. Only when the initial institutional set-up guarantees enough control on fiscal authority on the part of the workers, the institutional dynamics will tend to transition away from the *extractive regime*. Interestingly, this transition will generally induce the formation of a sizeable bourgeoisie. As well, it is also the case that a larger bourgeoisie at the initial conditions favors the transition away from the *extractive* regime.

The detailed analysis of this society follows. Workers, group $i = 1$, are in proportion $1 - \lambda$ the population distribution between workers and elites. In particular, this is the case even if the relative power of workers is unaffected by their relative size (or even relative income) in society.

⁴³Note that in this society, therefore, political groups (workers and elites) are not aligned with cultural groups (bourgeois and aristocrats, inside the elite). The example is then a special case of the class of societies introduced in Section 5 and therefore follows the notational structure laid out there.

⁴⁴This is not substantial to the analysis. It is just for the sake of variation.

⁴⁵The survival constraint can be binding only for workers, as members of the elites are endowed with initial resources which we postulate are enough for survival.

and members of the elites, group $i = 2$, in proportion λ . Members of the elite carry one of two possible cultural traits, $j = a, b$: i) the *bourgeois*, in proportion $q^{2b} = q$ of the total elite size λ , have the same preferences as workers; ii) the *aristocrats* are instead in proportion $q^{2a} = 1 - q$ of the elite and have preferences with extreme disutility for work.

All agents have preferences over a consumption good c^{ij} and labor effort a^{ij} , where $i = 1, 2$ indexes the group and j the cultural trait.⁴⁶ The production technology converts effort one-to-one in the consumption good. Let $\beta^1 = \beta$ denote the institutional weight of the workers, and p , the policy choice, represent the tax rate on workers' output, a^1 . Let T denote the lump sum fiscal transfer received by each member of the elite, by budget balance. Let finally \bar{c} denote the subsistence level required for survival. .

Preferences are represented by the following utility functions, respectively for workers and elites:

$$\begin{aligned} u^1(a^1, T^1, p) &= u(a^1(1-p) + s^1 + T^1) + \theta^1 v(1 - a^1) \\ u^{2j}(a^{2j}, T^2, p) &= u(a^{2j} + s^2 + T^2) + \theta^{2j} v(1 - a^{2j}) \end{aligned}$$

Our characterization of the distinction between the political groups (workers and elites) and the cultural groups (bourgeois and aristocrats) in terms of cultural values and technologies requires that:

the parameter θ^{2j} , representing the preference for leisure of the elites, satisfies $\theta^{2a} > \theta^{2b} = \theta^1$;

initial resources s^i satisfy: $s^1 = 0$, $s^2 = s > \bar{c}$;

tax transfers T^i satisfy $T^1 = 0$, $T^2 = T$.

Again we assume that the aristocrats have extreme preferences for leisure $\theta^{2a} > \frac{u'(s)}{v'(1)} > 1 = \theta^1$ so that again they never work, $a^{2a} = 0$.

In this society, the labor effort exerted by workers, $a^1(p)$, is non-monotonic in the tax rate p , depending on whether the survival constraint is binding, as shown in Figure 3. When the survival constraint is not binding, $a^1(p)$ is decreasing in p , because of the disincentive effects of the tax rate on effort.

⁴⁶Abusing notation the apex j is omitted for workers, $i = 1$, since they are culturally homogeneous.

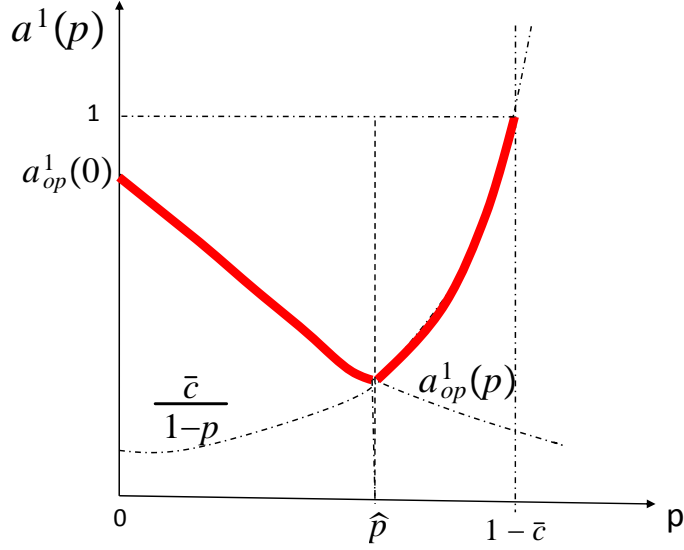


Figure 3: Transition away from extractive institutions
Worker's effort

When instead the survival constraint is binding (in the *extractive* regime),⁴⁷ $a^1(p) = \frac{\bar{c}}{1-p}$ for $p \in [\hat{p}, 1 - \bar{c}]$; that is, $a^1(p)$ is increasing in p . As for the labor effort choice of the bourgeoisie, $a^{2b}(T) \geq 0$ is decreasing in the transfer level T . Aristocrats continue not exerting any effort (for any value of $T \geq 0$). The per-capita fiscal transfer to the members of the elite is set to balance the budget of the fiscal institutions: $T =: \frac{1-\lambda}{\lambda} pa^1$.

The *societal equilibrium policy* $p(\beta, q)$, and the *societal commitment policy* $p^{com}(\beta, q)$ are illustrated in Figure 4. When the institutional weight of the workers is low enough, below a threshold $\underline{\beta}(q)$, at the *societal equilibrium*, the fiscal authorities tax the workers to a level that forces them to an extractive regime where the survival constraint \bar{c} is binding.⁴⁸ Indeed, when workers are at the survival constraint, more extractive institutions will not necessarily reduce their labor effort, as workers will always have to exert enough effort to satisfy the survival constraint. On the other hand, when workers are not at survival, the elites might have an incentive to establish less-extractive institutions, to indirectly commit on a lower tax rate, in turn inducing workers to extend an higher production effort, as in the society studied in the previous section. If

⁴⁷The policy space is assumed bounded in such a way as to always make survival of the workers feasible.

⁴⁸The maximal feasible tax rate is $p = 1 - \bar{c}$. At this rate workers have to supply their full time endowment $a^1 = 1$ to maintain their consumption level at the survival limit.

workers are sufficiently powerful, therefore, their behavior is the same as in the previous section: $p(\beta, q) > 0$, and declining in β , for $\beta \in (\underline{\beta}(q), \bar{\beta}(q))$; while $p = 0$ for $\beta \geq \bar{\beta}(q)$.

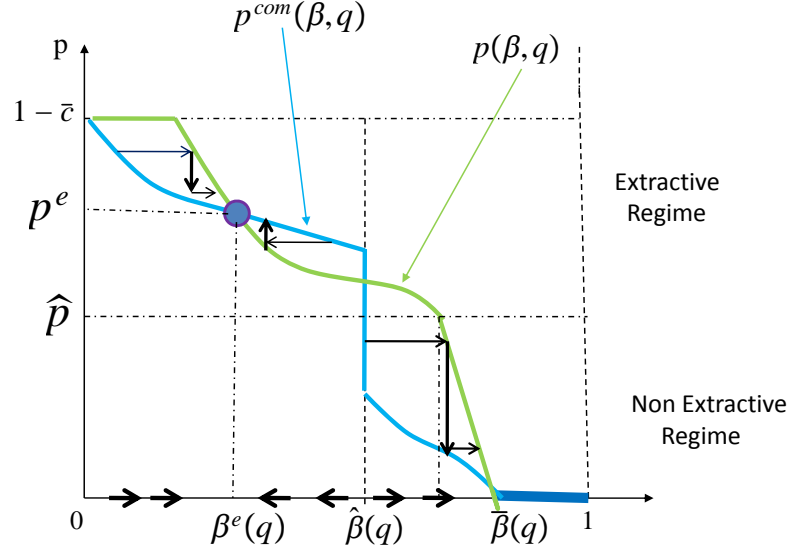


Figure 4: Transition away from extractive institutions
Equilibrium policies and institutional dynamics

As the *societal equilibrium policy*, the optimal policy at the *societal equilibrium with commitment*, is decreasing in the fiscal authority of workers β and is $= 0$ when $\beta \geq \bar{\beta}(q)$. Furthermore, when β is small enough, $p^{com}(\beta, q)$ is also high enough so that workers are kept at subsistence and the societal equilibrium with commitment is in the *extractive* regime. The transition away from the *extractive* regime occurs at $\beta = \hat{\beta}(q)$ at the *societal equilibrium with commitment*, a lower β than at the *societal equilibrium*, as in the first case the distortionary effects of taxation are internalized.

Not surprisingly, when the optimal policy at the *societal equilibrium with commitment* induces a non-extractive regime, $\beta > \hat{\beta}(q)$, it is the case that $p^{com}(\beta, q) < p(\beta, q)$. Indeed, without commitment the fiscal authorities do not internalize the negative effect of taxation on the tax base and therefore induce an equilibrium tax that is inefficiently high. This is not the case, however in the *extractive* regime. In this regime, in fact the effect of taxation on the effort of workers (and therefore on the tax base) is positive. This tends to make the *societal equilibrium policy* $p(\beta, q)$ too low compared to the *societal equilibrium policy with commitment* $p^{com}(\beta, q)$. On the other hand, in this regime it is also the case that the fiscal authorities do not internalize

the fact that a larger value of p makes workers deviate more from their optimal effort choice in order to satisfy their survival constraint. Such distortionary effect on workers' welfare tends to make $p(\beta, q)$ too high compared to $p^{com}(\beta, q)$. When β is very low, the distortionary effect on the workers' welfare dominates: indeed pressed by a high value of p , workers have to choose a^1 very close to the maximal possible value, in order to satisfy their survival constraint. The welfare distortionary of the survival constraint on the workers' labor supply is therefore maximal and $p(\beta, q)$ is too high compared to $p^{com}(\beta, q)$. As β increases, the positive effect of p on the tax base kicks in and turns to be larger, so that $p^{com}(\beta, q) > p(\beta, q)$. By continuity of the equilibrium policy functions $p(\beta, q)$ and $p^{com}(\beta, q)$ in the range $(0, \hat{\beta}(q))$, there is a point $\beta = \beta^e(q)$ where the two curves cross, as depicted in Figure 4. But as β increases even more, to the point where the fiscal authorities select taxes p which do not constrain workers at survival, the distortionary effects of taxation turn to have the negative usual impact on the tax base. As we noted, in fact, in this case, $p^{com}(\beta, q) < p(\beta, q)$ (it turns out that this happens discontinuously, as in the figure).

Institutional dynamics. From the previous discussion, the non-ergodic behavior of the institutional dynamics is clear. Fixing a cultural distribution $0 < q < 1$, for all initial value $\beta_0 \in [0, \hat{\beta}(q))$, the institutional dynamics converge to a unique steady state $\beta = \beta^e(q)$ and the society ends up in an *extractive* regime with low political representation of the workers who are maintained at their survival constraint by extractive taxation on the part of the elites.⁴⁹ Conversely for initial values $\beta_0 \in (\hat{\beta}(q), \bar{\beta}(q)]$, the institutional dynamics are very different. The weight of the workers on the institutional setting converge to the unique steady state $\beta = \bar{\beta}(q)$, characterized by no taxation, in a non-extractive regime.⁵⁰

⁴⁹Interestingly in the *extractive* regime, higher taxation may actually increase the efficiency of the rent extraction process as the survival constraint prevents the traditional disincentives on labor supply to kick in. This local effect is arguably instrumental in maintaining such an extractive regime for workers. This is reminiscent of an argument in Clark (2009, chapter 2), suggesting that policies that would otherwise appear as having inefficiency costs in a non extractive world, on the contrary may find some efficiency rationale under extractive conditions.

⁵⁰The dynamics from $\beta_0 = \hat{\beta}(q)$ are indetermined. Also, for initial values $\beta_0 > \bar{\beta}(q)$, the institutional weight of the workers is already large enough to induce no taxation and therefore no distortions. Institutions do not change and stay at their initial value $\beta_t = \beta_0$ for all $t > 0$.

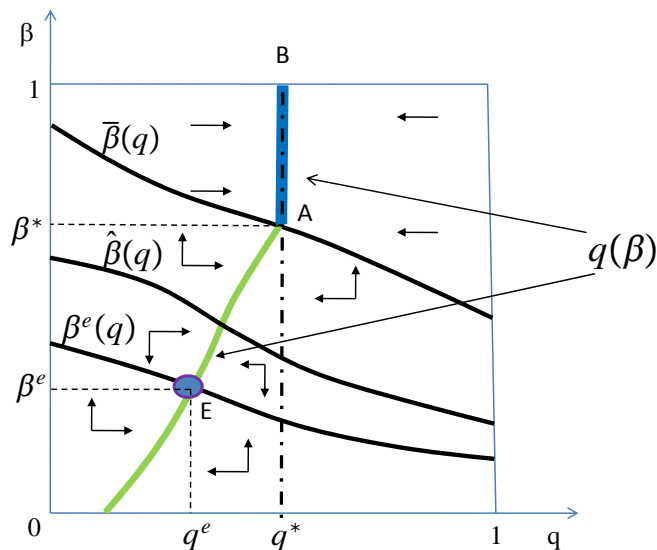


Figure 5: Transition away from extractive institutions
Joint dynamics

Cultural dynamics. The dynamics of cultural evolution within the elite are driven by the relative incentives to socialization $\Delta V^b(p)/\Delta V^a(p)$, which is generally⁵¹ decreasing in p . Indeed, aristocratic norms are more likely to be transmitted than those of the bourgeoisie the larger the rents of the elites. Since equilibrium taxation is a decreasing function of the institutional weight of workers, β , the more fiscal authority the workers possess in society, the larger the diffusion of (the norms of) the bourgeoisie inside the elite, and hence in society.

Joint evolution of culture and institutions. The joint cultural and institutional dynamics of this society are concisely represented in the phase diagram in Figure 5. The relevant portion of the institutional stationary manifold $\beta(q)$ is decreasing in q : in this society, in fact, the workers are at least in part supported by the bourgeoisie in their preferences over fiscal policy and as a consequence, when the fraction of the elite with bourgeois values is larger, the institutions support a no-tax policy even with less power to the workers. The cultural stationary manifold, $q(\beta) \in (0, 1)$, is instead as in the society studied in the previous section: an upward sloping curve in the region $\beta \in [0, \beta(q)]$ and a vertical line $q = q^*$ in the region $\beta \geq \beta(q)$ for which there is no

⁵¹More precisely, this is the case when the tax rate \hat{p} at which an *extractive* regime is triggered is below the tax rate that maximizes total tax revenues.

redistribution, $T = p = 0$. This is essentially for the same reasons: a higher β leads to a lower p and hence to greater socialization gains to the bourgeoisie and greater q in the long run.

Differently from the case of the society studied in the previous section, therefore, culture and institutions are not cultural complements in this society. Furthermore, the joint evolution of culture and institutions does not display a unique stationary state, but rather two: an *extractive* state, (β^e, q^e) , and a stationary state with no-tax, characterized by $\beta \geq \beta^*$ and $q = q^*$.

The dynamics of culture and institutions in this society will in general be *non-ergodic*: which stationary state they will converge to in the long-run depends on initial conditions. A transition away from *extractive* institutions is not inevitable in this society, as higher taxes do not decrease the fiscal rents of the elites when workers are at or around the survival constraint. *Extractive* institutions are therefore not any more undermined by their own inefficiency and could be supported in the long-run. Whether *extractive* institutions are supported in the long-run, or whether the dynamics transition away, depends on the political control the elites exert on the fiscal authority in society, but also on their distribution by cultural trait (that is, the relative size of the bourgeoisie, which is partly aligned with workers' interests). When the initial institutional set-up ensures enough control on fiscal authority on the part of the workers, the dynamics will tend to transition away from the *extractive regime*. But a larger bourgeoisie at the initial conditions also favors the transition away from the *extractive* regime. Formally, the basin of attraction of the $(\beta \geq \beta^*, q^*)$ stationary states comprises all (β, q) strictly above the line $\hat{\beta}(q)$. It is larger in β for higher q and it is also larger in q for higher β .

Importantly, even though culture and institutions are not cultural complements in this society, along a transition away from *extractive* institutions, with a powerful elites and small bourgeoisie (that is, in the region above $\hat{\beta}(q)$ and on the left of $q(\beta)$ in Figure 5, the dynamics display the devolution of power to workers jointly with the formation of a sizeable bourgeoisie (β and q both increase along the path).

5.2 Civic Culture and institutions

In this example we study *civic culture*, that is, a society where a fraction of the population is endowed with intrinsic motivations which induce actions that may have beneficial effects on other members of the society. We study in particular conditions under which the cultural and institutional dynamics in this society favors or hinders the spread of the *civic culture*.⁵²

Consider a society populated by two groups, workers and members of the elite, as in the previous example. Both workers and members of the elite are endowed with the same technology which transforms labor into private consumption goods. Fiscal institutions collect lump-sum taxes to finance the provision of a public good, whose consumption is valued by both groups. The

⁵²See also Ticchi, Verdier and Vindigni (2013), and Besley and Persson (2016) for specific analyses of the interactions between political culture and political institutions.

provision of the public good, however, creates opportunities for corruption that benefit exclusively the elite. We can therefore think of the elite as of a *caste of bureaucrats*. The preferences of a fraction of the workers are shaped by a *civic culture*, which motivates them to exert a participation effort, complementary to the provision of the public good, as well as a monitoring effort to fight corruption, costly to the elite. *Civic participation* involves e.g., contributing privately to public goods, creating social associations, volunteering in social activities. *Civic control* creates transparency, by monitoring the government in its public good provision process.

In this society, therefore, public good provision is associated to different externalities on society. On the one hand, it stimulates the civic participation of a fraction of the workers, a positive externality on society as a whole. On the other hand, public good provision induces corruption and the reaction of a fraction of workers against it, a positive externality on workers with no civic culture and a negative externality on the elite.

Institutions lack commitment; that is, fiscal authorities choose lump-sum taxes to finance public good provision without internalizing the effects of civic culture in society. Public good provision can be larger or smaller at equilibrium than efficient, depending on whether the positive or the negative externalities of civic culture dominate. As a consequence, the institutional dynamics lead to a stationary balanced allocation of power between workers and the elite.

Most interestingly, in this society, culture and institutions may act as substitutes. Indeed, on the one hand, the incentives to transmit civic culture are generally increasing in the political representation of workers in society; on the other hand, the larger is the spread of civic culture in the population of workers, the smaller are the incentives to design institutional changes devolving power to the workers, as the beneficial effects of civic culture are already present. In this society, therefore, an exogenous institutional change which endows with more political power citizens could see its effects mitigated by the induced cultural dynamics associated to the spread of civic culture in the population.

Formally, let workers be group $i = 1$ and the elite be $i = 2$, in fractions $\lambda^1 = 1 - \lambda^2 = \lambda$. Workers belong to one of two cultural groups. The first, $j = c$ in proportion q , is characterized by civic culture; the second group is passive with respect to the civic society, $j = p$. All individuals (workers and elite) are endowed with a fixed amount of resources, s . Lump-sum taxes are raised to finance public expenditures, g . In the process of providing for a public good, a fraction $\mu > 0$ of public expenditures leaks into corruption, generating diverted rents $T = \mu g$ that benefit exclusively the members of the elite. The residual share of public good expenditures is used to provide public good, $G = (1 - \mu) g$.

Workers can exert two types of efforts, *civic participation* and *civic control*. Let worker's j participation (resp. control) effort be denoted e^{1j} (resp. a^{1j}). Societal civic participation effort is then $E = \lambda [q \cdot e^{1c} + (1 - q) \cdot e^{1p}]$, while societal civic control effort is $A = \lambda [q \cdot a^{1c} + (1 - q) \cdot a^{1p}]$. Societal civic participation effort E produces a society wide externality which augments each in-

dividual's endowment by $\kappa \cdot E$, $\kappa > 0$. Societal civic control effort A increases the transaction costs associated to corruption activities: the consumption associated to T units of diverted rents is $(1 - \theta A)T$, with $0 < \theta < 1$.⁵³ The government policy is total public expenditures g , financed by lump-sum taxes in the same amount.

Workers' preferences are as follows:

$$\begin{aligned} U^{1c}(c^{1c}, G, a^{1c}, e^{1c}, T) &= c^{1c} + v(G) - (\alpha \cdot T)(1 - a^{1c}) - C(a^{1c}) + G \cdot e^{1c} - \Phi(e^{1c}) \\ U^{1p}(c^{1p}, G, a^{1p}, e^{1p}, T) &= c^{1p} + v(G) - C(a^{1p}) - \Phi(e^{1p}); \end{aligned}$$

where $c^{1c} + v(G)$ is the direct utility of private consumption and the public good; $-(\alpha \cdot T)(1 - a^{1c})$ is the intrinsic motivation for civic control; $C(a^{1c})$ is the utility cost of undertaking civic control; $G \cdot e^{1c}$ is the intrinsic motivation to contribute e^{1c} to civic participation; while $\Phi(e^{1c})$ is the disutility cost of civic participation.⁵⁴ Members of the elite have standard preferences over consumption and the public good:

$$U^2(c^2, G) = c^2 + v(G).$$

Policy choice $p = g$ depends on the workers' efforts only through $\theta \cdot A$ and $\kappa \cdot E$. Therefore, since the contribution of each worker effort to societal efforts E and A is negligible, passive workers always choose not to exert any effort, $a^{1p} = e^{1p} = 0$, and workers with civic culture contribute according to their intrinsic motivations. In fact, since both G and T increase in g , e^{1c} and a^{1c} also increase in g .

Under some reasonable regularity conditions, $p(\beta, q)$ and $p^{com}(\beta, q)$ are as in Figure 6: downward sloping in β , for any q . More specifically, when the character of civic culture is not too unbalanced in terms of civic participation, workers with civic culture are less in favor of large public expenditures than the elite. As a consequence, an increase in β would tend to reduce the size of the public expenditures at the *societal equilibrium*, $p(\beta, q)$, and the *societal commitment*, $p^{com}(\beta, q)$. For the same reason, at a given value of β , an increase in the fraction of workers with civic culture, q , would have the same effect on public expenditures. Most importantly, $p(\beta, q)$ crosses $p^{com}(\beta, q)$ from above at some interior point $\hat{\beta}(q)$. Indeed, $p^{com}(\beta, q)$ is the policy choice once all externalities in society are internalized. But the negative externality, via $\theta(A)$, is born out only by elite members, while the positive externality, via E , is enjoyed by the whole society. As a consequence, when the political power of the elite is large (i.e., β small), internalizing the negative externality dominates the society's political objectives and $p^{com}(\beta, q) < p(\beta, q)$. Conversely, when the weight of the elite is small, internalizing the positive externality dominates and consequently $p^{com}(\beta, q) > p(\beta, q)$.

⁵³Effort costs are normalized so that $\theta A < 1$.

⁵⁴See the Online Appendix for details, assumptions, and functional forms.

Institutional dynamics. For all initial value β_0 the institutional dynamics converge to a unique steady state $\beta = \hat{\beta}(q)$ and political power is shared between the workers and the elite; see Figure 6.

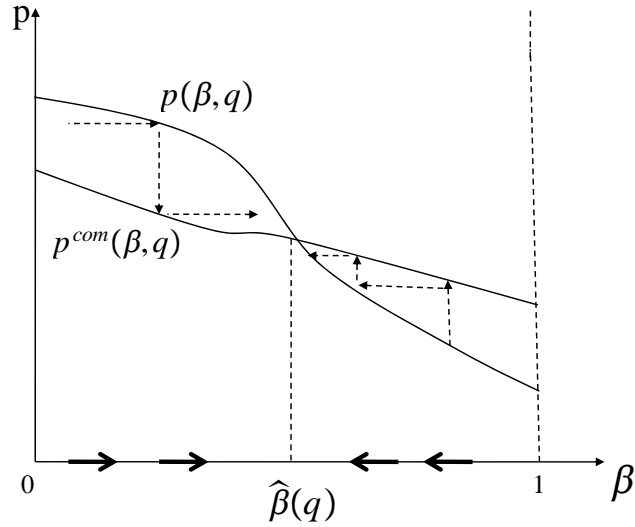
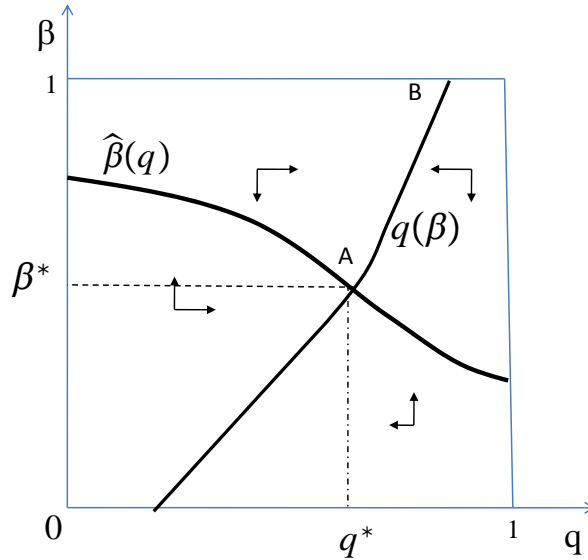


Figure 6: Civic Culture and institutions
Institutional dynamics

Importantly, in this society, the political power of workers at the stationary state, $\hat{\beta}(q)$, is decreasing in the cultural predominance of civic culture, q . This is because societal civic control A increases in q and A substitutes for formal political power. More in detail, at the stationary state $\hat{\beta}(q)$, $p^{com}(\hat{\beta}(q), q) = p(\hat{\beta}(q), q)$ and the positive and negative externalities associated to public expenditures balance out at the margin in the government objective function. An increase in q would lead to fewer public expenditures, as workers with civic culture are more concerned than the rest of society by corruption. To restore the equilibrium, institutional dynamics move then in the direction of re-introducing larger public expenditures and hence of reducing the political power of workers, β .

Cultural dynamics. The elite is culturally homogenous and hence displays no cultural dynamics. The cultural dynamics within workers are determined by the relative incentives to transmit civic culture, $\Delta V^{1c}(p)/\Delta V^{1p}(p)$, as they depend on the equilibrium policy instrument p . When civic participation e^{1c} is less sensitive to public good provision than civic monitoring a^{1c} ,

$\Delta V^{1c}(p)/\Delta V^{1p}(p)$ is decreasing in p . As the societal equilibrium $p(\beta, q)$ is itself a decreasing function of β and q , the relative incentives to transmit civic culture increase with both β and q in society. As a consequence, $q(\beta)$ is upward sloping in β : the formal delegation of power to the workers tends to induce a larger diffusion of civic culture between workers; see Figure 7.

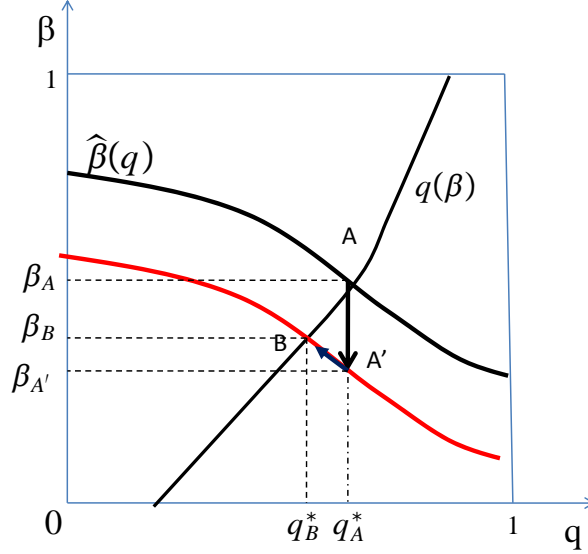


Figures 7: Civic Culture and institutions
Joint dynamics

Joint evolution of culture and institutions. The joint evolution of culture and institutions is also illustrated in Figure 8. The stationary state of the joint dynamics is (β^*, q^*) . At (β^*, q^*) , the two manifolds $\beta(q)$ and $q(\beta)$ have slopes of opposite signs. As a consequence, culture and institutions are substitute in this society and the cultural multiplier is negative (see Proposition 8): the effect of an exogenous shock which changes the political power of workers in some direction would be mitigated by the ensuing cultural dynamics.

As a way of illustration, Figure 8 describes the effects of an increase in the coefficient κ , which, other things equal, increases the positive externality associated to civic participation E . A change in κ triggers a higher demand for public expenditures, and therefore some institutional dynamics biased against the workers' group. This institutional change in turn reduces the relative incentives to transmit civic culture and leads to a reduction of q . As civic culture is reduced, there is less civic control effort against corruption in society. This in turn calls for some institutional change returning some formal power to workers, mitigating therefore the initial institutional impact of

the shock to κ .



Figures 8: Civic culture and institutions
Comparative dynamics

Interestingly, depending on the relative speeds of the dynamics of culture and institutions, the dynamics of adjustment to the shock may not be monotonic. Suppose for instance that institutions adjust much faster than culture, so that the adjustment dynamics lies on $\hat{\beta}(q)$. In this case, the shock on κ , after having induced β to jump downwards (with q constant at q^*), has q decrease and β increase along the adjustment path.

5.3 Property rights and conflict

In this example we study *property rights*, that is, a society where socio-economic interactions consist of agents contesting the component of each other's resource endowment which is not protected by the legal system. We study in particular conditions under which the cultural and institutional dynamics in this society, between groups with different propensions to act violently, favors or hinders the development of a legal system for the protection of *property rights*.

Consider a society populated by two groups, culturally differentiated by their propensity to act in conflict. One group is more prone to violence, e.g., because it is adopting a *culture of honor*.⁵⁵

⁵⁵Along the lines of the specific groups described by Nisbett (1993), Cohen and Nisbett (1994), or more recently Grosjean (2014).

In this society, agents are matched randomly in a contest for their own endowment. After two agents match, their relative effort determines the probability that each of them succeeds in the contest, hence winning the fraction of the endowment of the opponent which is not protected by property rights. The group which is more prone to violence has a lower cost of effort at the margin and hence tends to engage in more violent conflict at equilibrium, winning a larger fraction of the endowment contests.

Property right protection, e.g., by means of a legal system, is the main policy variable. They reduce the incentives to engage in violent conflict at equilibrium, for both groups. Property rights are therefore valuable in terms of efficiency because the effort employed by each group in conflict represents a negative externality on the other group. They are always favored by the group which is less prone to violence and they are favored also by the more prone to violence group when its fraction in the population is large enough. A larger fraction of conflict-prone agents in society, in fact, hurts both groups ex-ante: it induces a larger rent dissipation for the conflict-prone agents and also a larger probability of extortion (loss of endowment) for the others.

The dynamics of culture and institutions in this society display several complex features, including, notably, an interesting form of hysteresis. More specifically, societies where the more prone to conflict group is relatively powerful in terms of its control of institutions and hence of property rights policy, but small as a fraction of the population, will rely on limited property rights protection and will tend to display no institutional change. When however this group is relatively large, it develops enough incentives to devolve institutional power to the other group to induce more property right protection. It is then possible that, in a society where the conflict-prone are powerful and institutions are stuck in their control, the cultural dynamics work to un-do this equilibrium. Indeed the induced diffusion a conflict-prone culture will go through a threshold, after which endogenous institutional dynamics are triggered. Such institutional changes lead to the devolution of power and more property rights protection. Relatedly, in this society, a temporary institutional shock may trigger persistent (indeed, long-run) effects on the dynamics of culture and institutions, e.g., irreversibly drive the system into a new long run trajectory of the institutional and cultural dynamics.

In this society, political and cultural groups are aligned. Formally, let group $i = 1$ be the group which is more prone to violent conflict. Agents in group 1 have a marginal cost of violent effort to c^1 , smaller than the marginal cost c^2 which characterizes agents in group 2. Denote for convenience $\alpha = (c^2 - c^1) / c^1$.

Each agent's endowment of the consumption good, prior to the contest, is $s > 0$. The policy variable, p , represents the fraction of each agent's endowment s which is protected in the contest, a measure of the extent to which property rights are protected in society. Let a^{hk} denote the violent effort exerted by an agent h when matching with an agent k .⁵⁶ The probability of agent

⁵⁶Note that this example represents an extension of the general analysis in that a^j is a multi-dimensional vector;

h winning the contest is $\frac{a^{hk}}{a^{hk}+a^{kh}}$.⁵⁷ The winner of the contest appropriates of $2(1-p)s$ units of the consumption good.

Agents observe the opponent type before choosing their effort, that is, the contest is a complete information game. The Nash equilibrium effort of an agent of type i in his contest with an agent of type j is:

$$a^{ij} = 2(1-p)s \frac{c^j}{(c^i + c^j)^2}.$$

Matching is random, so that any agent will match a conflict-prone agent with probability q , which represents the fraction of such agents in society. Let the ex-ante expected payoff for agents of group $i = 1, 2$ at equilibrium be denoted $\Omega_i(p, q)$. It is decreasing in q , for any i , as a larger fraction of conflict-prone agents in society hurts both groups ex-ante: it induces a larger rent dissipation for the conflict-prone agents and a larger probability of extortion (loss of endowment) for the others. On the other hand, while $\Omega_2(p, q)$ is always increasing in p , $\Omega_1(p, q)$ is increasing in p only for a large enough fraction q . Indeed conflict-prone agents gain as a consequence of better property rights protection only when their fraction in the population is large enough. Finally, assume that implementing a level p of property rights protection requires a resource cost $C(p)$ satisfying standard convexity properties.

Institutional dynamics. The policy schedules $p(\beta, q)$ and $p^{com}(\beta, q)$ are represented in Figure 9. When $\beta \geq q$, the societal equilibrium involves no property rights protection and $p(\beta, q) = 0$. On the contrary, when $\beta < q$, some protection of property right is implemented, with $p(\beta, q) > 0$. Moreover, in such a case, $p(\beta, q)$ is a decreasing function of β and is increasing in q . The larger the political power of the conflict-prone group, the smaller the level of property rights protection, as such group benefits less from this protection. On the other hand, the larger the fraction of the conflict-prone individuals in society, the larger the social need for a reduction of violent effort, otherwise dissipated in resource contests, and hence the larger the social need for protection of property rights.

the same methods apply however.

⁵⁷This is the case if $a^{hk}, a^{kh} > 0$; while the probability of winning is $1/2$ if $a^{hk} = a^{kh} = 0$.

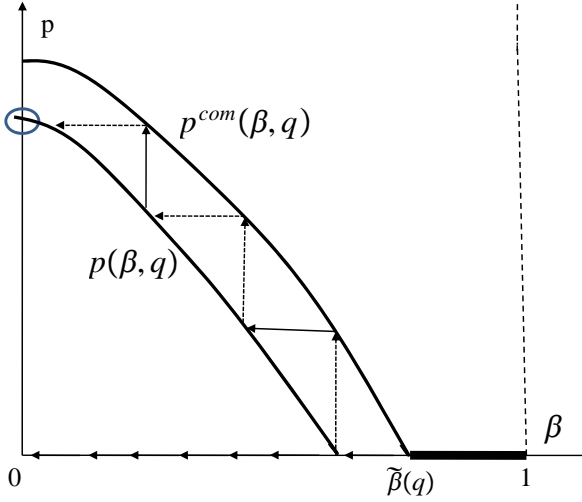


Figure 9: Property rights and conflicts
Institutional dynamics

It is also the case, in this society, that, $p(\beta, q) \leq p^{com}(\beta, q)$: for any (β, q) , a larger extent of property rights protection reduces the excessive (and inefficient) violent effort undertaken into the contests. In conclusion, the dynamics of institutions are simply characterized: for any given q , there exists a $\tilde{\beta}(q)$ such that i) if $\beta_0 > \tilde{\beta}(q)$, then $\beta_{t+1} = \beta_t = \beta_0$; ii) if instead $\beta_0 < \tilde{\beta}(q)$, then β_t converges towards $\beta = 0$.

Cultural dynamics. Conflict-prone agents have positive incentives $\Delta V^1(\beta, q)$ to transmit their trait, but such incentives decrease with their fraction q in society. The incentives for non conflict-prone agents, $\Delta V^2(\beta, q)$, are also positive. They are increasing with the fraction q of conflict prone agents in the population. A larger value of q in fact has two opposing effects on the incentive to socialization of non conflict-prone individuals: it reduces their expected payoff when matched with conflict-prone agents, thereby reducing their incentives to transmit their own trait; but at the same time, a larger q also increases the cost of effort for non conflict-prone agents whose children turn out to be conflict-prone and undertake effort a^{11} when facing other conflict-prone agents in a contest. It turns out that this last effect dominates. As a consequence, the cultural dynamics has a unique interior stationary state and furthermore $q(\beta) < 1/2$.

Joint dynamics of culture and institutions. Consider the case in which $\alpha = \frac{c_2}{c_1} - 1$ is large enough,

so that conflict-prone agents have a significant advantage in conflict.⁵⁸ In this case, when initial conditions (β_0, q_0) are in the highlighted region in the figure and the conflict-prone group is powerful but relatively small, there are no institutional dynamics; see Figure 10.

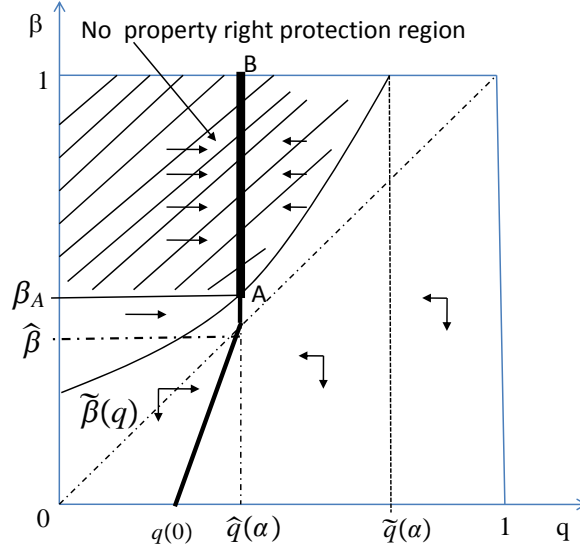


Figure 10: Property rights and conflicts
Joint dynamics (α large enough)

On the other hand, outside of this region of initial conditions the institutional dynamics evolve towards increasing the political power of the non conflict-prone group, inducing more property rights protection. Indeed the system converges towards an institutional set-up giving no power to the conflict-prone group ($\beta = 0$) and to the distribution of the population $q(0) < \hat{q}(\alpha)$.

Interestingly, when the conflict-prone is initially very powerful, $\beta_0 \leq \beta_A$, even a small fraction of conflict-prone individuals in society can be ultimately self-defeating in terms of institutional dynamics. While the system does not exhibit any institutional change, in this case, the underlying cultural dynamics tend to favor the socialization of the conflict-prone agents towards $\hat{q}(\alpha)$. As soon as q_t passes the threshold of $\tilde{\beta}^{-1}(\beta_0)$, endogenous institutional dynamics are triggered which induce the implementation of more extensive property rights and institutions biased towards the

⁵⁸When instead $\alpha = \frac{c_2}{c_1} - 1$ is not too large, the marginal effort costs c_i are similar across groups and the dynamics are not very interesting. In this case, in fact, property rights are protected for any initial conditions. The joint dynamics of culture and institutions converge to a stationary state characterized by institutions giving all power to the non conflict-prone group and hence a maximal protection of property rights.

non conflict-prone group. As a consequence, q_t regresses towards the long run steady state $q(0)$ and the conflict-prone group ends-up with no power ($\beta = 0$).

Similar non-monotonic dynamics of culture and institutions in this society may manifest themselves also as interesting forms of hysteresis as a consequence of an exogenous institutional shock, whereby temporary shock may trigger persistent (indeed, long-run) effects on the dynamics of culture and institutions. A shock that e.g., gives more formal power to the non conflict-prone group might irreversibly take the system into a new long run trajectory of the institutional and cultural dynamics. Suppose for instance that the society has settled to a point like point A in Figure 10 with no property rights and $\hat{q}(\alpha)$. Then a reduction of β below β_A , would lead an endogenous institutional response towards further power to the non conflict-prone group. This in turn would trigger reinforcing cultural dynamics favoring this group. An successive opposite institutional shock of similar amplitude would then not bring back the system towards to region with no property right protection. Indeed even when/if the conflict prone group regains some formal power, the cultural dynamics might have irreversibly driven the system to a region where property rights are protected and there are less individual conflicts for the contest of resources:

6 Conclusions

Motivated by the recent literature on the causal explanations of economic growth and prosperity, this paper proposes a simple theoretical perspective to analyze how culture and institutions evolve, interact, and jointly determine socio-economic outcomes. Institutional change obtains as a process through which existing social and political power structures are designed to resolve fundamental socio-economic externalities. Cultural dynamics are instead driven by cultural transmission processes at the population level.

Our approach allows a simple and easily applicable description of the joint interactions between culture and institutions. Particularly, we provide conditions under which cultural and institutional dynamics tend to strengthen each other in a complementary way, or on the contrary, tend to mitigate each other as substitutes in terms of their effects on socio-economic aggregate variables. Depending on whether culture and institutions are complements or substitutes, exogenous historical accidents propagating over the joint dynamics induced by institutions and culture may have magnified or mitigated effects on long run socioeconomic outcomes. Importantly, our discussion indicates the extent of the comparative dynamic bias that can be generated by neglecting one of the two dynamics, when the other one is affected by an exogenous shock (the *cultural and institutional multipliers*).

Conceptually, our framework also suggests that in general the joint evolution of culture and institutions is highly non-linear. This feature has a number of implications, including the non ergodic character of the underlying dynamic processes between culture and institutions and com-

plex phenomena like hysteresis and oscillations. In other words, the dynamics of culture and institutions are prone to display, for instance, sensitivity of equilibrium trajectories to initial conditions, existence of irreversibility and thresholds effects, and non-monotonicity of cultural and institutional changes over transition paths. From an empirical point of view, these phenomena appear consistent with the great diversity of development experiences encountered across the world. As well, they suggest that even a careful causal analysis (e.g., by means instrumental variables) may be at a disadvantage over more structural analyses of the data which consider explicitly the joint evolution of culture and institutions.⁵⁹

Overall, our analysis underlines the fact that the search for a deep and unique origin for the long-term development can be quite an arduous and even sterile undertaking. Focusing more systematically on the positive or negative interactions between culture and institutions along the development process might result more fruitful in terms of historical understanding as well as in terms of policy implications.

⁵⁹An interesting recent example of such an approach explicitly testing for the joint evolution of culture and institutions is Murrell and Schmidt, (2011), in the context of the cultural and institutional changes that happened in seventeenth century England. They construct institutional measures of evolution of formal legal institutions using citations of cases and statutes appearing in later legal decisions. They also measure the spread of a "whig" political culture which emphasized the virtues of freedom and the necessity of constraints on the monarchy, using the frequency of use of the word "freedom" in contemporary publications, and test for the possibility of cointegrating relationships between culture and institutions. They interpret their results using a model of social learning that explicitly describes the relationship between cultural diffusion and word frequency.

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Appendix A: Results on the dynamical system (5,8)

In this Appendix we study in some detail the dynamics of our economy. We study then the dynamics of $(\beta_t, q_t) \in [0, 1]^2$. The fundamental dynamics equation, as reported in the text as equations (5,8), are conveniently re-written to define the maps $f : [0, 1]^2 \rightarrow [0, 1]$ and $g : [0, 1]^2 \rightarrow [0, 1]$ as follows: the following:

$$\beta_{t+1} = f(\beta_t, q_t) := \begin{cases} \beta \text{ such that } p^{com}(\beta_t, q_{t+1}) = p(\beta, q_{t+1}) & \text{if it exists,} \\ \begin{cases} 1 & \text{if } P(\beta_t, q_{t+1}) > 0, \forall 0 \leq \beta \leq 1 \\ 0 & \text{if } P(\beta_t, q_{t+1}) < 0 \forall 0 \leq \beta \leq 1 \end{cases} & \text{else} \end{cases}$$

$$q_{t+1} - q_t = g(\beta_t, q_t) := q_t(1 - q_t)D(\beta_{t+1}, q_{t+1})$$

We impose Assumptions 1-3 and we further assume for regularity that all maps, $P(\beta, q)$, $D(\beta, q)$ are smooth.

We shall study the dynamical system in the continuous time limit, where the change in β_t and q_t between time t and $t + dt$ are, respectively, λdt and μdt , for $dt \rightarrow 0$.⁶⁰

$$\begin{aligned} \dot{\beta} &= \lambda [f(\beta, q) - \beta] \\ \dot{q} &= \mu g(\beta, q) \end{aligned}$$

given the initial conditions (β_0, q_0) .

The dynamics of β given q .

Lemma A. 1 *Under Assumptions 1-3, $f : [0, 1]^2 \rightarrow [0, 1]$ is a continuous function in $(\beta, q) \in [0, 1]^2$.*

Proof. First of all note that when $p(\beta_{t+1}, q_{t+1}) = p^{com}(\beta_t, q_t)$ is not satisfied for any β_{t+1} , for some (q_t, q_{t+1}) , the assumption that $p(\beta, q)$ is monotonic implies that β_{t+1} is $= 0$ or $= 1$, depending on the sign of $p^{com}(\beta_{t+1}, q_{t+1}) - p(\beta_t, q_t)$. In the continuous time limit $q_{t+1} = q_t = q$ and hence, in this case, trivially, f maps continuously $(\beta, q) \in [0, 1]^2$ into $\{0\}$.

Consider equation (5), again. We show that β_{t+1} is a continuous function of β_t, q_t, q_{t+1} when $p(\beta_{t+1}, q_{t+1}) = p^{com}(\beta_t, q_t)$ is satisfied. To this end note that the assumed monotonicity in β of $p(\beta, q)$ implies that, when $p(\beta_{t+1}, q_{t+1}) = p^{com}(\beta_t, q_t)$ is satisfied, we can write $\beta_{t+1} = p^{-1}(p(\beta_t, q_t), q_{t+1})$ and hence $\beta_{t+1} = p^{-1}(p^{com}(\beta_t, q_t), q_t, q_{t+1})$, a continuous function. Again, in the continuous time limit $q_{t+1} = q_t$ and hence we can construct a continuous function $f : [0, 1]^2 \rightarrow \mathbb{R}$ such that $\dot{\beta}_t = f(\beta_t, q_t)$.

⁶⁰As is well known, discrete time dynamics may generate complex dynamic behaviors that are difficult to characterize and go beyond the points we want to emphasize about the co-evolution between culture and institutions.

Finally, it is straightforward to see that as $p^{com}(\beta_{t+1}, q_{t+1}) - p(\beta_t, q_t)$ crosses 0 $\beta_{t+1} = p^{-1}(p^{com}(\beta_t, q_t), q_t, q_{t+1})$ converges continuously to 0 or 1 depending on the direction of the crossing so as to preserve continuity. ■

Let the $\beta : [0, 1] \rightarrow [0, 1]$ map $q \in [0, 1]$ into the stationary states of f ; that is, $\beta : [0, 1] \rightarrow [0, 1]$ satisfies

$$0 = f(\beta, q), \text{ for any } \beta \in \beta(q)$$

Lemma A. 2 *Under Assumptions 1-3, the map $\beta : [0, 1] \rightarrow [0, 1]$ is an non empty and compact valued upper-hemi-continuous correspondence with connected components.*

Proof. The proof is a direct consequence of the continuity of f proved in Lemma A.1. ■

Let $P(\beta, q) := p^{com}(\beta, q) - p(\beta, q)$. We consider only the regular case in which $P(\beta, q) \neq 0$ at the vertices of $[0, 1]^2$, leaving the simple but tedious analysis of the singular cases to the reader. Also, we say that q is a regular point of $\beta \in \beta(q)$ if any stationary state $\beta \in \beta(q)$ satisfies that property that $\frac{\partial P(\beta, q)}{\partial \beta} \neq 0$; that is if $p(\beta, q)$ and $p^{com}(\beta, q)$ intersect transversally. The characterization of $\beta : [0, 1] \rightarrow [0, 1]$ depends crucially on the topological properties of the zeros of $P(\beta, q)$. Let $\pi : [0, 1] \rightarrow [0, 1]$ map q into the stationary states β such that $P(\beta, q) = 0$; that is, the map π satisfies $P(\pi(q), q) = 0$.

Proposition A. 1 *Under Assumptions 1-3, the dynamics of β as a function of $q \in [0, 1]$ has the following properties,*

1. $P(0, q) > 0, P(1, q) < 0$, for any $q \in [0, 1]$, and $p(\beta, q)$ is increasing; or $P(0, q) < 0, P(1, q) > 0$, for any $q \in [0, 1]$, and $p(\beta, q)$ is decreasing. For any given regular $q \in [0, 1]$ there exist an odd number of regular stationary states $\beta \in \pi(q)$; furthermore $\beta = 0, 1$ are also stationary states for given $q \in [0, 1]$. The stability properties of the regular stationary states alternate with the smallest and the larger being always locally stable; the boundaries $\beta = 0, 1$ are locally unstable for all $q \in [0, 1]$.
2. $P(0, q) < 0, P(1, q) > 0$, for any $q \in [0, 1]$, and $p(\beta, q)$ is increasing; or $P(0, q) > 0, P(1, q) < 0$, for any $q \in [0, 1]$, and $p(\beta, q)$ is decreasing. For any given $q \in [0, 1]$ there exist an odd number of regular stationary states $\beta \in \pi(q)$; furthermore $\beta = 0, 1$ are also stationary states for given $q \in [0, 1]$. The stability properties of the regular stationary states alternate with the smallest and the larger being always locally unstable; the boundaries $\beta = 0, 1$ are locally stable.
3. $P(0, q) < 0, P(1, q) < 0$, for any $q \in [0, 1]$. For any given $q \in [0, 1]$ there exist either none or an even number of regular stationary states $\beta \in \pi(q)$; furthermore $\beta = 0$ is also a

stationary state for given $q \in [0, 1]$. The stability properties of the regular stationary states alternate with the smallest always locally unstable; the boundary $\beta = 0$ is locally stable.

4. $P(0, q) > 0$, $P(1, q) > 0$, for any $q \in [0, 1]$. For any given $q \in [0, 1]$ there exist either none or an even number of regular stationary states $\beta \in \pi(q)$; furthermore $\beta = 1$ is also a stationary state for given $q \in [0, 1]$. The stability properties of the regular stationary states alternate with the smallest always locally stable; the boundary $\beta = 1$ is locally stable.
5. $P(0, q)$ and/or $P(1, q)$ change sign with $q \in [0, 1]$. The characterization obtained above then can be repeated for each sub-interval of $[0, 1]$ in which the Brouwer degree of the manifold $\pi(q)$ is invariant (see the proof). We leave the tedious categorization of all possible cases to the reader.

The dynamics of q given β .

Lemma A. 3 Under Assumptions 1-3, $g : [0, 1]^2 \rightarrow [0, 1]$ is a continuous function in $(\beta, q) \in [0, 1]^2$.

Proof. The proof is an immediate consequence of the continuity of $D(\beta_t, q_t)$, imposed in Assumption 8. ■

Let the $q : [0, 1] \rightarrow [0, 1]$ map $\beta \in [0, 1]$ into the stationary states of g ; that is, $q : [0, 1] \rightarrow [0, 1]$ satisfies

$$0 = g(\beta, q), \text{ for any } q \in q(\beta)$$

Lemma A. 4 Under Assumptions 1-3, the map $q : [0, 1] \rightarrow [0, 1]$ is a non empty and compact valued upper-hemi-continuous correspondence with connected components. It contains $q(\beta) = 0$ and $q(\beta) = 1$, for any $0 \leq \beta \leq 1$.

Proof. The proof is a direct consequence of the continuity of g , proved in Lemma A.1, and of the fact that $g(0, \beta) = g(1, \beta) = 1$, for any $0 \leq \beta \leq 1$. ■

As for the dynamics of culture given institutions, consider the regular case in which $D(\beta, q) \neq 0$ at the vertices of $[0, 1]^2$, leaving the simple but tedious analysis of the singular cases to the reader. We say that β is a regular point of $q \in q(\beta)$ if any stationary state $q \in q(\beta)$ satisfies that property that $\frac{\partial D(\beta, q)}{\partial q} \neq 0$; that is if $d^i(\beta, q)$ and $d^j(1 - \beta, 1 - q)$ intersect transversally. The characterization of $q : [0, 1] \rightarrow [0, 1]$ depends crucially on the topological properties of the zeros of $D(\beta, q)$. Let $\sigma : [0, 1] \rightarrow [0, 1]$ map β into the stationary states q such that $D(\beta, q) = 0$; that is, the map σ satisfies $D(\sigma(\beta), \beta) = 0$.

Proposition A. 2 Under Assumptions 1-3, the dynamics of q as a function of $\beta \in [0, 1]$ has the following properties,

1. $D(\beta, 0) < 0, D(\beta, 1) > 0$, for any $\beta \in [0, 1]$. For any given regular $\beta \in [0, 1]$ there exist an odd number of regular stationary states $\beta \in \sigma(\beta)$. By Lemma 4, $q = 0, 1$ are also stationary states for given $\beta \in [0, 1]$. The stability properties of the regular stationary states alternate starting with the $q = 0$ being stable and ending with $q = 1$ also being stable. If the dynamics supports a unique interior stationary state q^* , then it is unstable.
2. $D(\beta, 0) > 0, D(\beta, 1) < 0$, for any $q \in [0, 1]$. For any given regular $\beta \in [0, 1]$ there exist an odd number of regular stationary states $\beta \in \sigma(\beta)$. By Lemma 4, $q = 0, 1$ are also stationary states for given $\beta \in [0, 1]$. The stability properties of the regular stationary states alternate starting with the $q = 0$ being unstable and ending with $q = 1$ also being unstable. If the dynamics supports a unique interior stationary state q^* , then it is stable.
3. $D(\beta, 0) < 0, D(\beta, 1) < 0$, for any $\beta \in [0, 1]$. For any given regular $\beta \in [0, 1]$ there exist either none or an even number of regular stationary states $\beta \in \sigma(\beta)$. By Lemma 4, $q = 0, 1$ are also stationary states for given $\beta \in [0, 1]$. The stability properties of the regular stationary states alternate starting with the $q = 0$ being stable and ending with $q = 1$ being unstable.
4. $D(\beta, 0) > 0, D(\beta, 1) > 0$, for any $\beta \in [0, 1]$. For any given regular $\beta \in [0, 1]$ there exist either none or an even number of regular stationary states $\beta \in \sigma(\beta)$. By Lemma 4, $q = 0, 1$ are also stationary states for given $\beta \in [0, 1]$. The stability properties of the regular stationary states alternate starting with the $q = 0$ being unstable and ending with $q = 1$ being stable.
5. $D(\beta, 0)$ and/or $D(\beta, 1)$ change sign with $\beta \in [0, 1]$. The characterization obtained above then can be repeated for each sub-interval of $[0, 1]$ in which the Brouwer degree of the manifold $\sigma(q)$ is invariant (see the proof). We leave the tedious categorization of all possible cases to the reader.

Proof. Under Assumptions 1-3, $D(\beta, q)$ is smooth and (β, q) lie in the compact set $[0, 1]^2$. $\sigma(q)$ is a dimension-1 smooth manifold with boundary, by a general version Implicit Function Theorem; see e.g. Milnor (1965), lemma 4, p. 13. The statement is then proved, closely along the lines of the proof of proposition A.1, using the full characterization of dimension-1 manifolds and Brouwer degree theory, thinking of $D(\beta, q)$ as a homothopy function varying β . We leave the details to the reader. ■

The joint dynamics of (β, q) .

The dynamical system (5,8), even under Assumptions 1-3, is impossible to study in general. We can however show that at least one stationary state always exists and characterize sufficient

conditions for the existence of an interior stationary state. To this end we re-state here more formally Proposition 5 in the text.

Proposition A. 3 *Under Assumptions 1-3 the dynamical system (5,8) has at least one stationary state. Furthermore, if the Brouwer degree of both $\pi(q)$ and $\sigma(\beta)$ is ± 1 , the dynamical system has at least one interior stationary state.*

Proof. The proof of the existence of a stationary state is a direct consequence of the characterization of $\beta(q)$ and $q(\beta)$ in Lemmata A.2, A.4.

The proof of the existence of an interior stationary state under the Brouwer degree conditions is a consequence of the *Jordan curve theorem*, which we state in the following for completeness:⁶¹

A curve J in \mathbb{R}^2 which is the image of an injective continuous map of a circle into \mathbb{R}^2 has two components (an "inside" and "outside"), with J the boundary of each.

Figure A.1 represents a *Jordan curve J* on the plane.

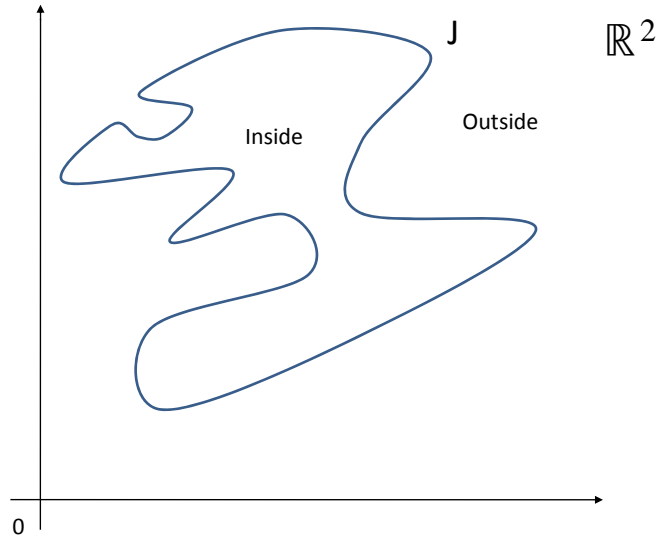


Figure A.1: Jordan Curve J in the (non negative) plane

Consider the compact space $[0, 1]^2 \subset \mathbb{R}^2$, in which (β, q) lay. By Lemma 4, the map $q(\beta)$ contains the boundaries $q = 0, 1$ as well as the map $\sigma(\beta)$ which, in the case its Brouwer degree is ± 1 , is homeomorphic to the compact interval $[0, 1]$. The map $\pi(q)$ is also homeomorphic to the compact interval $[0, 1]$ in the case its Brouwer degree is ± 1 .

⁶¹The theorem is a standard result in algebraic topology; see Hatcher (2002) p. 169 for a proof.

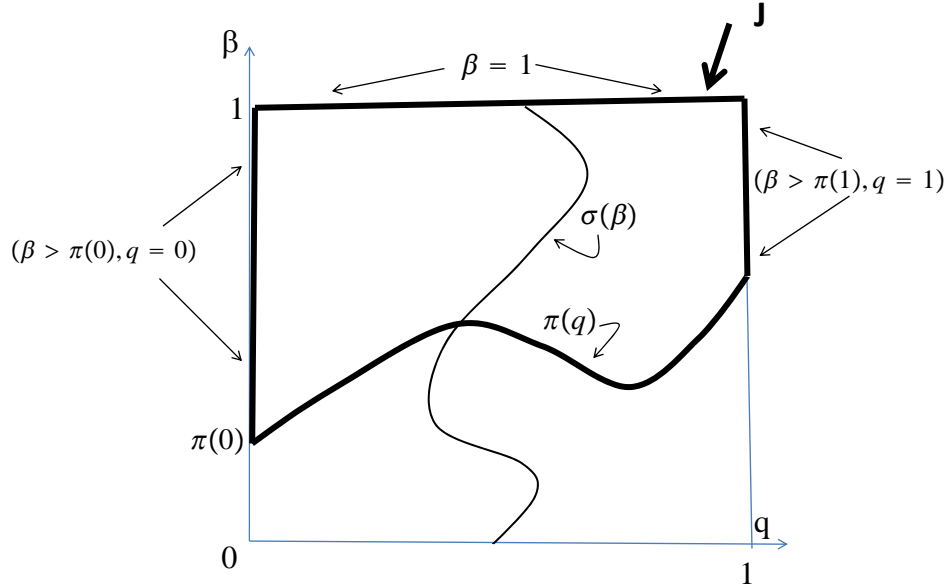


Figure A.2: Jordan Curve J in $[0,1]^2$ as constructed in the proof

We can therefore construct a Jordan curve J composed of $\pi(q)$, $(\beta > \pi(0), q = 0)$, $(\beta > \pi(1), q = 1)$, $\beta = 1$. Since $\sigma(\beta)$ connects the $\beta = 1$ and $\beta = 0$ its has a component inside and one outside the curve J . Furthermore, $0 < \sigma(\beta) < 1$, by construction. The *Jordan curve theorem* then guarantees that $\pi(q)$ and $\sigma(\beta)$ cross in the interior of $[0, 1]^2$; see Figure A.2 for a graphical representation of the construction. ■

Note that Proposition A.1 and A.2 provide conditions, respectively on $P(\beta, q)$ and $D(\beta, q)$, guaranteeing that the Brouwer degree of $\pi(q)$ and $\sigma(\beta)$ is ± 1 . Also, the analysis leading to Proposition A.3 can be extended to dynamical systems in which the Brouwer degrees of $\pi(q)$ and $\sigma(\beta)$ are not invariant.

The following study of the dynamics of the system is obtained, for simplicity, under Assumptions (4-5). Assumption 4 implies that $q(\beta) = \hat{q}(p)$ with $p = p(\beta, q)$, with some notational abuse, where $\hat{q}(p) \in [0, 1]$ is the unique solution of the following equation

$$\frac{\Delta V^1(p)}{\Delta V^2(p)} = \frac{q}{1-q}.$$

Assumption 5 implies that the policy instrument p affects the optimal private actions, a^i , independently of the economy-level aggregates a and q . This in turn implies that cultural intolerences ΔV^i depend only on the equilibrium policy level p . As usual, we denote the partial derivative of a variable x on another variable y as $\partial x / \partial y = x_y$.

The linearized local dynamics around the interior steady state (β^*, q^*) can then easily be obtained by

$$\begin{pmatrix} \dot{\beta} \\ \dot{q} \end{pmatrix} = \begin{pmatrix} \lambda \left[\frac{p_\beta^{com} - p_\beta}{p_\beta} \right]_{(\beta^*, q^*)} & \lambda \left[\frac{p_q^{com} - p_q}{p_\beta} \right]_{(\beta^*, q^*)} \\ -\mu G q^* (1 - q^*) \cdot \hat{q}_p \cdot p_\beta & \mu G q^* (1 - q^*) [1 - \hat{q}_p \cdot p_q] \end{pmatrix} \begin{pmatrix} \beta \\ q \end{pmatrix} \quad (11)$$

where $G = -(\Delta V^1(p(\beta^*, q^*)) + \Delta V^2(p(\beta^*, q^*))) < 0$.

The local stability of the interior steady state (β^*, q^*) of (??) is obtained under the standard Hessian conditions:

$$\begin{aligned} \left[\frac{p_\beta - p_\beta^{com}}{p_\beta} \right]_{(\beta^*, q^*)} &> 0 \\ 1 - [p_q \cdot \hat{q}_p]_{(\beta^*, q^*)} &> 0 \\ \left[(1 - p_q \cdot \hat{q}_p) \cdot \left[\frac{p_\beta - p_\beta^{com}}{p_\beta} \right] + \hat{q}_p \cdot (p_q - p_q^{com}) \right]_{(\beta^*, q^*)} &> 0 \end{aligned} \quad (12)$$

Dynamic Complementarity and Substitution between institutions and culture

Lemma A. 5 *Under Assumption 5, institutional and cultural dynamics are complementary at a locally stable interior steady state (β^*, q^*) if*

$$\frac{dP(\beta^*, q^*)}{dq} \text{ has the same sign as } \left[\frac{d \left(\frac{\Delta V^1(p)}{\Delta V^2(p)} \right)}{dp} \right]_{p(\beta^*, q^*)}; \quad (13)$$

they are instead substitute if the signs are opposite.

Proof. Institutional and cultural dynamics are complementary at (β^*, q^*) when

$$\frac{d\beta(q)}{dq} \text{ and } \frac{dq(\beta)}{d\beta} \text{ have the same sign.} \quad (14)$$

Differentiating,

$$\frac{d\beta(q)}{dq} = -\frac{(p_q - p_q^{com})}{p_\beta - p_\beta^{com}}, \quad \frac{dq(\beta)}{d\beta} = \frac{\hat{q}_p p_\beta}{1 - p_q \cdot \hat{q}_p}.$$

Thus, condition (14) is equivalent to

$$\frac{d\beta(q)}{dq} \cdot \frac{dq(\beta)}{d\beta} \geq 0$$

or

$$-\frac{(p_q - p_q^{com})}{p_\beta - p_\beta^{com}} \cdot \frac{\hat{q}_p p_\beta}{1 - p_q \cdot \hat{q}_p} \geq 0;$$

Given the Hessian conditions for local stability, (12), at an interior locally stable steady state (β^*, q^*) , this condition is equivalent to

$$[(p_q^{com} - p_q) \cdot \hat{q}_p]_{(\beta^*, q^*)} \geq 0.$$

Recalling that the cultural manifold $q(\beta)$ is obtained from

$$\frac{\Delta V^1(p)}{\Delta V^2(p)} = \frac{q}{1-q} \text{ and } p = p(\beta, q)$$

and that $P(\beta, q) := p^{com}(\beta, q) - p(\beta, q)$, differentiating,

$$[(p_q^{com} - p_q) \cdot \hat{q}_p]_{(\beta^*, q^*)} = \left[P_q(\beta, q) \cdot \left[\frac{d\left(\frac{\Delta V^1(p)}{\Delta V^2(p)}\right)}{dp} \right]_{p(\beta, q)} (1-q)^2 \right]_{(\beta^*, q^*)}.$$

Therefore, institutional and cultural *dynamics are complementary* at a locally stable interior steady state (β^*, q^*) when P_q and $\frac{d\left(\frac{\Delta V^1(p)}{\Delta V^2(p)}\right)}{dp}$ have the same sign at (β^*, q^*) . Obviously they are dynamic substitute otherwise. ■

Oscillations and Cycles

Proposition 6. *Proof.* Consider first an interior steady state (β^*, q^*) of (11) that is locally stable. The local stability conditions ensure that the trace $T < 0$ and that the determinant $\Delta > 0$. Dampened oscillations (a stable spiral steady state equilibrium) require $T^2 < 4\Delta$. With respect to dynamical system (5,8), this last condition is translated into:

$$\left[\lambda \frac{p_\beta^{com} - p_\beta}{p_\beta} + \mu G^* q^* (1 - q^*) [1 - \hat{q}_p \cdot p_q] \right]^2 < 4\lambda \mu G^* q^* (1 - q^*) \left[\frac{p_\beta^{com} - p_\beta}{p_\beta} [1 - \hat{q}_p \cdot p_q] + \hat{q}_p \cdot (p_q^{com} - p_q) \right];$$

or, after manipulations,

$$\left[\lambda \frac{p_\beta^{com} - p_\beta}{p_\beta} - \mu G^* q^* (1 - q^*) [1 - \hat{q}_p \cdot p_q] \right]^2 < 4\lambda \mu G^* q^* (1 - q^*) \hat{q}_p \cdot (p_q^{com} - p_q),$$

with $G^* = -(\Delta V^1(p^*) + \Delta V^2(p^*)) < 0$. Using (??), $G^* q^* (1 - q^*) = -\frac{\Delta V^1(p^*) \Delta V^2(p^*)}{\Delta V^1(p^*) + \Delta V^2(p^*)}$ and hence the condition for dampened oscillations becomes:

$$\left[\lambda \frac{p_\beta^{com} - p_\beta}{p_\beta} + \mu \frac{\Delta V^1(p^*) \Delta V^2(p^*)}{\Delta V^1(p^*) + \Delta V^2(p^*)} [1 - \hat{q}_p \cdot p_q] \right]^2 < -4\lambda \mu \frac{\Delta V^1(p^*) \Delta V^2(p^*)}{\Delta V^1(p^*) + \Delta V^2(p^*)} \cdot \hat{q}_p \cdot (p_q^{com} - p_q). \quad (15)$$

The left hand side of this inequality is positive. It follows then that there are no dampening oscillations in cultural and institutional change when institutions and culture are dynamic complements; i.e., when $[\widehat{q}_p \cdot (p_q^{com} - p_q)]_{(\beta^*, q^*)} > 0$ at the interior locally stable steady state (β^*, q^*) .

Conversely we have dampening oscillations when (15) is satisfied. If culture and institutions are substitutes, $|\widehat{q}_p \cdot (p_q^{com} - p_q)| = -\widehat{q}_p \cdot (p_q^{com} - p_q)$. In this case, non-monotonic dynamics in culture and institutions obtain when

$$\frac{\lambda}{\mu} \left[\frac{p_\beta^{com} - p_\beta}{p_\beta} + \frac{\mu}{\lambda} \frac{\Delta V^{1*} \Delta V^{2*}}{\Delta V^{1*} + \Delta V^{2*}} [1 - \widehat{q}_p \cdot p_q] \right]^2 < 4 \frac{\Delta V^{1*} \Delta V^{2*}}{\Delta V^{1*} + \Delta V^{2*}} |\widehat{q}_p \cdot (p_q^{com} - p_q)|. \quad (16)$$

Using the local stability conditions for the Hessian at (β^*, q^*) ,

$$\begin{aligned} \frac{p_\beta^{com} - p_\beta}{p_\beta} &= -a < 0 \\ \frac{\Delta V^{1*} \Delta V^{2*}}{\Delta V^{1*} + \Delta V^{2*}} [1 - \widehat{q}_p \cdot p_q] &= b > 0 \\ 4 \frac{\Delta V^{1*} \Delta V^{2*}}{\Delta V^{1*} + \Delta V^{2*}} |\widehat{q}_p \cdot (p_q^{com} - p_q)| &= M > 0. \end{aligned}$$

Denoting $x = \mu/\lambda$, the relative rate of change between culture and institutions, condition (16), can be written as

$$(-a + bx)^2 < Mx. \quad (17)$$

Simple examination of this condition reveals that (17) is satisfied when $x \in (x_- ; x_+)$, with

$$x_\pm = \frac{(2ab + M) \pm \sqrt{(2ab + M)^2 - 4(ab)^2}}{2b^2} > 0.$$

As a consequence, non-monotonic dynamics of institutions and culture around the locally stable steady state (β^*, q^*) are obtained when institutions and culture are dynamic substitutes and the relative rate of change between culture and institutions is neither too low, neither too high. ■

Proposition 7. *Proof.* Assume that the steady state institutions, β^* , are locally stable when culture remains constant at $q = q^*$ along the institutional dynamics; and, conversely, assume that the cultural steady state $q = q^*$ is locally stable when the institutional context remains constant along the cultural dynamics. Formally,

$$\begin{aligned} \left[\frac{p_\beta^{com} - p_\beta}{p_\beta} \right]_{(\beta^*, q^*)} &< 0 \\ 1 - [p_q \cdot \widehat{q}_p]_{(\beta^*, q^*)} &> 0 \end{aligned} \quad (18)$$

Suppose that conditions (18) are satisfied at an interior steady state (β^*, q^*) . Then with enough regularity of the policy functions p^{com} and p , there exists a connected neighborhood of

(β^*, q^*) such that the trace $T = \lambda \left[\frac{p_\beta^{com} - p_\beta}{p_\beta} \right] - \mu G q^* (1 - q^*) [1 - \widehat{q}_p \cdot p_q]$ does not change sign on that domain. The Bendixson Negative Criterion precludes then, in this case, the existence of local periodic orbits or limit cycles around (β^*, q^*) in that domain.

Note that when (18) are globally satisfied for all $(\beta, q) \in [0, 1] \times [0, 1]$, it is not possible to get globally periodic orbits and limit cycles for dynamical system (5,8). Indeed given that in the simple connected domain $D = [0, 1] \times [0, 1]$, the sign of the trace $T = \lambda \left[\frac{p_\beta^{com} - p_\beta}{p_\beta} \right] + \mu G q^* (1 - q^*) [1 - \widehat{q}_p \cdot p_q]$ is always strictly negative, the Bendixson Negative Criterion again precludes the existence of periodic orbits of (11) in this domain. ■

Cultural Multiplier

Proposition 8. *Proof.* Recall the required normalizations:

$$p_\beta = \frac{\partial p(\beta^*, q^*, \gamma)}{\partial \beta} > 0, \quad p_\gamma^{com} - p_\gamma = \frac{\partial P(\beta^*, q^*, \gamma)}{\partial \gamma} > 0 \quad (19)$$

The comparative statics on (β^*, q^*) on the parameter are then easily obtained by differentiation of (??). After tedious computations,

$$\begin{aligned} \frac{d\beta^*}{d\gamma} &= \frac{(p_\gamma^{com} - p_\gamma) + (p_q^{com} - p_q) \widehat{q}_p \frac{p_\gamma}{1 - p_q \widehat{q}_p}}{(p_\beta - p_\beta^{com}) - (p_q^{com} - p_q) \widehat{q}_p \frac{p_\beta}{1 - p_q \widehat{q}_p}} \\ \frac{dq^*}{d\gamma} &= \frac{\widehat{q}_p p_\beta}{1 - p_q \widehat{q}_p} \cdot \frac{(p_\gamma^{com} - p_\gamma) + (p_q^{com} - p_q) \widehat{q}_p \frac{p_\gamma}{1 - p_q \widehat{q}_p}}{(p_\beta - p_\beta^{com}) - (p_q^{com} - p_q) \widehat{q}_p \frac{p_\beta}{1 - p_q \widehat{q}_p}}. \end{aligned}$$

Consider now the impact of a change in γ on institutional change, fixing q to its pre-shock value. Differentiating the first equation in (??),

$$\left(\frac{d\beta^*}{d\gamma} \right)_{q=q^*} = \frac{(p_\gamma^{com} - p_\gamma)}{(p_\beta - p_\beta^{com})} > 0.$$

The last inequality is a consequence of stability condition (12) $(p_\beta - p_\beta^{com})/p_\beta > 0$ coupled with condition (19). moreover,

$$\frac{d\beta^*}{d\gamma} = \left(\frac{d\beta^*}{d\gamma} \right)_{q=q^*} + \frac{(p_q^{com} - p_q) \widehat{q}_p}{(p_\beta - p_\beta^{com})} \frac{\frac{(p_\gamma^{com} - p_\gamma)}{(p_\beta - p_\beta^{com})} p_\beta + p_\gamma}{1 - p_q \widehat{q}_p} \frac{1 - \frac{(p_q^{com} - p_q)}{(p_\beta - p_\beta^{com})} \widehat{q}_p \frac{p_\beta}{1 - p_q \widehat{q}_p}}{1 - p_q \widehat{q}_p}.$$

Hence, the *Cultural multiplier* on institutional change μ , at (β^*, q^*) , $m = \left(\frac{d\beta^*}{d\gamma} \right) / \left(\frac{d\beta^*}{d\gamma} \right)_{q=q^*} - 1$, is positive if and only if,

$$\frac{(p_q^{com} - p_q) \widehat{q}_p}{(p_\beta - p_\beta^{com})} \frac{\frac{(p_\gamma^{com} - p_\gamma)}{(p_\beta - p_\beta^{com})} p_\beta + p_\gamma}{1 - p_q \widehat{q}_p} \frac{1 - \frac{(p_q^{com} - p_q)}{(p_\beta - p_\beta^{com})} \widehat{q}_p \frac{p_\beta}{1 - p_q \widehat{q}_p}}{1 - p_q \widehat{q}_p} > 0.$$

The stability condition, coupled with (19), implies that $(p_\beta - p_\beta^{com}) > 0$, $1 - p_q \widehat{q}_p > 0$, $1 - \frac{(p_q^{com} - p_q)}{(p_\beta - p_\beta^{com})} \widehat{q}_p \frac{p_\beta}{1 - p_q \widehat{q}_p} > 0$, and $\frac{(p_\gamma^{com} - p_\gamma)}{(p_\beta - p_\beta^{com})} p_\beta > 0$. When $p_\gamma > 0$, it follows that $\frac{(p_\gamma^{com} - p_\gamma)}{(p_\beta - p_\beta^{com})} p_\beta + p_\gamma > 0$. Therefore $m > 0$ if and only if $(p_q^{com} - p_q) \widehat{q}_p > 0$, which is the condition for the dynamics of institutions and culture to be dynamic complementary. ■

Decomposition of the Cultural Multiplier on an aggregate variable $A(p, q, a^1(p), a^2(p))$.

The effects of a shock can be decomposed as follows:

$$\frac{dA}{d\gamma} = \left\{ \underbrace{\left[A_p + (A_{a^1} a_p^1 + A_{a^2} a_p^2) \right] p_\beta}_{\text{direct effect}} + \underbrace{\left[A_q + [A_p + (A_{a^1} a_p^1 + A_{a^2} a_p^2)] p_q \right] \frac{\widehat{q}_p p_\beta}{1 - p_q \widehat{q}_p}}_{\text{indirect effect}} \right\} \frac{d\beta^*}{d\gamma}$$

The effect of γ on institutions will come from a direct effect as well as an indirect one. The direct effect in turn will be composed of two terms: a direct effect of the policy change induced by an institutional change p_β on the aggregate variable A (i.e., the term A_p), and the impact of changes in private actions $a^1(p)$ and $a^2(p)$ as induced also by the policy change p_β , the term $(A_{a^1} a_p^1 + A_{a^2} a_p^2) p_\beta$. The indirect effect of cultural evolution will come from the compositional effect of changing the cultural group sizes (A_q), plus again the change in policy and private actions $[A_p + (A_{a^1} a_p^1 + A_{a^2} a_p^2)] p_q$ which such a cultural compositional change induces.

Furthermore,

$$\left(\frac{dA}{d\gamma} \right)_{q=q^*} = [A_p + (A_{a^1} a_p^1 + A_{a^2} a_p^2)] p_\beta \cdot \left(\frac{d\beta^*}{d\gamma} \right)_{q=q^*}.$$

Recalling that the cultural multiplier on institutions is $m = \left[\left(\frac{d\beta^*}{d\gamma} \right) / \left(\frac{d\beta^*}{d\gamma} \right)_{q=q^*} - 1 \right]$,

$$m_A = \frac{dA}{d\gamma} / \left(\frac{dA}{d\gamma} \right)_{q=q^*} - 1 = \frac{A_q + [A_p + (A_{a^1} a_p^1 + A_{a^2} a_p^2)] p_q}{[A_p + (A_{a^1} a_p^1 + A_{a^2} a_p^2)] p_\beta} \frac{\widehat{q}_p p_\beta}{1 - p_q \widehat{q}_p} (1 + m);$$

and hence,

$$m_A = \left[\frac{A_q}{[A_p + (A_{a^1} a_p^1 + A_{a^2} a_p^2)]} + p_q \right] \frac{\widehat{q}_p}{1 - p_q \widehat{q}_p} (1 + m). \quad (20)$$

Appendix B: Extensions

We briefly discuss two main extensions to our analysis of the dynamics of culture and institutions in the text. First of all we consider the case in which $p(\beta, q_{t+1})$ is not necessarily monotonic, that is, when Assumption 2 is not satisfied. Second, we consider a general society in which political and cultural group are distinct, as delineated in Section 4 and in some of the example societies studied in Section 5.

Non-monotonic $p(\beta, q_{t+1})$.

Consider the case in which Assumption 2 is not imposed and hence $p(\beta, q_{t+1})$ can be non-monotonic. Then the dynamical system for β^i is characterized by the following implicit difference equation:

$$\beta_{t+1}^i = \begin{cases} \beta \text{ such that } p^{com}(\beta_t^i, q_{t+1}) = p(\beta, q_{t+1}) & \text{if it exists,} \\ \left[\begin{array}{l} \arg \max p(\beta, q_{t+1}) \text{ if } p^{com}(\beta_t^I, q_{t+1}) > p(\beta, q_{t+1}), \forall 0 \leq \beta \leq 1 \\ \arg \min p(\beta, q_{t+1}) \text{ if } p^{com}(\beta_t^I, q_{t+1}) < p(\beta, q_{t+1}), \forall 0 \leq \beta \leq 1 \end{array} \right] & \text{else} \end{cases} \quad (21)$$

In this case it is straightforward to show that the institutional dynamics might be under-determined, that is, equation 21 might define an implicit map $(\beta_t^i, q_t) \rightarrow \beta_{t+1}^i$ which is multi-valued in an open set of the domain. Furthermore, in this case, the dynamics of institutions can easily give rise to limit cycles. Consider for instance the example in Figure 1, with initial condition β_0 , where the path $\beta_1- > \beta_2- > \beta_1$ constitutes such a limit cycle for a particular selection of the solutions to $P(\beta, q) = 0$.

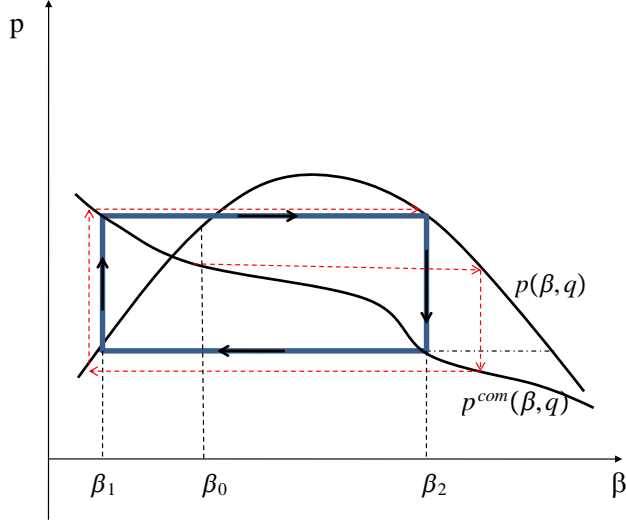


Figure A.3: Limit cycle (β_1, β_2) for $p(\beta, q)$ non-monotonic

Distinct political and cultural groups.

Consider the general society delineated in Section 4, where political and cultural groups are distinct. We briefly indicate here how the concepts, Assumptions, and Propositions in the text are extended to this society.

The *societal equilibrium given institutions β and cultural distribution q* is a tuple $\{a, p\}$ such that:

$$\begin{aligned} p &\in \arg \max_p \sum_i \beta^i \sum_j q^{ij} u^{ij}(a^{ij}, p; a, q) \\ a^{ij} &\in \arg \max u^{ij}(a^{ij}, p; a, q) \quad i \in I, j \in J. \end{aligned} \quad (22)$$

The *societal commitment equilibrium given institutions β and cultural distribution q* is a tuple $\{a^{com}, p^{com}\}$ such that:

$$\begin{aligned} \{a^{com}, p^{com}\} &\in \arg \max \sum_i \beta^i \sum_j q^{ij} u^{ij}(a^{ij}, p; a, q) \\ \text{s.t. } a^{ij} &\in \arg \max u^{ij}(a^{ij}, p; a, q), \quad i \in I, j \in J \end{aligned} \quad (23)$$

Restricting to dichotomous groups, that is $I = \{1, 2\}$ and $J = \{a, b\}$ the *societal equilibrium*, the *societal commitment equilibrium*, and the *societal optimum* can be denoted, respectively:

$$[a(\beta, q), p(\beta, q)]; [a^{com}(\beta, q), p^{com}(\beta, q)]; [a^{eff}(\beta, q), p^{eff}(\beta, q)]$$

Assumption 6 *Utility functions are sufficiently regular so that*

$a(\beta, q)$, $p(\beta, q)$, $a^{com}(\beta, q)$, $p^{com}(\beta, q)$ are continuous functions.

Assumption 7 *Utility functions are sufficiently regular so that $p(\beta, q)$ is monotonic in β .*

Adding an index t to denote time, institutions evolve as a solution to the following design problem:

$$\max_{\beta_{t+1}} \sum_{i \in I} \beta_t^i \sum_{j \in J} q_{t+1}^{ij} u^{ij} (a^{ij}(\beta_{t+1}, q_{t+1}), p(\beta_{t+1}, q_{t+1}); a(\beta_{t+1}, q_{t+1})) \quad (24)$$

Proposition 9 *Under Assumption 6-7, and given (q_t, q_{t+1}) , the dynamics of institutions β_t^i , $i \in I$, is governed by the following implicit difference equation:*

$$\beta_{t+1}^i = \begin{cases} \beta^i \text{ such that } p^{\text{com}}(\beta, q_{t+1}) = p(\beta_t, q_{t+1}) & \text{if it exists,} \\ \begin{cases} 1 & \text{if } p^{\text{com}}(\beta_t, q_{t+1}) > p(\beta, q_{t+1}), \forall 0 \leq \beta^i \leq 1 \\ 0 & \text{if } p^{\text{com}}(\beta_t, q_{t+1}) < p(\beta, q_{t+1}), \forall 0 \leq \beta^i \leq 1 \end{cases} & \text{else} \end{cases} \quad (25)$$

It is convenient to define $P(\beta, q) := p^{\text{com}}(\beta, q) - p(\beta, q)$.

Proposition 10 *Under Assumption 6-7, for any given q , the dynamics of institutions governed by (25) have at least one stationary state. An interior stationary states β^* obtains as a solution to $P(\beta, q) = 0$. The boundary stationary state $\beta^i = 1$ obtains when $P(\beta, q) |_{\beta^i=1} > 0$; while the boundary stationary state $\beta^i = 0$ obtains when $P(\beta, q) |_{\beta^i=0} < 0$.*

Proposition 11 *Under Assumption 6-7, for any given q , in the continuous time limit, the dynamics governed by (25) satisfies the following properties:*

if $P(\beta, q) > 0$ for any $\beta^i \in [0, 1]$, then $\beta^i = 1$ is a globally stable stationary state.

if $P(\beta, q) < 0$ for any $\beta^i \in [0, 1]$, then $\beta^i = 0$ is a globally stable stationary state;

any boundary stationary state is always locally stable;

if an interior stationary state β^ exists, it is locally stable if $\frac{\partial P(\beta^*, q)}{\partial \beta^i} < 0$.*

Cultural transmission implies:

$$\begin{aligned} P_t^{i,jj} &= d^{ij} + (1 - d^{ij})q_t^{ij} \\ P_t^{i,jj'} &= (1 - d^{ij})(1 - q_t^{ij}) \end{aligned}$$

$$V^{i,jj}(\beta_{t+1}, q_{t+1}) = u^{ij} (a^{ij}(\beta_{t+1}, q_{t+1}), p(\beta_{t+1}, q_{t+1}); a(\beta_{t+1}, q_{t+1}), q_{t+1}) \quad (26)$$

$$V^{i,j \neq j}(\beta_{t+1}, q_{t+1}) = u^{ij} (a^{ij'}(\beta_{t+1}, q_{t+1}), p(\beta_{t+1}, q_{t+1}); a(\beta_{t+1}, q_{t+1}), q_{t+1}) \quad (27)$$

Let $C(d^{ij})$ denote socialization costs. Direct socialization is then the solution to the following parental socialization problem:

$$\max_{d^{ij} \in [0,1]} -C(d^{ij}) + P_t^{i,jj} V^{i,jj}(\beta_{t+1}, q_{t+1}) + P_t^{i,jj'} V^{i,jj'}(\beta_{t+1}, q_{t+1}), \text{ s. t. } 1).$$

Calling $\Delta V^{ij}(\beta_{t+1}, q_{t+1}) = V^{i,jj}(\beta_{t+1}, q_{t+1}) - V^{i,j \neq j}(\beta_{t+1}, q_{t+1})$, the *cultural intolerance* of trait j in political group i , it follows that the direct socialization, with some notational abuse, has the form:

$$d^{ij} = d^{ij}(q_t, \Delta V^{ij}(\beta_{t+1}, q_{t+1})) = d^{ij}(\beta, q), \quad i \in I, j \in J \quad (28)$$

Assumption 8 *Utility and socialization cost functions are sufficiently regular so that $d^{ij} = d^{ij}(\beta, q)$ is continuous.*

Proposition 12 *Under Assumption 8, and given β_{t+1} , the dynamics of culture q_t^{ij} is governed by the following difference equation:*

$$q_{t+1}^{ij} - q_t^{ij} = q_t^{ij}(1 - q_t^{ij}) (d^{ij} - d^{ij'}). \quad (29)$$

evaluated at $d^{ij} = d^{ij}(q_t, \Delta V^{ij}(\beta_{t+1}, q_{t+1}))$ satisfying (28).

It is convenient to define $D^{ij}(\beta, q) := d^{ij}(\beta, q) - d^{ij'}(\beta, q)$.

Proposition 13 *Under Assumption 8, for any given β , the dynamics of institutions governed by (29) have at least the two boundary stationary states, $q^{ij} = 0$ and $q^{ij} = 1$. An interior stationary states $0 < q^{ij*} < 1$ obtains as a solution to $D(\beta, q) = 0$.*

Proposition 14 *Under Assumption 8, for any given β , in the continuous time limit, the dynamics governed by (29) satisfies the following properties:*

if $D^{ij}(\beta, q) > 0$ for any $q^{ij} \in [0, 1]$, then q_t^{ij} converges to $q^{ij} = 1$ from any initial condition $q_0^{ij} > 0$;

if $D^{ij}(\beta, q) < 0$ for any $q^{ij} \in [0, 1]$, then q_t^{ij} converges to $q^{ij} = 0$ from any initial condition $q_0^{ij} < 1$;

if $D^{ij}(\beta, 1) > 0$, then $q^{ij} = 1$ is locally stable ;

if $D(\beta, 0) < 0$, then $q^{ij} = 0$ is locally stable;

if an interior stationary state q^{ij} exists, and $\frac{\partial D^{ij}(\beta, q^{ij*})}{\partial q^{ij}} < 0$, it is locally stable.*

Consider the following assumption:

Assumption 9 *Socialization costs are quadratic:*

$$C(d^{ij}) = \frac{1}{2} (d^{ij})^2.$$

We then obtain:

Corollary 2 *Under Assumption 9,*

$$D^{ij}(\beta, q) = \Delta V^{ij}(\beta, q)q^{ij'} - \Delta V^{ij'}(\beta, q)q^{ij},$$

and hence interior steady states are characterized by solutions to:

$$\frac{\Delta V^{ij}(\beta, q)}{\Delta V^{ij'}(\beta, 1 - q)} = \frac{q^{ij}}{q^{ij'}} \quad (30)$$

Under Assumptions 6-8, the joint dynamics of institutions and culture is governed by the system (25,29), which we report here for convenience:

$$\begin{aligned} \beta_{t+1}^i &= \begin{cases} \beta^i \text{ such that } p^{com}(\beta, q_{t+1}) - p(\beta_t, q_{t+1}) & \text{if it exists,} \\ \left[\begin{array}{l} 1 \text{ if } p^{com}(\beta_t, q_{t+1}) > p(\beta, q_{t+1}), \forall 0 \leq \beta^i \leq 1 \\ 0 \text{ if } p^{com}(\beta_t, q_{t+1}) < p(\beta, q_{t+1}), \forall 0 \leq \beta^i \leq 1 \end{array} \right. & \text{else} \end{cases} \\ q_{t+1}^{ij} - q_t^{ij} &= q^{ij}(1 - q_t^{ij}) (d^{ij} - d^{ij'}), \text{ with } d^{ij} = d^{ij}(q_t, \Delta V^{ij}(\beta_{t+1}, q_{t+1})). \end{aligned}$$

Proposition 15 *Under Assumptions 6-8 the dynamical system (25,29) has at least one stationary state. Furthermore, if both the institutional and the cultural dynamics display an interior stationary state, respectively, for all $0 \leq q^{ij} \leq 1$ and all $0 \leq \beta \leq 1$, then the dynamical system (25,29) has at least one interior stationary state.*

Appendix C: Assumptions on fundamentals

In this Appendix we translate Assumptions 1-2 into restrictions on fundamentals.

Sufficient conditions for the existence and monotonicity of the societal equilibrium $p(\beta, q)$.

Without loss of generality, restrict $p \in [0, 1]$. The indirect utility function is

$$u^i(a^i, p; A, q);$$

where the individual private action $a^i \in [0, 1]$ and A is an aggregate population level index $A = A(a^1, a^2, p, q)$.

Assume $u^i(\cdot)$ is twice differentiable in $(a^i, p; A, q)$ and strictly concave in a^i (ie. $u_{11}^i < 0$). Assume also that the aggregator function $A(\cdot)$ is differentiable in (a^1, a^2, p, q) and such that the image of $[0, 1]^4$ by $A(\cdot)$ is an interval $[A_{\min}; A_{\max}]$. Finally, assume the following boundary conditions:

$$u_1^i(0, p; A, q) \geq 0, \quad u_1^i(1, p; A, q) \leq 0 \quad \text{for all } (p, A, q) \in [0, 1] \times [A_{\min}; A_{\max}] \times [0, 1].$$

These conditions and the fact that $u^i(\cdot)$ is a strictly concave function in a^i ensure that the optimal individual behavior for a given value of p and A is characterized by a continuous function $a^i(p, A, q) \in [0, 1]$ obtained from the First Order Condition:

$$u_1^i(a^i, p; A, q) = 0.$$

For given values of $p \in P$ and $q \in [0, 1]$, a Nash equilibrium in private actions a^{1N}, a^{2N} and aggregate index $A^N(p, q)$ is characterized by the solution of the following system:

$$a^{iN} = a^i(p, A^N, q) \quad \text{for } i \in (1, 2) \quad \text{and} \quad A^N = A(a^{1N}, a^{2N}, p, q),$$

which in turn translates into the following condition for A^N :

$$A^N = A(a^1(p, A^N, q), a^2(p, A^N, q), p, q). \tag{31}$$

The following sufficient conditions ensure the existence of a unique Nash equilibrium in private actions $a^{1N}(p, q)$, $a^{2N}(p, q)$, $A^N(p, q)$:

$$\begin{aligned} 1 - \sum_{i=1,2} A'_i \frac{u_{13}^i}{-u_{11}^i} &> 0 \quad \text{for all } (a^1, a^2, A, p, q) \\ A(a^1(p, A_{\min}, q), a^2(p, A_{\min}, q), p, q) &> A_{\min} \quad \text{for all } (p, q) \in [0, 1]^2 \\ A(a^1(p, A_{\max}, q), a^2(p, A_{\max}, q), p, q) &< A_{\max} \quad \text{for all } (p, q) \in [0, 1]^2. \end{aligned}$$

The first condition ensures that the function $\Gamma(x, p, q) = x - A(a^1(p, x, q), a^2(p, x, q), p, q)$ is increasing for all $(p, q) \in [0, 1]^2$. The second and the third conditions ensure that $\Gamma(A_{\min}, p, q) < 0 < \Gamma(A_{\max}, p, q)$; Together these conditions ensure the existence of a unique value $A^N(p, q)$ satisfying (31) and thus correspondingly a unique Nash equilibrium profile $a^{1N}(p, q), a^{2N}(p, q)$.

Moreover, differentiating,

$$\begin{aligned} \frac{dA^N}{dp} &= \frac{A_p + \sum_{j=1,2} A'_j \frac{u_{12}^j}{-u_{11}^j}}{\left[1 - \sum_{j=1,2} A'_j \frac{u_{13}^j}{-u_{11}^j}\right]} \\ \frac{da^{iN}}{dp} &= \frac{u_{12}^i}{-u_{11}^i} + \frac{u_{13}^i}{-u_{11}^i} \frac{A_p + \sum_{j=1,2} A'_j \frac{u_{12}^j}{-u_{11}^j}}{\left[1 - \sum_{j=1,2} A'_j \frac{u_{13}^j}{-u_{11}^j}\right]}. \end{aligned}$$

The condition for an interior *societal equilibrium* $p(\beta, q)$ is obtained from the First Order Conditions of the policymaker,

$$\beta u_2^1(a^1, p, A, q) + (1 - \beta) u_2^2(a^2, p, A, q) = 0.$$

After substitution of the Nash equilibrium private actions $a^{1N}(p, q), a^{2N}(p, q), A^N(p, q)$, this condition can be written as

$$\Psi(p, q, \beta) = 0; \tag{32}$$

with

$$\Psi(p, q, \beta) = \beta u_2^1(a^{1N}(p, q), p, A^N(p, q), q) + (1 - \beta) u_2^2(a^{2N}(p, q), p, A^N(p, q), q).$$

Moreover a corner societal equilibrium $p(\beta, q) = 0$ (resp. $p(\beta, q) = 1$) obtains when $\Psi(0, q, \beta) \leq 0$ (resp. $\Psi(1, q, \beta) \geq 1$).

A sufficient condition for the existence of a unique *societal equilibrium* $p(\beta, q)$ consists in the function $\Psi(p, q, \beta)$ being decreasing in p for all $q \in [0, 1]$. Given the smoothness assumptions on the functions $u^i(\cdot)$ and $A(\cdot)$ this is satisfied when the following condition holds:

$$u_{12}^i \frac{da^i}{dp} + u_{22}^i + u_{23}^i \frac{dA}{dp} < 0 \text{ for all } i \in (1, 2).$$

In turn, in terms of the fundamentals, this conditions becomes:

$$\frac{u_{12}^i}{-u_{22}^i} \left[\frac{u_{12}^i}{-u_{11}^i} + \frac{u_{13}^i}{-u_{11}^i} \frac{A_p + \sum_{j=1,2} A'_j \frac{u_{12}^j}{-u_{11}^j}}{\left[1 - \sum_{j=1,2} A'_j \frac{u_{13}^j}{-u_{11}^j}\right]} \right] + \frac{u_{23}^i}{-u_{22}^i} \frac{A_p + \sum_{j=1,2} A'_j \frac{u_{12}^j}{-u_{11}^j}}{\left[1 - \sum_{j=1,2} A'_j \frac{u_{13}^j}{-u_{11}^j}\right]} < 1;$$

or

$$\frac{(u_{12}^i)^2}{u_{22}^i u_{11}^i} + \left(\frac{u_{12}^i u_{13}^i}{u_{22}^i u_{11}^i} + \frac{u_{23}^i}{(-u_{22}^i)} \right) \frac{A_p + \sum_{j=1,2} A'_j \frac{u_{12}^j}{-u_{11}^j}}{\left[1 - \sum_{j=1,2} A'_j \frac{u_{13}^j}{-u_{11}^j} \right]} < 1 \quad \text{for } i \in (1, 2),$$

with $A_p = \partial A / \partial p$, and $A_j = \partial A / \partial a^j$. This condition is more likely to be satisfied when $|u_{11}^i|$ and $|u_{22}^i|$ are large enough.

To obtain a condition ensuring the monotonicity in β of the *societal equilibrium* $p(\beta, q)$, we differentiate (6), obtaining

$$\frac{\partial p}{\partial \beta} = \frac{u_2^1 - u_2^2}{-\Psi_p}.$$

Thus $p(\beta, q)$ is monotonic in β when $u_2^1 - u_2^2$ has a constant sign.

This condition simplifies if the preferences structure is characterized by some degree of separability:

$$\begin{aligned} u^i(a, p; A, q) &= u(a, p; A, q, \theta_i) \\ &= v(a, p, \theta_i) + H(p, A) \end{aligned}$$

and $\theta_1 > \theta_2$. Such preferences lead to

$$\begin{aligned} a^{1N} &= a(p, \theta_1) \\ a^{2N} &= a(p, \theta_2) \\ A^N &= A(a(p, \theta_1), a(p, \theta_2), p, q). \end{aligned}$$

A sufficient condition for the existence of a unique *societal equilibrium*, given that $u_{13}^j = 0$, is then

$$\frac{(v_{12}^i)^2}{[v_{22}^i + H_{pp}] v_{11}^i} + \left(\frac{H_{pA}}{-(v_{22}^i + H_{pp})} \right) \left[A_p + \sum_{j=1,2} A'_j \frac{v_{12}^j}{-v_{11}^j} \right] < 1 \quad \text{for } i \in (1, 2),$$

where $v_{kl}^i = v''_{kl}(a, p, \theta_i)$. But

$$\begin{aligned} u_2^1 - u_2^2 &= u_2(a^{1N}, p, A^N, q, \theta_1) - u_2(a^{2N}, p, A^N, q, \theta_2) \\ &= v_2(a(p, \theta_1), p, \theta_1) - v_2(a(p, \theta_2), p, \theta_2). \end{aligned}$$

Thus a sufficient conditions for the monotonicity of the *societal equilibrium* $p(\beta, q)$ is that $v_p(a(p, \theta), p, \theta)$ is monotonic in θ ; or, after manipulations, that $v_{ap} \frac{v_{a\theta}}{(-v_{aa})} + v_{p\theta}$ has a constant sign.

Consider as an example the following preference structure:

$$u(a, p, A, q, \theta) = (1 - p)a + \theta W(1 - a) + H(p, A);$$

with $W(\cdot)$ a strictly increasing and concave function, $A = qa^1 + (1 - q)a^2$, $H(p, A)$ concave in p . Then,

$$\begin{aligned} v_a &= (1 - p) - \theta W'(1 - a), \quad v_p = -a \\ v_{ap} &= -1, \quad v_{a\theta} = -W'(1 - a) \\ -v_{aa} &= -\theta W''(1 - a) \\ v_{p\theta} &= 0, \quad v_{pp} = 0. \end{aligned}$$

The sufficient condition for a well defined *societal equilibrium* $p(\beta, q)$ can be written as

$$\frac{1}{H_{pp}\theta_i W''} + \left(\frac{H_{pA}}{-(H_{pp})} \right) \left[\sum_{j=1,2} q_j \frac{1}{\theta_j W''} \right] < 1 \text{ text for } i = 1, 2.$$

When $H_{pA} > 0$, given that $-(H_{pp}) > 0$ and $\left[\sum_{j=1,2} q_j \frac{1}{\theta_j W''} \right] < 0$, this condition is satisfied when $\frac{1}{H_{pp}\theta_i W''} < 1$, which in turn holds when $1 < H_{pp}W''\theta_2$. This is satisfied when $H_{pp}W''$ is sufficiently large; that is, with enough concavity of W and H , respectively, in a and p .

When $H_{pA} < 0$, this sufficient condition can be rewritten as

$$\frac{1}{\theta_i} - H_{pA} \left[\sum_{j=1,2} q_j \frac{1}{\theta_j} \right] < H_{pp}W'',$$

which again will be satisfied when H_{pA} is bounded from below on the relevant domain, $[0, 1] \times [A_{\min}, A_{\max}]$ (i.e., $H_{pA} > -K$, with $K > 0$) and $H_{pp}W'' > (1 + K)/\theta_2$. This also is satisfied with enough concavity of W and H , respectively, in a and p .

Finally, the monotonicity of the *societal equilibrium* function $p(\beta, q)$ holds when $v_{ap} \frac{v_{a\theta}}{(-v_{aa})} + v_{p\theta}$ has a constant sign. But

$$v_{ap} \frac{v_{a\theta}}{(-v_{aa})} + v_{p\theta} = \frac{W'}{-\theta W''} > 0.$$

Thus the *societal equilibrium* function is monotonically increasing in β .

Sufficient conditions for the existence of the societal commitment equilibrium $p^{com}(\beta, q)$.

The *societal commitment equilibrium* given institutions β and cultural distribution q is obtained from the following maximization problem:

$$\begin{aligned} \max \quad & \beta u^1(a^{1N}, p; A^N, q) + (1 - \beta) u^2(a^{2N}, p; A^N, q) \\ \text{s.t.} \quad & a^{iN} = a^{iN}(p, q) \quad \text{for } i \in (1, 2) \quad \text{and } A^N = A^N(p, q) \end{aligned} \quad (33)$$

Let

$$\begin{aligned}\Omega(p, \beta, q) &= \beta u^1(a^{1N}(p, q), p; A^N(p, q), q) \\ &\quad + (1 - \beta) u^2(a^{1N}(p, q), p; A^N(p, q), q).\end{aligned}$$

The First Order Condition for an interior *societal commitment equilibrium* $p^{com}(\beta, q)$ can be written as

$$\begin{aligned}\Omega_p(p, \beta, q) &= \beta u_2^1(a^{1N}, p, A^N, q) + (1 - \beta) u_2^2(a^{2N}, p, A^N, q) \\ &\quad + (\beta u_3^1(a^{1N}, p, A^N, q) + (1 - \beta) u_3^2(a^{2N}, p, A^N, q)) \frac{dA^N}{dp} \\ &= 0.\end{aligned}$$

A sufficient (strong) condition is then,

$$\Omega_{pp}(p, \beta, q) < 0, \text{ for all } p \in [0, 1].$$

Differentiating,

$$\begin{aligned}\Omega_{pp}(p, \beta, q) &= \sum_{i=1,2} \beta^i (u_{12}^i \frac{da^{iN}}{dp} + u_{22}^i + u_{23}^i \frac{dA^N}{dp}) \\ &\quad + \sum_{i=1,2} \beta^i \left[u_{13}^i \frac{da^{iN}}{dp} + u_{23}^i + u_{33}^i \right] \frac{dA^N}{dp} \\ &\quad + \sum_{i=1,2} \beta^i u_3^i \frac{d^2 A^N}{dp^2}.\end{aligned}$$

Thus a sufficient condition for $\Omega_{pp}(p, \beta, q) < 0$ is that, for $i = 1, 2$,

$$u_{12}^i \frac{da^{iN}}{dp} + u_{22}^i + u_{23}^i \frac{dA^N}{dp} + \left[u_{13}^i \frac{da^{iN}}{dp} + u_{23}^i + u_{33}^i \right] \frac{dA^N}{dp} + u_3^i \frac{d^2 A^N}{dp^2} < 0.$$

Recall

$$\frac{da^{iN}}{dp} = \frac{u_{12}^i}{-u_{11}^i} + \frac{u_{13}^i}{-u_{11}^i} \frac{A_p + \sum_{j=1,2} A'_j \frac{u_{12}^j}{-u_{11}^j}}{\left[1 - \sum_{j=1,2} A'_j \frac{u_{13}^j}{-u_{11}^j} \right]}$$

and

$$\frac{dA^N}{dp} = \frac{A_p + \sum_{j=1,2} A'_j \frac{u_{12}^j}{-u_{11}^j}}{\left[1 - \sum_{j=1,2} A'_j \frac{u_{13}^j}{-u_{11}^j} \right]}.$$

Tedious manipulations show then that a sufficient condition for $\Omega_{pp}(p, \beta, q) < 0$ is that

$$D^i = \frac{(u_{12}^i)^2}{-u_{11}^i} + u_{22}^i + \left(2\left(\frac{u_{13}^i u_{12}^i}{-u_{11}^i} + u_{23}^i \right) + u_{33}^i \right) \frac{A_p + \sum_{j=1,2} A'_j \frac{u_{12}^j}{-u_{11}^j}}{\left[1 - \sum_{j=1,2} A'_j \frac{u_{13}^j}{-u_{11}^j} \right]}$$

$$+ \frac{(u_{13}^i)^2}{-u_{11}^i} \left[\frac{A_p + \sum_{j=1,2} A'_j \frac{u_{12}^j}{-u_{11}^j}}{\left[1 - \sum_{j=1,2} A'_j \frac{u_{13}^j}{-u_{11}^j} \right]} \right]^2 + u_3^i \frac{d^2 A^N}{dp^2}$$

is negative for $i = 1, 2$. Because of the term in $d^2 A^N / dp^2$, this involves complicated conditions on the third derivatives of the indirect preference functions. When preferences are separable, of the form

$$\begin{aligned} u^i(a, p; A, q) &= u(a, p; A, q, \theta_i) \\ &= v(a, p, \theta_i) + H(p, A), \end{aligned}$$

the expression D^i simplifies somewhat:

$$D^i = \frac{(v_{ap}^i)^2}{-v_{aa}^i} + v_{pp}^i + (2H_{pA} + H_{AA}) (A_p + \sum_{j=1,2} A'_j \frac{v_{ap}^j}{-v_{pp}^j})$$

$$+ H_A \frac{d^2 A^N}{dp^2}.$$

Therefore $\Omega(p, \beta, q)$ is strictly concave in p when $v(a, p, \theta_i)$ is sufficiently concave in (a, p) .