

## Online Appendix

### O.1 Multiple Investment Inputs Each Period

In this Appendix, we show how our model can be generalized to include multiple investments each period, where  $i_j$  would then reflect total investment expenditures in period  $j$  given optimal choices about different inputs each period.

Suppose human capital production depends on two inputs each period: purchased goods  $g_j$  and parental time  $\tau_j$  (scaled by effective parental human capital  $h_j^p \equiv \Gamma_{j+2}h^p$ ). Define the child's human capital production function as:

$$h = \theta f(x_1, x_2) \quad (26)$$

where

$$x_j = \chi_j(g_j, \tau_j h_j^p), \quad j = 1, 2.$$

Notice that this technology assumes parental human capital increases the productivity of parental time inputs in the same way it increases productivity in the labor market. This is analogous to the neutrality assumption of Ben-Porath (1967) only with respect to investments in child human capital rather than own human capital.

Next, consider maximizing per period human capital inputs  $x_j$  subject to total expenditure  $i_j$  that period. We assume input prices ( $p$  for the price of goods and  $w$  for the price of human capital) are stable across periods and that individuals are at an interior point in their time budget (i.e.  $\tau \in (\tau_{min}, \tau_{max})$ ). Define the following maximized period  $j$  input:

$$x_j^*(i_j; p, w) = \max_{g_j, \tau_j} \chi_j(g_j, \tau_j h_j^p) \quad \text{subject to} \quad pg_j + w\tau_j h_j^p = i_j \quad (27)$$

for total investment expenditures  $i_j$  in each period.

If we assume  $\chi_j(\cdot, \cdot)$  are homogeneous of degree 1, then we can write

$$x_j^*(i_j; p, w) = \tilde{x}_j(p, w)i_j,$$

where  $\tilde{x}(p, w)$  is the maximized output for a total expenditure of 1. Substituting this into equation (26) yields

$$h = \theta f(\tilde{x}_1(p, w)i_1, \tilde{x}_2(p, w)i_2).$$

Clearly, one can just re-write the production function in terms of total investment expenditures  $i_1$  and  $i_2$  as  $\tilde{f}(i_1, i_2) = f(\tilde{x}_1(p, w)i_1, \tilde{x}_2(p, w)i_2)$  where the  $\tilde{x}_j(p, w)$  are like technology parameters that depend on prices  $(p, w)$ .

For the CES production function  $f(x_1, x_2) = [ax_1^b + (1-a)x_2^b]^{d/b}$ , we have the following:

$$\begin{aligned}
h &= \theta \left[ a(\tilde{x}_1(p, w)i_1)^b + (1-a)(\tilde{x}_2(p, w)i_2)^b \right]^{d/b} \\
&= \theta \left\{ \left[ \left( \frac{a\tilde{x}_1^b}{a\tilde{x}_1^b + (1-a)\tilde{x}_2^b} \right) i_1^b + \left( \frac{(1-a)\tilde{x}_2^b}{a\tilde{x}_1^b + (1-a)\tilde{x}_2^b} \right) i_2^b \right] \left( a\tilde{x}_1^b + (1-a)\tilde{x}_2^b \right) \right\}^{d/b} \\
&= \theta \left( a\tilde{x}_1^b + (1-a)\tilde{x}_2^b \right)^{d/b} \left[ \left( \frac{a\tilde{x}_1^b}{a\tilde{x}_1^b + (1-a)\tilde{x}_2^b} \right) i_1^b + \left( 1 - \frac{a\tilde{x}_1^b}{a\tilde{x}_1^b + (1-a)\tilde{x}_2^b} \right) i_2^b \right]^{d/b} \\
&= \tilde{\theta} \left[ \tilde{a}i_1^b + (1-\tilde{a})i_2^b \right]^{d/b}
\end{aligned}$$

where

$$\begin{aligned}
\tilde{\theta} &= \theta [a\tilde{x}_1^b(p, w) + (1-a)\tilde{x}_2^b(p, w)]^{d/b} \\
\tilde{a} &= \frac{a\tilde{x}_1^b(p, w)}{a\tilde{x}_1^b(p, w) + (1-a)\tilde{x}_2^b(p, w)}.
\end{aligned}$$

Thus, if (i) parental time investment is unconstrained (i.e. at an interior point), (ii) parental human capital is equally productive in child development and the labor market, and (iii) within period investment functions  $\chi_j(\cdot, \cdot)$  are homogeneous of degree 1, then our CES human capital production function still represents the production process with  $i_j$  reflecting total investment expenditures in period  $j$ . The ‘technology’ parameters  $\tilde{\theta}$  and  $\tilde{a}$  now depend on input prices  $p$  and  $w$  in addition to true underlying technology parameters.

In general, variation in prices ( $w, p$ ) can affect both total factor productivity  $\tilde{\theta}$  and the relative productivity of early vs. late investments,  $\tilde{a}$ . Two interesting special cases yield variation in  $\tilde{\theta}$  alone.

First, variation in price levels (but not relative prices) will only affect  $\tilde{\theta}$ . For example, consider two sets of prices ( $p, w$ ) and ( $p', w'$ ) where  $\frac{w'}{w} = \frac{p'}{p} = \delta$ . In this case, it is easy to see that  $\tilde{x}'_j = \tilde{x}_j/\delta$ , so  $\tilde{\theta}' = \tilde{\theta}\delta^{-d}$  and  $\tilde{a}' = \tilde{a}$ .

Second, if both within-period production functions are identical, so  $\chi_j(\cdot, \cdot) = \chi(\cdot, \cdot)$  and  $\tilde{x}_j(p, w) = \tilde{x}(p, w)$  are independent of period  $j$ , then differences in input prices ( $p, w$ ) will generally lead to differences in  $\tilde{\theta} = \tilde{x}^d\theta$  but not  $\tilde{a}$ , which equals  $a$  regardless of ( $w, p$ ).

### Special Case: CES $\chi_j(\cdot, \cdot)$

Suppose  $\chi_j(g, \tau h^p) = [\psi_{jg}g^\phi + \psi_{j\tau}(\tau h^p)^\phi]^{1/\phi}$ . In this case, it is straightforward to show that

$$\tilde{x}_j(p, w) = \frac{\left( \psi_{jg} \left[ \left( \frac{\psi_{jg}}{\psi_{j\tau}} \right) \left( \frac{w}{p} \right) \right]^{\frac{\phi}{1-\phi}} + \psi_{j\tau} \right)^{1/\phi}}{p \left( \left[ \left( \frac{\psi_{jg}}{\psi_{j\tau}} \right) \left( \frac{w}{p} \right) \right]^{\frac{1}{1-\phi}} + \frac{w}{p} \right)}.$$

## O.2 Properties of the Value Function $V_3(\cdot, \cdot)$

In order to apply the proofs contained in CLP to our dynastic structure, we need to demonstrate that the properties of the lifecycle continuation utility are maintained with the dynastic value function. In particular, that  $V_3(a_3, h)$  is strictly increasing and strictly concave in both assets and human capital.

It is straightforward to apply the results in Stokey, Lucas, and Prescott (1989) (SLP) to show that the dynastic value function is unique, strictly increasing and strictly concave. We can rewrite the dynastic problem to be consistent with SLP:

$$V(a_3, h) = \max_{c_3, c_4, c_6, a_4, a_5, i'_1, a'_3, h'} \hat{U}(a_3, h, c_3, c_4, c_6, a_4, a_5, i'_1, a'_3, h') + \rho\beta^2 V(a'_3, h') \quad (28)$$

subject to:

$$\begin{aligned} Ra_3 + W_3(h) + y_3 - a_4 - i'_1 - c_3 - c'_1 &> 0, \\ Ra_4 + W_4(h) + y_4 + W_2 - a_5 - a'_3 - i'_2(i'_1, h') - c_4 - c'_2 &> 0, \\ Ra_5 + W_5(h) - R^{-1}c_6 &> 0, \\ a_4 &\geq -L_3, \\ a_5 &\geq -L_4, \\ a'_3 &\geq -L_2, \\ h' &= \theta' f(i'_1, i'_2), \end{aligned}$$

where

$$\begin{aligned} \hat{U}(a_3, h, c_3, c_4, c_6, a_4, a_5, i'_1, a'_3, h') &= u(c_3) + \beta u(c_4) + \beta^2 u(Ra_5 + W_5(h) - R^{-1}c_6) + \beta^3 u(c_6) + \\ &\rho[u(Ra_3 + W_3(h) + y_3 - a_4 - i'_1 - c_3) + \beta u(Ra_4 + W_4(h) + y_4 + W_2 - a_5 - a'_3 - i'_2(i'_1, h') - c_4)]. \end{aligned}$$

Note that  $i_2(i'_1, h')$  is the  $i'_2$  that satisfies,  $h' = \theta' f(i'_1, i'_2)$ .

The following assumptions (see SLP, chapter 4) hold for this problem:

A4.3 The state space  $(a_3, h)$  is a convex subset of  $R^2$  and the constraint set is non-empty, compact-valued and continuous.

A4.4 The function  $\hat{U}$  is bounded and continuous and  $\rho\beta^2 < 1$ . Because  $\hat{U}$  is derived from  $u$  it is bounded and continuous. The latter condition holds when  $\beta < 1$  and  $\rho < 1$ .

A4.5 The function  $\hat{U}$  is strictly increasing in  $a_3$  and  $h$ . It is clear that  $\hat{U}$  is strictly increasing in  $a_3$  and  $h$  because  $u'(\cdot) > 0$ , and arguments of  $u$  are increasing in  $a_3$  and  $h$ .

A4.6 The constraint set is monotone: As either state variable  $a_3$  or  $h$  increases, the set of possible choice variables contains the original set.

A4.7 The function  $\hat{U}$  is concave. Because  $u$  and  $f$  are concave,  $\hat{U}$  is concave.

A4.8 The constraint set is convex. Convexity of the constraint set follows because  $f$  is concave.

A4.9 The function  $\hat{U}$  is continuously differentiable with respect to  $a_3$  and  $h$ .

Given these assumptions, we have (see SLP, chapter 4):

**Theorem 4.6** If A4.3 and A4.4 hold there exists a unique  $V$ .

**Theorem 4.7** If A4.3-A4.6 hold,  $V$  is strictly increasing.

**Theorem 4.8** If A4.3-A4.4 and A4.7-A4.8 hold,  $V$  is strictly concave.

**Theorem 4.11** If A4.3-A4.4 and A4.7-A4.9 hold,  $V$  is continuously differentiable.

Therefore, there exists a unique  $V$ , that is strictly increasing, strictly concave, and continuously differentiable. We do not know if  $V$  is twice, continuously differentiable. What we need is that  $V$  is twice differentiable at an optimum (at least one-sided). If this is the case, then  $V_{22} < 0$ , due to the concavity of  $V$ .

### O.3 Calibration Sensitivity Analysis

We conduct a comprehensive sensitivity analysis for our calibration, counterfactual, and policy simulations. In particular, we re-calibrate our model imposing different assumptions about (i) the extent of dynamic complementarity (i.e. different values for  $b$ ), (ii) greater borrowing opportunities (i.e.  $\gamma = 0.5$ ), (iii) no effect of parental human capital on the child's ability (i.e.  $\pi_2 = 0$ ), and (iv) no unmeasured costs of high school (i.e.  $\zeta_1 = 0$ ). We also re-calibrate our model using a 'full' family income measure that adjusts for the possibility that mothers may work part-time in order to spend time investing in their children. In all cases, we repeat our main counterfactual and policy simulations with the restricted/new parameter sets obtained through the same simulated method of moments procedure.

Table O-1 reports the calibrated parameter values for all cases, while Table O-2 reports the mean weighted squared error (MWSE) for the different subsets of moments. Tables O-3 to O-6 report measures of investment and the fraction of families borrowing up to their limits in the

re-calibrated economies. Tables O-7 to O-12 reproduce the main results from our counterfactual and policy simulations for each calibration.

Section 5 of the paper discusses the analysis and findings for our results imposing different values for  $b$  (-0.5, 0, 0.5) and using the ‘full income’ measure in creating our sets of moments (iii)-(v) for investment and period 3 wage outcomes conditional on family income and maternal education. Here, we provide a brief discussion of results for the other three cases.

When re-calibrating the model fixing any single parameter (i.e.  $\gamma = 0.5$  or  $\pi_2 = 0$  or  $\zeta_1 = 0$ ), most other parameter estimates are quite similar to those of our baseline calibration (Table O-1). One exception is the smaller value for  $\gamma$  when imposing  $\zeta_1 = 0$ , implying fewer borrowing opportunities than in the baseline case. Given the importance of dynamic complementarity for many of our results, it is also worth noting the somewhat higher values (compared to the baseline calibration) for  $b$  when we impose  $\pi_2 = 0$  or  $\zeta_1 = 0$  and lower value when we set  $\gamma = 0.5$ . In terms of fit (Table O-2), imposing  $\gamma = 0.5$  produces a poor fit for late investments and wage distributions, while imposing  $\pi_2 = 0$  leads to a poor fit for early and late investments conditional on parental education and family income. Imposing  $\zeta_1 = 0$  fits slightly worse than the baseline for all moments, but is not particularly bad for any subset. In all cases, the investment ratios for children of college graduates vs. high school dropouts are comparable to the baseline calibration (Table O-5). The proportions of families up against their borrowing or transfer constraints (Table O-6) are also quite similar to those reported for the baseline calibration with one exception: far fewer old parents are borrowing constrained when imposing  $\gamma = 0.5$ . The fraction of young parents up against their borrowing limit is quite similar to the baseline case even with the much higher  $\gamma$ . Other parameters adjust to fit the data in a way that still yields a non-trivial fraction of borrowing constrained young parents.

Table O-7 reports the anticipated and unanticipated short-run effects of a \$10,000/year income transfer to old parents. In all cases, the effects of an anticipated transfer are much greater than an unanticipated transfer; however, the the differences are more modest when  $\pi_2 = 0$  or  $\zeta_2 = 0$  are imposed. These more muted differences are consistent with the greater substitutability implied by the higher estimated values for  $b$  in these cases, much as we see for the case imposing  $b = 0.5$ .

Tables O-8 and O-9 reproduce the counterfactual analyses aimed at understanding the importance of ability transmission and market frictions for intergenerational mobility. In all cases, child ability accounts for a comparable share of the investment gaps by parental income, while eliminating lifecycle borrowing constraints would have similar or stronger effects (compared to the baseline calibration). There is a greater discrepancy between calibration cases in the implied role of ability vs. market frictions when we simulate the economy with zero intergenerational ability correlation (Table O-9). Assuming greater opportunities for borrowing than estimated by our baseline calibration (imposing  $\gamma = 0.5$ ) produces a much greater role for ability transmission

relative to market frictions.

As shown in Tables O-10 and O-11, we obtain very similar effects of relaxing borrowing constraints (one-by-one or completely eliminating all constraints) for all of our restricted calibration sets, even when  $\gamma = 0.5$  is assumed. In all cases, completely eliminating all lifecycle borrowing constraints has substantial effects on investments and post-school earnings – much greater than the effects of relaxing any single borrowing limit by itself.<sup>80</sup>

Finally, Table O-12 reports the short-run effects of fiscally equivalent early and late investment subsidies. In all calibration cases, we consider the impacts of increasing  $s_1$  to 0.1, as well as increasing  $s_2$  by an amount that produces the same total expenditure on all investment subsidies. Our main conclusions hold for all parameterizations: (i) early investment subsidies have greater effects than late subsidies, and (ii) the effects of late subsidies are much greater when the subsidies are announced early so early investment can respond. Perhaps surprisingly, the effects of subsidies are greatest when  $\gamma = 0.5$ . They are smallest when  $\pi_2 = 0$ .

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<sup>80</sup>Note that Table O-10 studies the short-run effects of increasing borrowing limits by \$1,500 rather than \$2,500 as in the paper, because increasing borrowing limits (at one stage) by the latter amount (in two calibration cases) would extend them beyond the natural borrowing limits for some families due to subsequent constraints.

**Table O-1: Calibrated Parameter Values under Different Restrictions and Data Assumptions**

Parameter	Baseline	<b>b = 0</b>	<b>b = 0.5</b>	<b>b = -0.5</b>	<b><math>\gamma = 0.5</math></b>	<b><math>\pi_2 = 0</math></b>	<b><math>\zeta_1 = 0</math></b>	<b>full income</b>
<b>a</b>	0.58	0.53	0.62	0.50	0.52	0.55	0.60	0.59
<b>b</b>	0.26	<b>0</b>	<b>0.5</b>	<b>-0.5</b>	0.15	0.37	0.33	0.32
<b>d</b>	0.82	0.82	0.82	0.82	0.80	0.80	0.81	0.81
<b><math>\theta_1</math></b>	4.85	4.60	5.10	4.77	5.35	5.00	5.20	5.46
<b><math>\theta_2</math></b>	12.03	12.16	12.44	12.06	13.79	13.16	14.52	14.18
<b><math>\pi_0</math></b>	-0.88	-0.57	-0.90	-0.76	-0.89	-1.07	-0.66	-0.69
<b><math>\pi_1</math></b>	0.15	0.10	0.13	0.14	0.12	0.15	0.12	0.12
<b><math>\pi_2</math></b>	0.000019	0.000014	0.000019	0.000018	0.000016	<b>0</b>	0.000017	0.000010
<b><math>\zeta_1</math></b>	47.49	61.72	30.86	88.19	57.81	29.97	<b>0</b>	39.85
<b><math>\zeta_2</math></b>	760.73	726.46	719.28	571.99	857.38	808.31	809.12	888.53
<b>m</b>	9.90	9.96	9.90	9.92	9.93	9.85	9.81	9.90
<b>s</b>	0.71	0.70	0.71	0.75	0.74	0.71	0.77	0.71
<b><math>\rho</math></b>	0.86	0.85	0.84	0.87	0.84	0.86	0.83	0.85
<b><math>\gamma</math></b>	0.22	0.11	0.07	0.15	<b>0.5</b>	0.17	0.05	0.09

**Table O-2: Weighted Average Mean Squared Error for Calibration Sensitivity Analysis**

	Baseline	b = 0	b = 0.5	b = -0.5	$\gamma = 0.5$	$\pi_2 = 0$	$\zeta_1 = 0$	full income
<b>Moment subset:</b>								
$\Pr(i_2)$	0.0001	0.0002	0.0001	0.0026	0.0008	0.0000	0.0001	0.0002
$E(W_3   i_2), \text{Var}(W_3   i_2), \text{Cov}(W_3, W_4)$	0.6490	0.8221	0.6410	1.0850	0.8426	0.6545	0.6924	0.6485
$E(\Phi   i_2, W_3, W_4)$	0.0478	0.0597	0.0630	0.0442	0.0453	0.0882	0.0544	0.0650
$\Pr(i_2'   i_2, W_4)$	0.0046	0.0049	0.0058	0.0044	0.0047	0.0066	0.0055	0.0053
$E(W_3'   i_2, W_3, W_4)$	0.0282	0.0302	0.0233	0.0391	0.0237	0.0248	0.0321	0.0278
$\Pr(a_4 < 0)$	0.0336	0.0335	0.0339	0.0325	0.0260	0.0326	0.0347	0.0339
<b>All moments</b>	0.0089	0.0099	0.0093	0.0114	0.0106	0.0107	0.0097	0.0089

Notes: Values for subsets of moments reflect the weighted average MSE over that subset of moments only.



**Table O-3: Calibrated Education Distribution (Sensitivity Analysis)**

<b>Education</b>	<b>Baseline</b>	<b>b = 0</b>	<b>b = 0.5</b>	<b>b = -0.5</b>	<b><math>\gamma = 0.5</math></b>	<b><math>\pi_2 = 0</math></b>	<b><math>\zeta_1 = 0</math></b>	<b>full income</b>
<b>HS Graduate or More</b>	0.83	0.81	0.83	0.83	0.82	0.82	0.82	0.81
<b>Some College or More</b>	0.44	0.44	0.42	0.54	0.41	0.41	0.42	0.41
<b>College Graduate</b>	0.21	0.19	0.21	0.22	0.15	0.19	0.21	0.20

**Table O-4: Avg. Early Investment Factor Scores and Educational Attainment by Parental Education**

	Parental Education	Early investment Score	HS Grad. or More	Some College or More	College Grad.
<b>Baseline</b>	HS Dropout	-0.49	0.64	0.20	0.08
	HS Graduate	-0.40	0.81	0.27	0.08
	Some College	0.11	0.90	0.57	0.15
	College Graduate	0.67	0.94	0.82	0.63
<b>b = 0</b>	HS Dropout	-0.49	0.68	0.23	0.08
	HS Graduate	-0.40	0.78	0.30	0.08
	Some College	0.13	0.87	0.58	0.14
	College Graduate	0.67	0.91	0.76	0.58
<b>b = 0.5</b>	HS Dropout	-0.48	0.66	0.20	0.08
	HS Graduate	-0.39	0.80	0.27	0.08
	Some College	0.12	0.91	0.54	0.14
	College Graduate	0.67	0.93	0.79	0.62
<b>b = -0.5</b>	HS Dropout	-0.51	0.64	0.27	0.09
	HS Graduate	-0.41	0.78	0.35	0.09
	Some College	0.09	0.89	0.64	0.12
	College Graduate	0.67	0.94	0.84	0.65
<b><math>\gamma = 0.5</math></b>	HS Dropout	-0.47	0.65	0.20	0.07
	HS Graduate	-0.38	0.80	0.27	0.07
	Some College	0.14	0.92	0.56	0.12
	College Graduate	0.67	0.91	0.78	0.54
<b><math>\pi_2 = 0</math></b>	HS Dropout	-0.48	0.70	0.22	0.09
	HS Graduate	-0.39	0.78	0.29	0.09
	Some College	0.13	0.91	0.53	0.13
	College Graduate	0.67	0.91	0.69	0.56
<b><math>\zeta_1 = 0</math></b>	HS Dropout	-0.50	0.66	0.19	0.08
	HS Graduate	-0.40	0.79	0.23	0.07
	Some College	0.13	0.89	0.57	0.13
	College Graduate	0.67	0.93	0.85	0.66
<b>full income</b>	HS Dropout	-0.49	0.65	0.19	0.08
	HS Graduate	-0.40	0.79	0.24	0.07
	Some College	0.13	0.88	0.55	0.15
	College Graduate	0.67	0.93	0.79	0.63

**Table O-5: Average Investment Amounts by Parental Education (Sensitivity Analysis)**

	Parental Education	$i_1$	$i_2$	$i_2 + \zeta(i_2) - S_2(i_2)$
<b>Baseline</b>	All Levels	1888	8744	5629
	HS Dropout	770	4351	2671
	College Graduate	4600	18687	12304
<b>b = 0</b>	All Levels	1921	8369	5374
	HS Dropout	877	4660	2875
	College Graduate	4721	17305	11306
<b>b = 0.5</b>	All Levels	1296	8464	5375
	HS Dropout	523	4329	2612
	College Graduate	3463	18325	11946
<b>b = -0.5</b>	All Levels	3389	9641	6078
	HS Dropout	1521	4952	3039
	College Graduate	7315	19149	12186
<b><math>\gamma = 0.5</math></b>	All Levels	1229	7524	4924
	HS Dropout	575	4181	2604
	College Graduate	3200	16910	11345
<b><math>\pi_2 = 0</math></b>	All Levels	736	8137	5246
	HS Dropout	363	4842	2984
	College Graduate	2028	16547	10907
<b><math>\zeta_1 = 0</math></b>	All Levels	2055	8483	5462
	HS Dropout	877	4279	2586
	College Graduate	5227	19347	12857
<b>full income</b>	All Levels	1702	8337	5471
	HS Dropout	738	4362	2721
	College Graduate	4312	18388	12371

**Table O-6: Fraction Borrowing and Transfer Constrained (Sensitivity Analysis)**

		Fraction of Young Parents Constrained	Fraction of Old Parents Constrained	Fraction of Parents Transfer Constrained
<b>Baseline</b>	All Levels	0.12	0.14	0.00
	HS Dropout	0.13	0.06	0.01
	HS Graduate	0.20	0.17	0.00
	Some College	0.06	0.17	0.00
	College Graduate	0.01	0.14	0.00
<b>b = 0</b>	All Levels	0.13	0.18	0.00
	HS Dropout	0.13	0.05	0.00
	HS Graduate	0.21	0.17	0.00
	Some College	0.10	0.22	0.00
	College Graduate	0.03	0.25	0.00
<b>b = 0.5</b>	All Levels	0.13	0.18	0.00
	HS Dropout	0.16	0.06	0.00
	HS Graduate	0.23	0.18	0.00
	Some College	0.06	0.25	0.00
	College Graduate	0.00	0.21	0.00
<b>b = -0.5</b>	All Levels	0.10	0.16	0.00
	HS Dropout	0.09	0.03	0.00
	HS Graduate	0.18	0.13	0.00
	Some College	0.11	0.26	0.00
	College Graduate	0.01	0.18	0.00
<b><math>\gamma = 0.5</math></b>	All Levels	0.11	0.04	0.01
	HS Dropout	0.15	0.03	0.02
	HS Graduate	0.17	0.05	0.00
	Some College	0.05	0.05	0.00
	College Graduate	0.00	0.01	0.00
<b><math>\pi_2 = 0</math></b>	All Levels	0.12	0.13	0.00
	HS Dropout	0.16	0.06	0.01
	HS Graduate	0.19	0.14	0.00
	Some College	0.05	0.17	0.00
	College Graduate	0.00	0.14	0.00
<b><math>\zeta_1 = 0</math></b>	All Levels	0.15	0.21	0.00
	HS Dropout	0.10	0.03	0.00
	HS Graduate	0.27	0.22	0.00
	Some College	0.08	0.29	0.00
	College Graduate	0.01	0.25	0.00
<b>full income</b>	All Levels	0.13	0.19	0.00
	HS Dropout	0.11	0.04	0.00
	HS Graduate	0.24	0.21	0.00
	Some College	0.06	0.24	0.00
	College Graduate	0.01	0.25	0.00

**Table O-7: Short-Run Effects (% Change) of \$10,000 One-Time Transfer to Old Parents (Sensitivity Analysis)**

	Unanticipated or Anticipated	Avg. $i_1$	Avg. $i_2$	Some Coll+	Avg. $W_3$
<b>Baseline</b>	unanticipated	0.0	1.4	3.0	0.2
	anticipated	8.0	6.2	7.2	1.3
<b>b = 0</b>	unanticipated	0.0	2.8	4.9	0.3
	anticipated	7.5	6.5	6.7	1.2
<b>b = 0.5</b>	unanticipated	0.0	8.9	11.4	1.0
	anticipated	9.2	7.7	8.2	1.3
<b>b = -0.5</b>	unanticipated	0.0	0.2	0.0	0.0
	anticipated	5.6	4.7	3.6	1.1
<b><math>\gamma = 0.5</math></b>	unanticipated	0.0	0.9	0.5	0.1
	anticipated	7.8	6.2	7.3	1.0
<b><math>\pi_2 = 0</math></b>	unanticipated	0.0	6.6	11.1	0.8
	anticipated	10.5	6.9	7.5	1.1
<b><math>\zeta_1 = 0</math></b>	unanticipated	0.0	6.0	7.3	0.7
	anticipated	8.1	7.0	6.3	1.4
<b>full income</b>	unanticipated	0.0	5.5	6.9	0.6
	anticipated	8.8	7.4	7.6	1.4

Table O-8: Decomposition of Investment Gaps between Parental Income Quartiles 1 and 4 (Sensitivity Analysis)

	Investment Gaps			% Change Relative to Benchmark			
	Avg. i1	Avg. i2	Some Coll+	Avg. i1	Avg. i2	Some Coll+	
Baseline	Benchmark:						
	Unconditional	3057	7743	0.38			
	Conditional on child ability	2615	5924	0.28	-14.5	-23.5	-26.6
	Relax all borrowing limits:						
	Unconditional	3555	8174	0.33	16.3	5.6	-14.1
	Conditional on child ability	2480	3757	0.12	-18.9	-51.5	-67.6
b = 0	Benchmark:						
	Unconditional	3038	7447	0.42			
	Conditional on child ability	2591	5755	0.32	-14.7	-22.7	-23.8
	Relax all borrowing limits:						
	Unconditional	3452	6375	0.21	13.6	-14.4	-49.4
	Conditional on child ability	2254	2014	0.00	-25.8	-73.0	-100.0
b = 0.5	Benchmark:						
	Unconditional	2226	10351	0.54			
	Conditional on child ability	1904	8518	0.44	-14.5	-17.7	-18.5
	Relax all borrowing limits:						
	Unconditional	2837	7119	0.26	27.4	-31.2	-52.7
	Conditional on child ability	1818	1722	0.00	-18.3	-83.4	-100.0
b = -0.5	Benchmark:						
	Unconditional	4671	7613	0.34			
	Conditional on child ability	3855	5430	0.20	-17.5	-28.7	-39.5
	Relax all borrowing limits:						
	Unconditional	5270	8706	0.32	12.8	14.4	-6.5
	Conditional on child ability	3632	4260	0.10	-22.2	-44.0	-69.1
γ = 0.5	Benchmark:						
	Unconditional	1798	5079	0.26			
	Conditional on child ability	1519	3581	0.17	-15.5	-29.5	-36.3
	Relax all borrowing limits:						
	Unconditional	2285	6736	0.27	27.1	32.6	5.2
	Conditional on child ability	1658	3382	0.11	-7.8	-33.4	-59.6
π <sub>2</sub> = 0	Benchmark:						
	Unconditional	1195	7685	0.44			
	Conditional on child ability	984	5815	0.34	-17.7	-24.3	-23.6
	Relax all borrowing limits:						
	Unconditional	1462	5640	0.22	22.4	-26.6	-49.7
	Conditional on child ability	805	919	0.00	-32.6	-88.0	-100.0
ζ <sub>1</sub> = 0	Benchmark:						
	Unconditional	3569	9730	0.47			
	Conditional on child ability	3136	8087	0.39	-12.1	-16.9	-17.7
	Relax all borrowing limits:						
	Unconditional	4595	8943	0.34	28.8	-8.1	-27.5
	Conditional on child ability	3418	4499	0.15	-4.2	-53.8	-68.5
full income	Benchmark:						
	Unconditional	2870	9010	0.45			
	Conditional on child ability	2514	7435	0.37	-12.4	-17.5	-18.1
	Relax all borrowing limits:						
	Unconditional	3325	6838	0.23	15.8	-24.1	-49.0
	Conditional on child ability	2245	2262	0.01	-21.8	-74.9	-97.8

Table O-9: Intergenerational Ability and Investment Transmission (Sensitivity Analysis)

		Baseline	No effect of parental $h_3$ on child $\theta$	No correlation between parent and child $\theta$	Perfect correlation between parent and child $\theta$
Baseline	Intergen. corr. in $\theta$	0.31	0.31	0.00	1.00
	Intergen. corr. In $i_2$	0.52	0.46	0.29	0.85
	Intergen. corr. In lifetime earnings	0.29	0.26	0.19	0.44
$b = 0$	Intergen. corr. in $\theta$	0.24	0.24	0.00	1.00
	Intergen. corr. In $i_2$	0.45	0.40	0.26	0.84
	Intergen. corr. In lifetime earnings	0.27	0.25	0.17	0.47
$b = 0.5$	Intergen. corr. in $\theta$	0.29	0.29	0.00	1.00
	Intergen. corr. In $i_2$	0.50	0.44	0.27	0.85
	Intergen. corr. In lifetime earnings	0.29	0.26	0.18	0.44
$b = -0.5$	Intergen. corr. in $\theta$	0.30	0.30	0.00	1.00
	Intergen. corr. In $i_2$	0.50	0.44	0.25	0.85
	Intergen. corr. In lifetime earnings	0.30	0.27	0.18	0.46
$\gamma = 0.5$	Intergen. corr. in $\theta$	0.29	0.29	0.00	1.00
	Intergen. corr. In $i_2$	0.46	0.34	0.13	0.83
	Intergen. corr. In lifetime earnings	0.20	0.10	0.05	0.21
$\pi_2 = 0$	Intergen. corr. in $\theta$	0.28	0.28	0.00	1.00
	Intergen. corr. In $i_2$	0.42	0.42	0.24	0.82
	Intergen. corr. In lifetime earnings	0.23	0.23	0.14	0.45
$\zeta_1 = 0$	Intergen. corr. in $\theta$	0.31	0.31	0.00	1.00
	Intergen. corr. In $i_2$	0.55	0.49	0.33	0.84
	Intergen. corr. In lifetime earnings	0.33	0.30	0.22	0.47
full income	Intergen. corr. in $\theta$	0.27	0.27	0.00	1.00
	Intergen. corr. In $i_2$	0.51	0.47	0.33	0.83
	Intergen. corr. In lifetime earnings	0.30	0.28	0.21	0.48

**Table O-10: Short-Run Effects (% Change) of Increasing Borrowing Limits by \$1,500 (Sensitivity Analysis)**

	Relaxing Constraint on Young Parents					Relaxing Constraint on Old Parents				
	Avg. $i_1$	Avg. $i_2$	HS+	Some Coll+	Avg. $W_3$	Avg. $i_1$	Avg. $i_2$	HS+	Some Coll+	Avg. $W_3$
<b>Baseline</b>	1.7	1.1	-0.2	2.9	0.3	7.2	6.3	1.7	3.8	1.2
<b><math>b = 0</math></b>	2.3	1.6	0.5	2.2	0.3	5.7	5.8	3.6	4.5	1.0
<b><math>b = 0.5</math></b>	1.0	1.2	0.5	1.8	0.2	7.2	6.8	3.2	3.6	1.1
<b><math>b = -0.5</math></b>	1.2	0.9	0.6	1.8	0.2	4.7	4.3	0.0	1.3	1.0
<b><math>\gamma = 0.5</math></b>	2.2	1.2	-0.8	2.7	0.2	8.4	7.0	0.8	0.5	1.0
<b><math>\pi_2 = 0</math></b>	0.9	1.1	0.8	2.1	0.2	11.1	6.3	1.8	2.4	1.0
<b><math>\zeta_1 = 0</math></b>	2.5	1.6	0.6	2.6	0.4	6.6	5.4	0.7	2.9	1.1
<b>full income</b>	2.5	1.6	0.7	2.8	0.3	7.1	6.7	4.3	4.2	1.1



**Table O-11: Short-Run Effects (% Change) of Fully Relaxing All Borrowing Limits (Sensitivity Analysis)**

	<b>Avg. <math>i_1</math></b>	<b>Avg. <math>i_2</math></b>	<b>HS+</b>	<b>Some Coll+</b>	<b>Avg. <math>W_3</math></b>
<b>Baseline</b>	72.5	63.2	12.5	31.0	11.7
<b>b = 0</b>	80.7	71.5	17.1	44.9	12.8
<b>b = 0.5</b>	96.9	82.4	15.7	54.7	13.7
<b>b = -0.5</b>	48.7	45.8	4.8	12.6	10.1
<b><math>\gamma = 0.5</math></b>	68.1	59.5	11.9	25.8	8.5
<b><math>\pi_2 = 0</math></b>	116.6	69.9	21.4	41.5	11.4
<b><math>\zeta_1 = 0</math></b>	76.9	66.9	4.7	35.8	13.1
<b>full income</b>	88.7	86.0	16.9	68.0	14.9

**Table O-12: Short-Run Effects (% Change) of Early and Late Investment Subsidies (Sensitivity Analysis)**

	Policy	Avg. $i_1$	Avg. $i_2$	HS+	Some Coll+	Coll Grad	Avg. $W_3$
Baseline	Announced early:						
	$s_1 = 0.10$	63.6	22.5	0.8	13.5	43.2	6.5
	$s_2 = 0.026$	13.0	25.9	15.9	17.7	39.3	3.6
$b = 0$	Announced late:						
	$s_2 = 0.026$	0.0	15.4	15.9	15.4	15.1	1.6
	Announced early:						
$b = 0.5$	$s_1 = 0.10$	48.7	16.8	0.6	7.7	36.2	4.9
	$s_2 = 0.028$	10.5	19.3	17.6	10.4	29.8	2.6
	Announced late:						
$b = -0.5$	$s_2 = 0.028$	0.0	9.6	17.6	8.3	6.2	0.9
	Announced early:						
	$s_1 = 0.10$	72.3	8.8	0.4	3.3	18.7	4.2
$\gamma = 0.5$	$s_2 = 0.015$	3.8	13.4	12.9	5.7	20.9	1.5
	Announced late:						
	$s_2 = 0.015$	0.0	12.6	12.9	5.6	19.0	1.3
$\pi_2 = 0$	Announced early:						
	$s_1 = 0.10$	43.1	21.8	-0.4	8.4	48.0	6.9
	$s_2 = 0.048$	16.6	22.5	18.7	6.1	42.5	4.1
$\zeta_1 = 0$	Announced late:						
	$s_2 = 0.048$	0.0	3.8	18.6	0.0	0.0	0.3
	Announced early:						
full income	$s_1 = 0.10$	107.2	44.8	0.5	16.6	112.5	8.7
	$s_2 = 0.024$	38.9	46.4	16.8	24.4	95.3	6.0
	Announced late:						
$\zeta_1 = 0$	$s_2 = 0.024$	0.0	13.0	16.8	21.4	0.0	1.3
	Announced early:						
	$s_1 = 0.10$	86.6	13.1	-0.2	3.0	30.9	3.9
full income	$s_2 = 0.012$	9.4	12.7	14.1	4.1	20.1	1.6
	Announced late:						
	$s_2 = 0.012$	0.0	8.9	14.1	3.9	10.5	0.9
full income	Announced early:						
	$s_1 = 0.10$	52.9	13.0	0.2	9.4	23.7	5.2
	$s_2 = 0.022$	6.5	18.7	17.0	11.8	26.1	2.4
full income	Announced late:						
	$s_2 = 0.022$	0.0	15.5	17.0	11.0	18.9	1.6
	Announced early:						
full income	$s_1 = 0.10$	59.1	14.8	0.8	9.8	27.2	5.1
	$s_2 = 0.021$	8.2	18.6	17.5	10.7	26.4	2.4
	Announced late:						
full income	$s_2 = 0.021$	0.0	14.5	17.5	9.4	17.5	1.4