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Marriage, Divorce and Wage Uncertainty along the Life-cycle

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Marriage, Divorce and Wage Uncertainty along the Life-cycle

PRELIMINARY

Edoardo Ciscato[♭]

June 19, 2018

Abstract

The American family underwent important transformations in the last decades. Mating patterns changed, college graduates and high earners marry with each other more and more frequently. On the other hand, those at the bottom of the wage and schooling distributions have become more and more likely to stay single, and, once married or cohabiting, more likely to break up. This increasing gap in family achievements has important implications for both income and consumption inequalities, as well as intergenerational mobility. In this paper, I aim to quantify the importance of the marriage market as a channel of inequality, both at household and individual level. I build on the matching literature and set up a model of marriage, divorce and remarriage along the life-cycle in order to reproduce the afore-mentioned aggregate trends and understand the underlying drivers. In the model, risk-averse agents get married in order to benefit from joint public good expenditure, but economic gains from marriage are volatile due to labor market shocks. I show that the underlying structure of preferences and of the meeting technology are identified with matched data on the distribution of couples' and singles' traits, jointly with data on newlyweds and divorcees. I propose an estimation method based on indirect inference and estimate the model with PSID data. Preliminary findings suggest that differences in the productivity of household public good expenditure appear as a key driving force behind differentials in the odds of staying single, mating patterns, and, ultimately, household income inequality.

Keywords: marriage market, divorce, inequality, life-cycle, wage uncertainty, search and matching.

JEL Classification: D13, J11, J12.

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1 Introduction.

In the last decades, Americans experienced important changes in family life, and social scientists have been looking at aggregate marriage trends in order to understand what is going on at the household level. In a time when women caught up with men in terms of educational achievements, educational assortativeness has increased on the marriage market (Greenwood et al., 2014; Eika et al., 2014). From Gary Becker's standpoint, this suggests that gains from marriage have changed, and in particular that the structure of complementarities between spouses' inputs has changed. Chiappori et al. (2017) argue that the increased importance of the spouses' human capital for family public good investments, and particularly for childrearing, explains the increased taste for homogamy on the marriage market. This is consistent with time-use trends for highly educated couples, where both spouses are found to spend more and more time with their children. Hence, understanding mating patterns matters to explain both cross-sectional household income inequality and intergenerational mobility.

However, not only the answer to the question *who marries whom* has changed: nowadays, there is another key question that needs to be addressed, *who marries, and who does not*. In the last decades, changes in family life did not affect people equally along the wage and schooling distribution. Lundberg and Pollak (2014) and Carbone and Cahn (2014) both provide a complete picture of this emerging gap in family achievements: lower educated individuals are much less likely to be married than college graduates and, once married, more likely to divorce. While those at the bottom of the distribution are more likely to choose cohabitation over marriage, their overall probability of remaining single is still higher than for those at the top. Similar differences are observed along the wage distribution, with high earners being more likely to be married and experiencing lower divorce rates. Autor et al. (2017) look at regional labor market shocks expected to hit men particularly hard with respect to women: these shocks have negative consequences on marriage, fertility and children's welfare. Finally, Greenwood et al. (2016) arguably provide the most complete theoretical framework to explain the marriage-market-related channels through which changes in household technology, preferences, and labor market conditions propagate and create larger inequalities. The works mentioned in this paragraph all contain a key takeaway: they all point at the possibility that, nowadays, economic gains from marriage are too low, too volatile, and possibly nonexistent, for a non-negligible part of the American population.

Yet, these have not been the only changes: the median age of first marriage, after hitting a historic low in the post-war period, has been increasing. Together with life expectancy, divorce rates for older couples and remarriage rates have also been growing, and people can now reasonably expect to marry more than once in their lifetime (Browning et al., 2014; Kennedy and Ruggles, 2014). It is time to question whether the idea of the marriage market as a static, once-in-a-lifetime assignment problem is still in touch with reality.

All these phenomena call for a new model of the marriage market where matching is dynamic - in the sense that *timing* matters and *remarriage* is allowed - and occurs in a world of uncertainty where people age and experience labor market shocks. In this paper, I build on the matching literature and bring in new modeling ingredients in order to study marriage, divorce and remarriage decisions along the life-cycle. The model aims to rationalize the main marriage market trends observed in the data and described in the previous paragraphs. First, it has to explain sorting patterns along age, human capital and wages. Second, it has to reproduce the differentials in the likelihood of being single conditionally on one's current traits. Third, it also has to explain the degree of marriage instability observed in the data, i.e., the marriage and divorce rates for different groups of individuals. Fourth, it has to be consistent with microeconomic behavior at the household level, and particularly with female labor supply.

The model outlined in this paper is empirically tractable, and the main parameters driving the key equilibrium outcomes can be shown to be identified with data on stocks, flows, and labor supply. The empirical strategy draws from a recent work by Goussé et al. (2017). I estimate the model on a subsample of individuals born between 1956 and 1965 from the Panel Study of Income Dynamics data (PSID). In this draft, I present some preliminary results that show, consistently with Chiappori et al. (2017), that families where both parents are college graduates have stronger incentives to invest in public good. I intend to produce reliable estimates on which to base a counterfactual exercise to single out the role of the primitives of the model - household preferences, wage and schooling differentials, wage uncertainty - on the equilibrium outcome, and particularly on female labor supply, inequalities in terms of household income and household public good expenditure.

The relationship between mating patterns and inequality has been largely discussed in the literature[1]. However, most works so far focus on the relationship between the cross-sectional composition of the married population in prime or middle adulthood and the cross-sectional household income distribution. In this paper, I aim to assess these differences along the whole life-cycle of a cohort of individuals, in order to understand how the timing of marriage and divorce amplify or smooth inequalities as the cohort gets older. This also allows to compute inequalities in terms of permanent income. Finally, note that this structural approach also allows to analyze inequality at the individual level, since the model links marriage market behavior with intra-household decisions about how to share the household budget (see Lise and Seitz, 2011).

2 Data: the PSID.

The empirical analysis is conducted with data from the Panel Study of Income Dynamics (PSID). Most trends mentioned in the introduction can also be observed in the PSID sample. I mainly work with a sample including people born between 1956 and 1965.

2.1 Sample Selection.

I keep observations from the SRC and Census samples, thus excluding the Immigrant and Latino samples. The empirical analysis focuses on individuals born between 1956 and 1965. I follow this cohort up to the 50th birthday, the last available wave of the PSID being in 2015. Since 1997, a new PSID wave was published every two years: consequently, even for waves prior to 1997, I only keep odd years in the panel. My longitudinal sample is made of about 1,200 sampled men and women

[1] Early theoretical frameworks to study marriage, income inequality and intergenerational transmission of human capital include Fernández and Rogerson (2001) and Fernández et al. (2005). Earl empirical assessments of the relevance of mating patterns on inequality include Burtless (1999) and Cancian and Reed (1998). More recently, Greenwood et al. (2014) and Eika et al. (2014), mentioned in the main text, revived interest in this topic.

per year. The panel is balanced for sampled individuals. However, temporary non-sample individuals living with the sampled do not enter the sample themselves, and thus they are not followed once they quit the household.

For descriptive purpose, I also use another sample of individuals born between 1946 and 1955. Finally, always for descriptive purpose, I also build another sample, a series of cross-sections including households with single male heads aged between 25 and 50, single female heads aged between 23 and 48, and married couples where the wife is aged between 23 and 48 (regardless of the age of the husband).

2.2 Main Variables.

The construction of some variables is of particular interest.

- *Conjugal status*: PSID respondents are associated with a household identifier and a "relationship-to-head" variable. If a person is head of household and not living with a partner, I consider him/her to be single. In addition, if he or she is living in a household and is not head, nor partner of the head, I also consider him/her to be single[2]. I identify couples through the presence of an individual who claims to be the head's partner. Heads are mostly men, and partners are mostly women. The latter are labeled as either legal wives, female cohabitors who have lived with the head at least 12 months, or first-year cohabitors. In the few cases where the woman is the head, her partner is labeled as either legal husband, or first-year cohabitor. Whenever an adult head and an opposite-sex partner of any of the types listed above are present in the same household, I consider them to be a couple. Whenever the partner is the legal spouse, I consider them to be married.

- *New couples*: changes in conjugal status occurring between two PSID waves help identify the formation of new couples. When a respondent is single in wave $t-1$ and in a relationship in wave t, a new couple is formed. If the two partners are legally married, then here it is a married couple. Transitions from cohabitation to marriage are also observed in this way.

- *Breakups*: in the same way, if a couple is observed living together in wave t and at least one of the two partners is observed living alone in $t+1$, then the couple is about to break up. If the couple was married, the breakup is actually a divorce: I do not make a difference between divorce and separation as the legal duties of divorcees are not taken into account in the analysis. Note how this way of identifying dissolving couples allows to gather information on both spouses' characteristics. In fact, a drawback of most restrospective marital history datasets is that they do not contain detailed information about former partners. In spite of this, there is an attrition issue due to the fact that, typically, only one spouse is followed

- *Education*: I divide respondents into two categories, those with a college degree and those without. Education is assumed to be time-invariant, and corresponds to the maximum level observed in the age bracket considered.

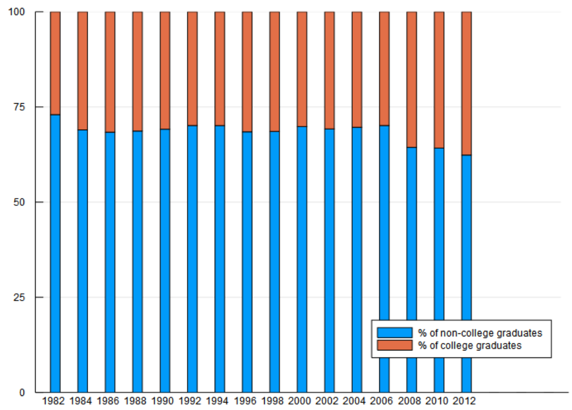
- *Hourly wage*: to build hourly wage, I use information on the year prior to the interview. Hourly wage rates are obtained by dividing the (deflated) total yearly labor income by the total yearly

[2]Secondary families are few in the sample.

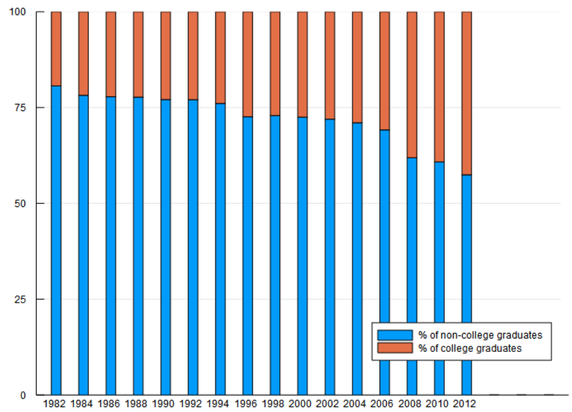
number of hours worked (see e.g. Rupert and Zanella, 2015). While information on labor income and labor supply is always available for surveyed heads and partners, it is never available for other members of the household: hence, for some sampled individual-year observations, labor market information is missing, especially for earlier waves when individuals are young and might still be living with their parents.

2.3 Descriptive Statistics.

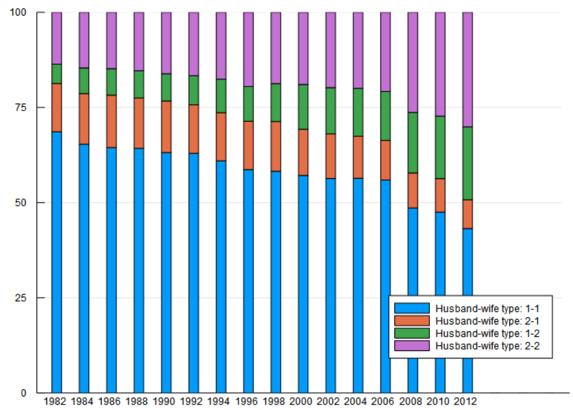
From PSID data, it is possible to glance how the percentage of college graduates increased faster for women than for men (see graph 2.1). The joint distribution of partners' college degrees for married and cohabiting couples suggests two key facts: i) the percentage of couples where both partners hold a college degree approximately doubled in 30 years; ii) the number of women that "marry down" approximately tripled. These changes can be explained, at least in part, by the shifts in the marginals, i.e., the overall increase in the number of college graduates and the reversal of the gender gap.



(a) Percentage of college graduates among men



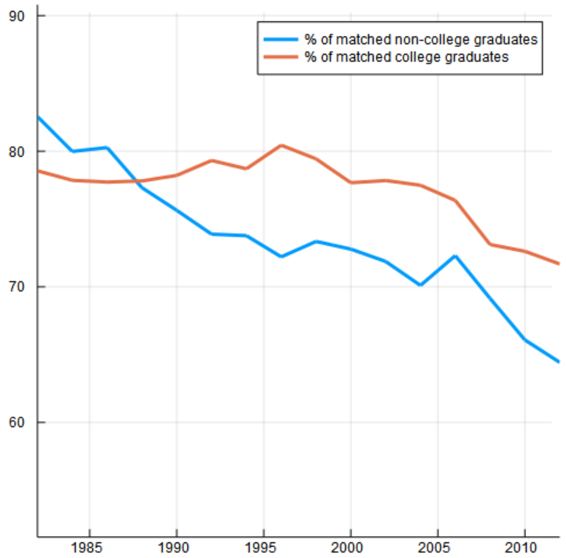
(b) Percentage of college graduates among women



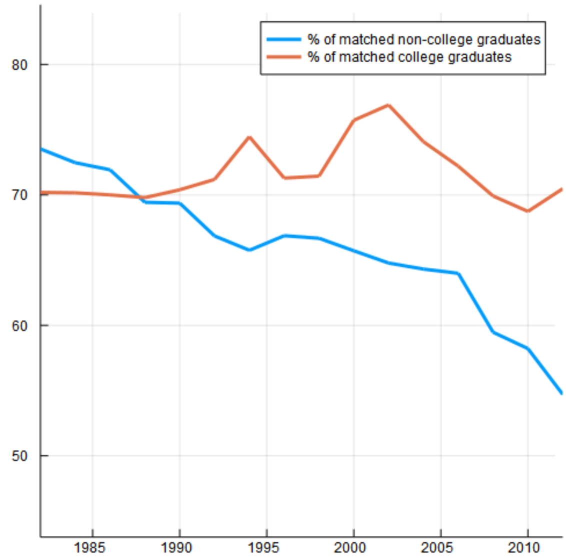
(c) Married and cohabiting couples by partners' college degrees (1 corresponds to no college degree, 2 to college degree)

Figure 2.1: Distribution of schooling attainments

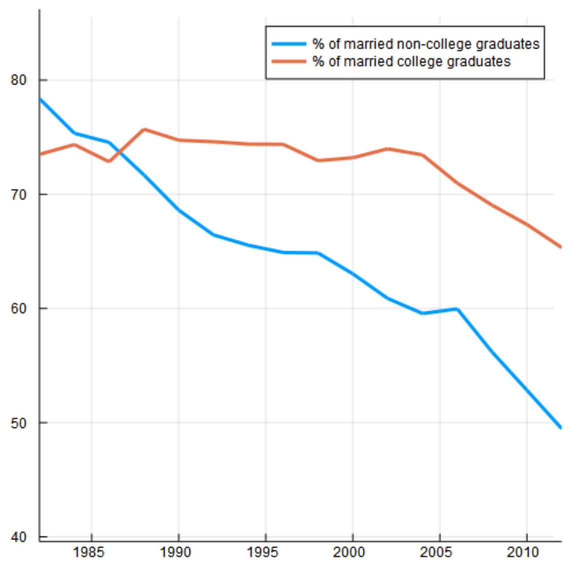
The set of graphs 2.2 shows that the drop in the number of individuals engaged in a relationship was much stronger for the less educated. Starting from a situation of substantial parity in the 1980s, low educated men and women have progressively lost ground since then. The gap is wider when considering legally married couples, suggesting that a larger share of non-college graduates is choosing cohabitation over marriage. While college-educated women did not seem to experience any decrease after all - at least when including cohabiting couples - those without a college degree saw their probability of being in a couple decrease by almost 25% and their probability of being married by about 38%.



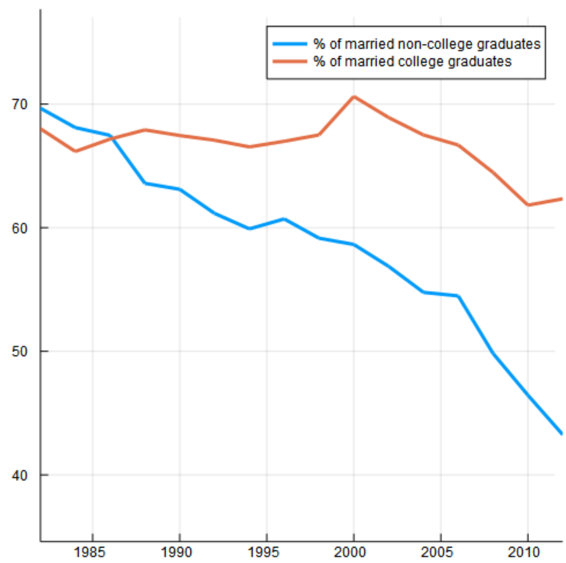
(a) Men in a couple



(b) Women in a couple



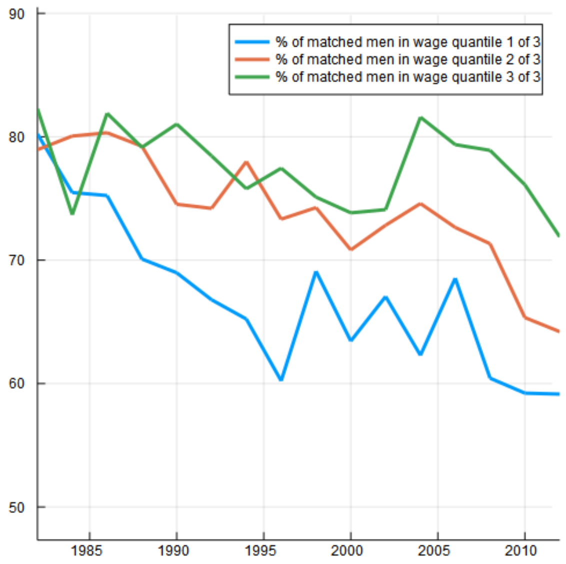
(c) Married men



(d) Married women

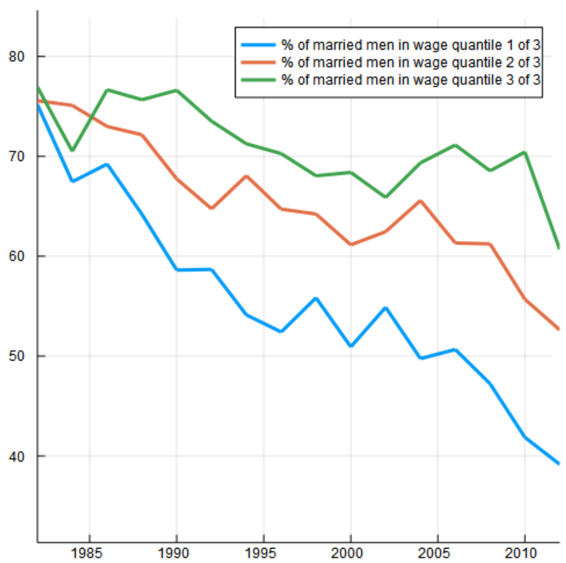
Figure 2.2: Percentage of matched individuals by education

Similar trends can be observed along the wage distribution for a subsample of employed individuals (see set 2.3). While in the early 1980s the odds of being cohabiting or married were very similar for men in the upper, middle and lower third of the wage distribution, the gap progressively widened over time, and is especially large when looking at married couples. Men in the lower third of the wage distribution were 46% less likely to be married in 2013 relatively to 1983. Trends for women are similar, but the gap is not as wide. However, employment rate for women is smaller.

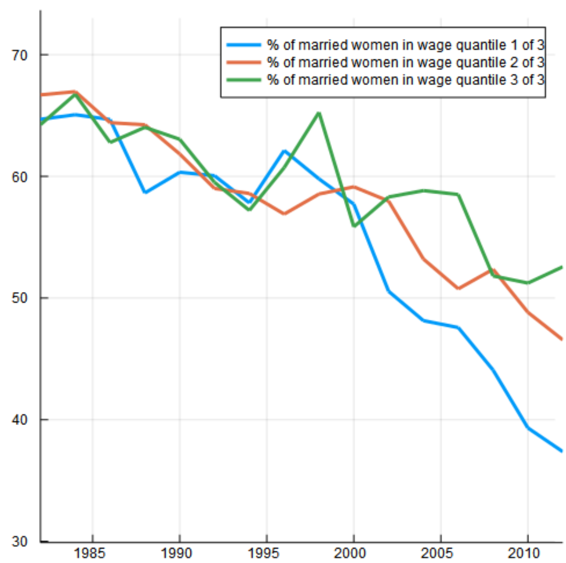


(a) Men in a couple

(b) Women in a couple



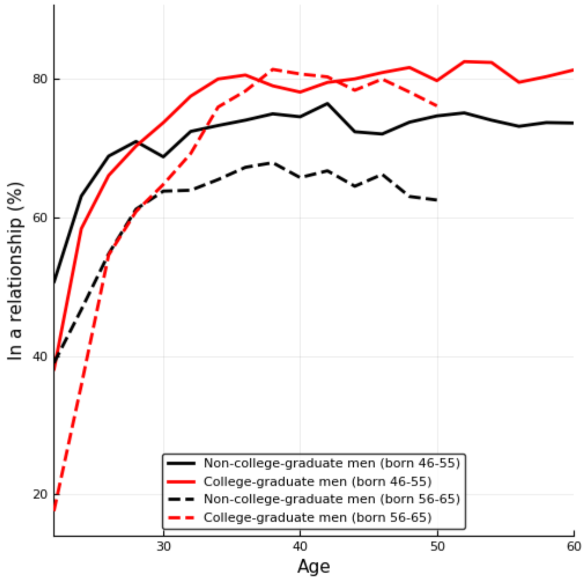
(c) Married men



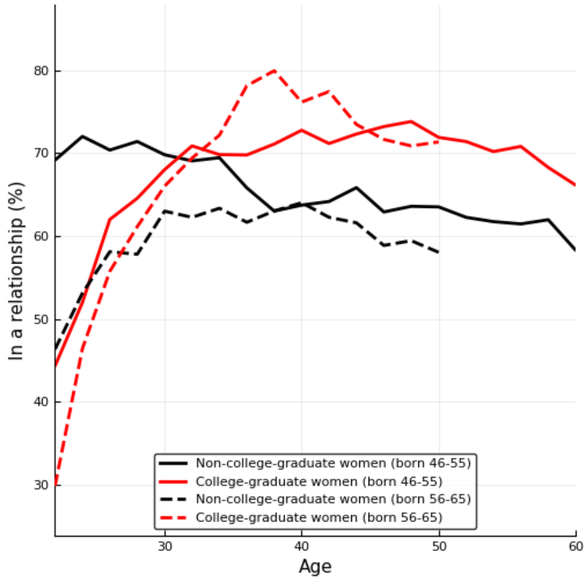
(d) Married women

Figure 2.3: Percentage of matched individuals by wage quantile

Plots 2.4 show how differentials in the odds of being single change with age, education, and across cohorts. Overall, men marry later, but while the college-educated born between 1956 and 1965 are, by age 35, as likely to be in a couple as those born between 1946 and 1955, the non-college-educated born between 1956 and 1965 fail to keep up with those born 10 years earlier. For women, the picture is similar. Interestingly, note that trends are all slightly downsloping starting from middle adulthood, suggesting that women may struggle to find a new partner when older.



(a) Men in a couple



(b) Women in a couple

Figure 2.4: Percentage of individuals in a couple by sex, age and education for given cohort.

Graph 2.5 shows how the likelihood of ending a relationship evolved over time. Divorce rates are computed as the number of divorces in the last two years divided by the relative "population at risk", i.e., the number of married couples as measured at the beginning of the two-year period. Similarly, breakup rates are computed as the number of (married or cohabiting) couples ending their relationship divided by the relative population at risk. The reader may notice that these rates are higher than those that are often presented in other studies (e.g. Kennedy and Ruggles, 2014): this is due to several reasons, primarily because divorce rates usually consider the number of divorces per year, and not every two years, but also because this paper does not treat divorce and separation separately, and also focuses on people in prime and middle adulthood. The trends are noisy, as the sample is small. However, they show that the rate of divorce might have increased only slightly in the last 30 years. When considering cohabiting couples, the rate of breakup increased by about 33%.

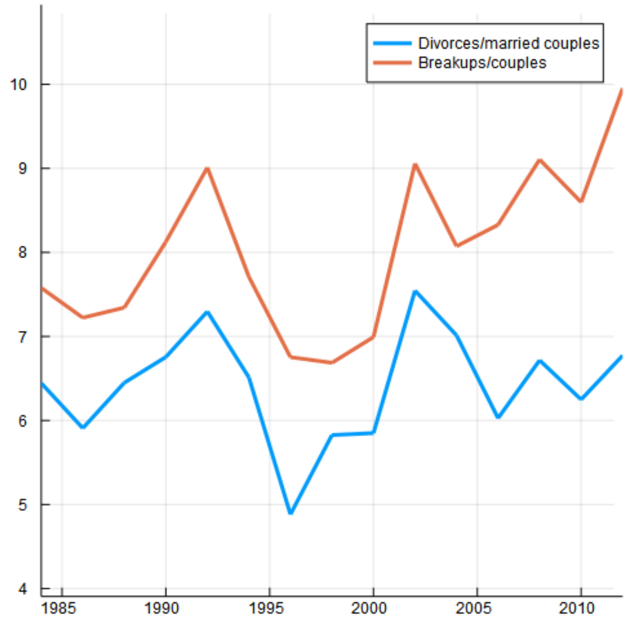
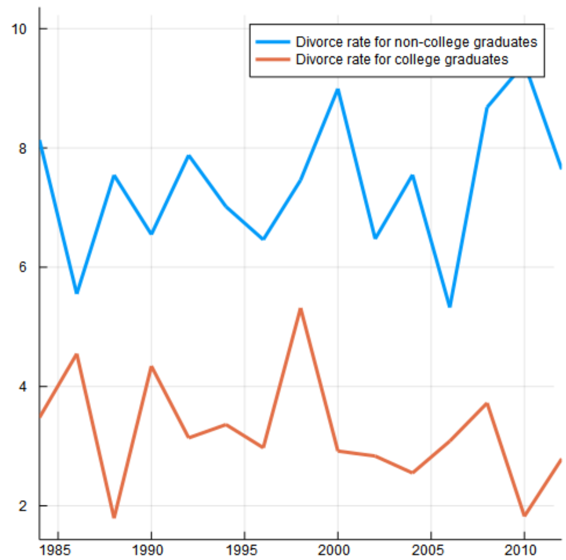
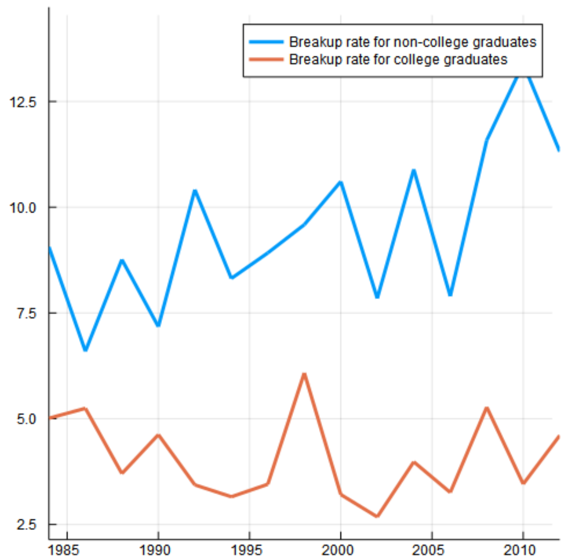


Figure 2.5: Rate of breakup and divorce

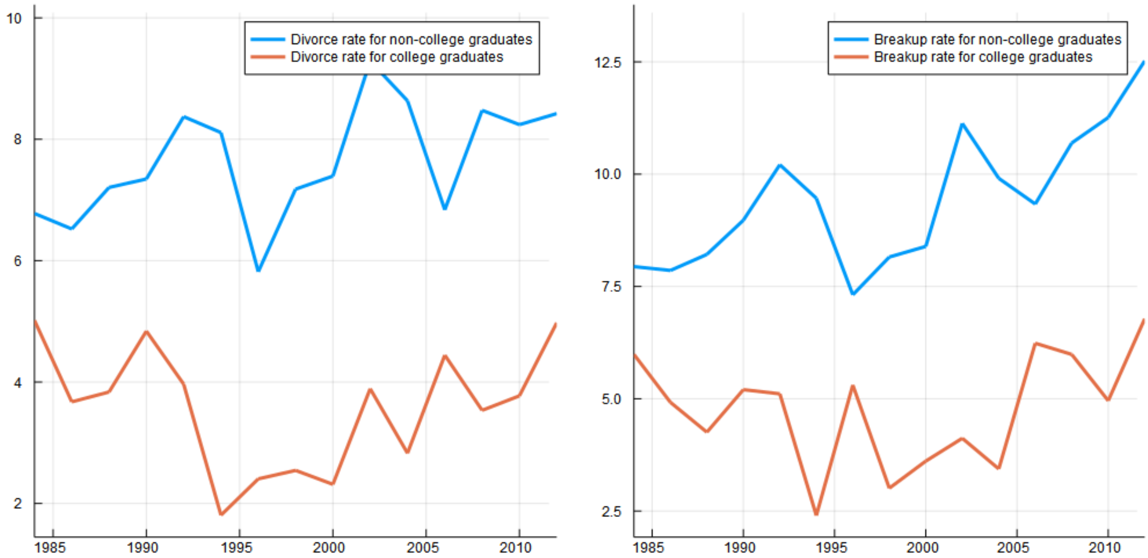
Trends decomposed by partners' schooling levels in figure 2.6 show that the low educated have always been at higher risk of breakup and divorce. However, the likelihood of ending a relationship has been increasing relatively to the high educated. Breakup rates have been outstandingly growing for non-college graduates, have reverted to 1980s levels for college graduate women after a drop in the 1990s, and have been actually decreasing for college graduate men. Divorce rates follow similar trends.



(a) Divorce rate, men



(b) Breakup rate, men



(c) Divorce rate, women

(d) Breakup rate, women

Figure 2.6: Divorce and breakup rates by education

3 The Model.

The theoretical framework extends the original two-sided search-and-matching model by Shimer and Smith (2000) in the vein of Goussé et al. (2017). Household production includes a random parameter that is drawn upon a meeting between two singles of opposite sex, who then decide whether to marry or not. Divorce is endogenized by allowing this random parameter to be updated over time. In contrast with previous literature, I introduce some key elements that allow to bring the empirical analysis to the next level. First, I introduce aging: agents get older as time goes by, and age influences their odds of marriage, divorce and remarriage. Second, I assume agents are risk-averse and experience wage shocks along the life-cycle: wage uncertainty partly explains marital instability, as economic gains from marriage may disappear following a wage shock.

Before turning to the setup, let me anticipate some key implications of these assumptions. On the one hand, the model is one of overlapping generations, where only some strong assumptions on market entry and exit allow the possibility of a steady-state equilibrium. On the other hand, life-cycle dynamics do not require the wage process to be stationary, nor new marriages to be outbalanced by an equal amount of divorces for a given group of agents: (exogenous) market exit allows to preserve stationarity even if different groups within the same cohort take strongly diverging paths in terms of both earnings and family achievements.

3.1 Heterogeneous Agents and Aging.

Men are *ex-ante* heterogeneous and are associated with a publicly observable *type* i, a vector comprising the following elements:

- a time-invariant human capital type h_i;
- age a_i;

- current wage w_i, which changes over time according to a random process described in the next subsection.

Similarly, the type of a woman is given by $j = (h_j, a_j, w_j)$. The men's (women's) set of types is named $\mathcal{I}$ ($\mathcal{J}$). Note that the time subscript t is unnecessary, since I will focus on the steady-state equilibrium: time only matters from an individual point of view due to aging and wage dynamics, but is not associated with an aggregate state of the world.

Aging individuals face an exogenous probability of exiting the marriage market. If a man's age is equal to a in t, he will exit the market with probability $1 - \psi_m(a_{i,t})$. If he stays in the market, he grows one-year older, so that his new age is given by $a' = a + 1$. A similar process governs women's aging, with a different vector of survival probabilities $\psi_f(a_j)$. In addition, assume agents enter the market at age $\underline{a}_g$, $g \in \{m, f\}$, and eventually leave with probability one at $\bar{\bar{a}}_g$, i.e., $\psi_g(\bar{\bar{a}}_g) = 0$. Agents discount future at rate $1/\beta - 1$.

3.2 Wage Process

Wages follow a Markov process, so that a man transits from type i to type i' with probability $\pi_{i,i'} \equiv Pr(i|i')$, where $a_{i'} = a_i + 1$. The definition of $\pi_{i,i'}$ also implies that the corresponding transition matrix is allowed to depend on the full vector i. In practice, an agent draws his future wage from a distribution that depends not only on his current wage, but also on his human capital type h_i and his age a_i. Analogous considerations hold for women, whose probability of transiting from j to j' is given by $\pi_{j,j'}$.

Note that the wage process does not need to be stationary due to life-cycle dynamics being taken into account. Importantly, for a given cohort of people, mean wages are allowed to depend on age, and cross-sectional wage dispersion may increase along the life-cycle. While I allow for time-invariant traits and age to directly influence the wage's conditional distribution, I do not attempt to decompose wages in multiple factors, and in particular to distinguish between a permanent and a temporary component as it is common in the literature (see Meghir and Pistaferri, 2011). Therefore, wage shocks need to be interpreted as permanent.

Finally, note that the assumption that the wage process is fully exogenous - and in particular that it is not affected by the agent's marital status - is likely to be highly counterfactual. Joint household labor supply decisions are likely to have an impact on the human capital accumulation, especially if household specialization plays an important role as a motive to marry. These important limitations are discussed in the conclusion.

3.3 Marital Status and Aggregate Stocks.

It is useful to define aggregate measures that count the number of individuals by type in the population. Define p_m over $\mathcal{I}$ and p_f over $\mathcal{J}$ as the marginal distributions of men's and women's types respectively. In order for marginal distributions to be constant over time, I impose that in each period there is an inflow of young men $p_m(i)$, $a_i = \underline{a}_m$, and young women $p_f(j)$, $a_j = \underline{a}_f$, whose composition is constant over time, which I formalize as follows:

$$p_m(i) \perp\!\!\!\perp t \text{ if } a_i = \underline{a}_m \tag{3.1}$$

$$p_f(j) \perp\!\!\!\perp t \text{ if } a_j = \underline{a}_f \tag{3.2}$$

In addition, the total number of outflows must be equal to total number of inflows by sex, so that the gender ratio is constant:

$$\int_{a_i=\underline{a}_m} p_m(i)di = \int (1-\psi_m(a_i))p_m(i)di \tag{3.3}$$

$$\int_{a_j=\underline{a}_f} p_f(j)dj = \int (1-\psi_f(a_j))p_f(j)dj. \tag{3.4}$$

In any period t, an agent is either married or single. To keep track of aggregate stocks in the marriage market, I introduce measures n_m over $\mathcal{I}$, n_f over $\mathcal{J}$ and m over $\mathcal{I} \times \mathcal{J}$ that count male singles, female singles and couples respectively. The *matching* (n_m, n_f, m) must respect the usual accounting restrictions:

$$p_m(i) = n_m(i) + \int m(i,j)dj \tag{3.5}$$

$$p_f(j) = n_f(j) + \int m(i,j)di. \tag{3.6}$$

Finally, I define the following aggregate counts: $N \equiv \iint m(i,j)didj$, $N_m \equiv \int n_m(i)di$, $N_f \equiv \int n_f(j)dj$, the total number of couples, male singles, and female singles respectively. Note that conditions (3.5) and (3.6) need to hold regardless of whether the economy is at the stationary state. However, throughout this paper I will not deal with the model's out-of-steady-state dynamics.

3.4 Household Problem.

Couples need to choose an allocation (q_m, q_f, l_f, Q), where q_m and q_f are the husband's and the wife's private consumption shares respectively, l_f is the wife's labor supply, and Q is public good consumption. In addition, I normalize $0 \leq l_f \leq 1$ and impose that l_f is chosen from a discrete set $\mathcal{L}$, while the husband's labor supply is fixed to $\max \mathcal{L}$. Singles only have access to the private good: single men and women consume q_m^0 and q_f^0 respectively, and their labor supply is also fixed to $\max \mathcal{L}$ regardless of their sex.

The problem is *static*, choice variables in t do not affect the future state of the couple, nor of the spouses taken individually. In particular, savings and human capital accumulation are not considered: extensions are discussed in the conclusion. Hence, a woman's private budget only depends on her current private income, and her individual budget constraint is given by $w_j l_f = q_f + t_f$, where t_f is her contribution to the household public good when she is married. Similarly, a man's budget constraint is given by $w_i = q_m + t_m$, so that the total expenditure on public good is given by $Q = t_m + t_f$. Note that t_f is unrestricted, and that $t_f < 0$ implies that the husband is actually transferring money into the wife's pocket.

Single women enjoy utility flows $v_f^0(j) \equiv \max_{q_f^0} u_f^0(q_f^0; j)$, and single men enjoy $v_m^0(i)$. In contrast, married agents are associated with utility functions $u_g(q_g, Q, l_f; i, j)$, $g \in \{m, f\}$, and make *collective* decisions about the level of public good expenditure and the wife's labor supply (Chiappori, 1988, 1992). The amount l_f of hours worked by the wife is the result of bargaining due to the presence of both potential public benefits of selecting a stay-at-home wife and the wife's private incentives reflecting her personal time-use taste. The collective choice of the allocation implies a *sharing rule* consisting of both public good expenditure shares (t_m, t_f) and the wife's labor supply l_f.

The collective problem can be thought of as a two-step decision-making process: once the couple has reached an agreement on the sharing rule and on the wife's labor supply, each spouse is free to spend the rest of his/her private income on individual consumption (Blundell et al., 2005). It is thus convenient to define the following *conditional indirect utility functions*:

$$\phi_m(t_m, t_f, l_f; i, j) \equiv u_m(w_i - t_m, t_m + t_f, l_f) \tag{3.7}$$

$$\phi_f(t_m, t_f, l_f; i, j) \equiv u_f(w_j - t_f, t_m + t_f, l_f). \tag{3.8}$$

Now assume that ϕ_m and ϕ_f verify the Transferable Utility property. A well-known consequence of this assumption is that, in a static collective framework, the demand for public good, named $Q^*(l_f; i, j)$, does not depend on the sharing rule. In other words, while the spouses disagree on the respective contributions (t_m, t_f), they perfectly agree on the share of household budget to spend on the public good. In particular, the couple will choose a Q such that the allocation lies on the Pareto frontier, conditionally on the choice of l_f.

A second implication is that the $|\mathcal{L}|$ feasible Pareto frontiers do not intersect[3]. The choice of l_f also involves some (separable) random taste shocks that will be described in the next section, but the key idea is that the couple is more likely to choose l_f on the outermost frontier. The underlying intuition is that, while the wife might be "unhappy" about the selected l_f, she can always be compensated through a more favorable division of the public good expenditure. These Pareto frontiers can be described as follows: each level of female labor supply $l_f \in \mathcal{L}$ is associated with a constant $\Gamma(l_f; i, j)$, which stands for the level of the $-45°$ line obtained after transforming the Pareto frontier as follows:

$$\Gamma(l_f; i, j) = g_m(\phi_m(t_m, t_f, l_f; i, j)) + g_f(\phi_f(t_m, t_f, l_f; i, j)), \tag{3.9}$$

where g_m and g_f are continuous and increasing functions. In practice, the choice of g_m and g_f is meant for t_m and t_f to add up to the constant $Q^*(l_f; i, j)$. The existence of g_m and g_f is a necessary and sufficient condition for the TU property to hold (Chiappori and Gugl, 2014; Demuynck and Potoms, 2018).

However, another key implication of the Transferable Utility property is that it is impossible to recover the sharing rule (t_m, t_f), i.e., the exact point on the Pareto frontier chosen by the spouses. The solution $(t_m^*(l_f; i, j), t_f^*(l_f; i, j))$ can only be pinned down if the distribution of power within the household is known. As suggested by Becker (1973), the sharing rule within the couple responds to shifts in supply

[3]Note that, in presence of discrete labor supply l_f, this also means that preferences must be separable with respect to private good consumption and labor supply, although they do not need to be separable with respect to public good consumption and labor supply.

and demand of mates of a given type in the marriage market. While the presence of search frictions has important implications in this regard, the intuition is still largely valid in search-and-matching models.

3.5 Splitting Rule.

Exactly as in the seminal paper by Shimer and Smith (2000), the presence of search frictions relieves the pressure of the marriage market on a matched couple. For instance, consider a man who proposes to a woman and offers her a marriage contract that makes her just indifferent between marrying him and keeping on searching. Due to search frictions, she cannot turn to another similar man and agree on a better deal where she extracts a slightly larger share of surplus. In fact, waiting might be too costly, and, if the surplus is positive, the woman does have an incentive to accept. Yet, the marriage market still plays a crucial role in determining whether there is a set of allocations which made both candidates better off together than singles, and both the meeting and the household technology are the key primitives explaining the equilibrium sorting patterns. In some markets, marriage surplus can be small and competition might greatly reduce the role of within-couple bargaining; in others, the reverse can be true.

In other words, a crucial implication of the presence of search frictions is that it is not possible to pin down Pareto weights from matching behavior. In order to understand how the couple selects the optimal sharing rule $t_g^*(l_f;i,j)$, let me first introduce the relevant marital payoffs at the time of bargaining. These are given by the present discounted value of all expected flows in the future, once accounted for the possibility of breakups. Define $(W_m(t_m,t_f,l_f;i,j), W_f(t_m,t_f,l_f;i,j))$ as the *present discounted value of marriage* for a man and a woman in a couple (i,j) under sharing rule (t_m,t_f) and with labor supply l_f. In addition, define $(V_m^0(i), V_f^0(j))$ as the respective *present discounted value of singlehood*. The latter constitute the bargaining breakpoint, and are taken as exogenous at this stage. Call $(t_m^*(l_f;i,j), t_f^*(l_f;i,j))$ the solution to the bargaining process for a couple (i,j) conditional on labor supply l_f. The respective marriage payoffs are

$$V_m(l_f;i,j) \equiv W_m(t_m^*(l_f;i,j), t_f^*(l_f;i,j), l_f;i,j) \tag{3.10}$$

$$V_f(l_f;i,j) \equiv W_f(t_m^*(l_f;i,j), t_f^*(l_f;i,j), l_f;i,j) \tag{3.11}$$

and are chosen according to the following *surplus splitting rule*:

$$V_f(l_f;i,j) - V_f^0(j) = \theta(i,j)\left(V_f(l_f;i,j) - V_f^0(j) + V_m(l_f;i,j) - V_m^0(i)\right) \equiv \theta(i,j)S(l_f;i,j) \tag{3.12}$$

where S is the *total marriage surplus*. In other words, in a couple (i,j), the ratio of the wife's surplus to the husband's surplus is a constant $\theta(i,j)/(1-\theta(i,j))$. Note how it is necessary to introduce a further bargaining element to the framework in order to close to model, which results in further exogenous parameters to deal with. In contrast to Shimer and Smith (2000) and Goussé et al. (2017), who assume couples select a sharing rule through Nash bargaining, I use a simpler rule which turns out to be easier to deal with in terms of computations. As concerns the bargaining parameter $\theta(i,j)$, note that one could think of it as a parametric function of spouses' traits, and even let it depend on some (time-invariant) *distribution factors* in the spirit of Browning and Chiappori (1998). Eventually, $\theta(i,j)$ matters in order to determine demand function for private goods and leisure time, and thus maintains a key role in the empirical analysis.

It is now useful to remark that $S(l_f;i,j)$ can be used as a sufficient statistic for couples to make labor supply decision. First note that $W_m(t_m,t_f,l_f;i,j) = \phi_m(t_m,t_f,l_f;i,j)+C_m(i,j)$ and $W_f(t_m,t_f,l_f;i,j) = \phi_f(t_m,t_f,l_f;i,j)+C_f(i,j)$, where C_m and C_f are, respectively, the continuation values for the husband and the wife. The latter do not depend on current labor supply due to the household problem being static. While the sum $\phi_m(t_m,t_f,l_f;i,j) + \phi_f(t_m,t_f,l_f;i,j)$ does *not* necessarily provide an implicit representation of the Pareto frontier[4], the surplus splitting rule (3.12) implies that

$$S(l_f;i,j) > S(l'_f;i,j) \Leftrightarrow \mathit{\Gamma}(l_f;i,j) > \mathit{\Gamma}(l'_f;i,j). \tag{3.13}$$

3.6 Divorce Decisions.

When period t is about to end, a couple (i,j) draws a new type (i',j'). Hence, the couple needs to rebargain conditionally on the new draw (i',j'). Rebargaining leads to a new static collective problem as the one described in the previous section. However, the new proposed allocation has to respect the spouses' individual rationality constraints. In other words, if the spouses are able to find an agreement such that both are still better off together than apart, then they shift to a new allocation and stay married for another period. If instead there exists no allocation such that both are better off together, they divorce.

However, the new proposed household allocation, and ultimately the presence of gains from marriage, also depends on a vector of match-quality parameters $\zeta \in \mathbb{R}^{|\mathcal{L}|+1}$. In particular, each of the first $|\mathcal{L}|$ elements of ζ is linked with a given level l_f, so that $\zeta(l_f)$ shifts the Pareto frontier corresponding to l_f, while the last element of ζ, named $\zeta(d)$, shifts the breakpoint. The vector ζ is assumed to be drawn from a multivariate GEV distribution allowing for correlation between its elements (Train, 2009). In practice, I assume a nested logit structure so that it is convenient to separate the divorce decision from the labor supply decision.

First, consider the proposed agreement for labor supply l_f. The proposed allocation is the one that lies on the outermost frontier once the shifters have been taken into account:

$$l_f^*(i,j) = \arg\max_{l_f \in \mathcal{L}} \{S(l_f;i,j) + \zeta(l_f)\}. \tag{3.14}$$

The logit framework yields that the probability of selecting a given level l is equal to:

$$\ell(l;i,j) \equiv \frac{\exp(S(l;i,j)/\sigma_\ell)}{\sum_k \exp(S(k;i,j)/\sigma_\ell)} \tag{3.15}$$

where σ_ℓ is the scale factor associated with the *nest* of alternative labor supply choices. It is also useful to write down the following expected surplus, which I will often refer to as *inclusive surplus* in reference to the GEV literature:

[4] It only does so in the case where u_g is quasilinear in private consumption q_g. In this case, functions g_m and g_f required to obtain a linear representation of the Pareto frontiers as in (3.9) are simple linear functions.

$$\bar{S}(i,j) \equiv \sigma_\ell \ln \sum_k \exp(S(k;i,j)/\sigma_\ell). \tag{3.16}$$

Finally, note that, as in Shimer and Smith (2000), Transferable Utility implies efficient divorce. Both spouses agree on divorce when gains from marriage, once accounted for random taste shocks, are negative. From splitting rule (3.12), it is also clear that both spouses' rationality constraints are satisfied when $S(l_f;i,j)$ for a labor supply choice l_f. The taste shock $\zeta(d)$ is drawn independently from the other elements of ζ and shifts the breakpoint. The divorce rule can be written down as follows:

$$\max_{l_f \in \mathcal{L}} \{S(l_f;i,j) + \zeta(l_f) - \zeta(d)\} > 0. \tag{3.17}$$

The choice between nests is associated with a scale factor σ, so that the probability of divorce is equal to:

$$1 - \alpha(i,j) \equiv \frac{1}{1 + (\sum_k \exp(S(k;i,j)/\sigma_\ell))^{\sigma_\ell/\sigma}} = \frac{1}{1 + \exp(\bar{S}(i,j)/\sigma)}. \tag{3.18}$$

A concluding remark for this section. In this model, it is assumed that, in each period, the couple rebargains, decides whether to stay together, and shifts to a new allocation, conditionally on a joint draw for (i', j', ζ'). Hence, this is a model of no commitment, which implies that gains from marriage do not come from risk-sharing, but only from the presence of a public good. Risk-averse agents would prefer to commit to a given sharing rule in order to smooth out future shocks. However, when spouses are allowed to divorce, only limited commitment devices are feasible. Note that, if a commitment device were available to agents in this model, divorce would still be efficient, but sorting patterns would change as expectations over future allocations would be internalized in the present discounted value of marriage. Ligon et al. (2002) and Mazzocco (2007) exhaustively discuss the role of commitment in risk-sharing, while Voena (2015) and Reynoso (2017) represent two applications of limited commitment devices in presence of divorce.

3.7 Value of Marriage.

Now that labor supply and divorce decisions have been fully described, it is possible to write down Bellman equations that recursively characterize the equilibrium marital payoffs for given reservation values $(V_m^0(i), V_f^0(j))$.

$$\begin{aligned} V_m(l_f;i,j) =& v_m(l_f;i,j) + \psi_m(i)\beta \int V_m^0(i')\pi_{i,i'} di' + \\ &+ \psi_m(i)\psi_f(j)\beta \iint (1-\theta(i',j')) \ln(1+\exp(\bar{S}(i',j')/\sigma))\pi_{i,i'}\pi_{j,j'} di' dj'. \end{aligned} \tag{3.19}$$

$$\begin{aligned} V_f(l_f;i,j) =& v_f(l_f;i,j) + \psi_f(j)\beta \int V_f^0(j')\pi_{j,j'} dj' + \\ &+ \psi_m(i)\psi_f(j)\beta \iint \theta(i',j') \ln(1+\exp(\bar{S}(i',j')/\sigma))\pi_{i,i'}\pi_{j,j'} di' dj'. \end{aligned} \tag{3.20}$$

Enforcing the splitting rule (3.12), equations (3.19) and (3.20) yield a system of $|\mathcal{I}|\times|\mathcal{J}|\times|\mathcal{L}|$ equations with as many unknowns. It is possible to solve the system by first computing S for couples where at least one spouse has age $\bar{\bar{a}}_g$, and then solving backwards by computing S for couples where at least one spouse has age $\bar{\bar{a}}_g - 1$, and so on.

3.8 Meetings.

At the beginning of period t, the population of singles is given by measures $n_m(i)$ and $n_f(j)$. Singles make their household choices as described in section 3.4, and need to wait for the next "market opening" to look for a partner. At the end of time t, thereafter denoted $t+$, singles draw their new type and go on the marriage market. They are joined by a new young cohort of agents, who enter the market at age $\underline{a}_g$ as singles. The measure of singles participating to the marriage market at the end of period t is given by:

$$n_{m,+}(i') \equiv \begin{cases} \int \psi_m(i) n_m(i) \pi_{i,i'} di' & \text{if } a_{i'} > \underline{a}_m \\ p_m(i') & \text{if } a_{i'} = \underline{a}_m \end{cases} \tag{3.21}$$

$$n_{f,+}(j') \equiv \begin{cases} \int \psi_f(j) n_m(j) \pi_{j,j'} dj' & \text{if } a_{j'} > \underline{a}_f \\ p_f(j') & \text{if } a_{j'} = \underline{a}_f \end{cases} \tag{3.22}$$

Search for a partner is costless and singles of different sex meet each other randomly. Meetings between singles of different sex occur according to a rate of arrival $\lambda(i,j)$. The probability of meeting a specific type is also proportional to the total number of single agents of that type. For instance, a single man of type i meets a single woman of type j with probability $\lambda(i,j)n_{f,+}(j)/N_f$. Hence, the *meeting function* $M(i,j)$, which gives the number of meetings between agents of type (i,j) per period, can be defined as

$$M(i,j) = \lambda(i,j) n_{m,+}(i) n_{f,+}(j). \tag{3.23}$$

The meeting function needs to respect some theoretical restrictions recently summarized by Jaffe and Weber (2017). First, note that the total number of meetings per period cannot exceed $\min\{N_m, N_f\}$. The total number of meetings is given by:

$$\iint \lambda(i,j) n_{m,+}(i) n_{f,+}(j) di dj = \lambda_0 N_m N_f \tag{3.24}$$

where $\lambda_0 \leq \min\{1/N_m, 1/N_f\}$ and can be interpreted as an average rate of arrival. At the type level, the meeting function must be such that the adding-up constraint holds for every couple (i,j), i.e., meetings must be "reciprocated": the way in which $M(i,j)$ is written ensures that this adding-up constraint is always respected.

3.9 Marriage Decisions and Value of Singlehood.

Upon a meeting, a man and a woman observe each other's type and draw a vector $\zeta_0 \in \mathbb{R}^{|\mathcal{L}|+1}$ from a multivariate GEV distribution. The parameters of such distribution are allowed to differ from those

that determine divorce decisions. In particular, the scale parameter associated with the choice between nests, i.e., between marriage and singlehood, is equal to $\sigma_0 \neq \sigma$. In addition, the elements of ζ_0 have mean $-\sigma_0\kappa$: this parameter can be interpreted as a sunk cost of marriage, possibly due to the cost of dating and gathering information on the partner.
As in equation (3.18), the logit framework yields the following probability of getting married:

$$\alpha_0(x,y) = 1 - \frac{1}{1+\exp(\bar{S}(i,j)/\sigma_0 - \kappa)}. \tag{3.25}$$

The value of being single for a man i and a woman j is given, respectively, by:

$$\begin{aligned} V_m^0(i) =& v_m^0(i) + \beta\psi_m(i)\int V_m^0(i')\pi_{i,i'}di' + \\ &+ \beta\psi_m(i)\underbrace{\iint \lambda(i',j')(1-\theta(i',j'))\ln(1+\exp(\bar{S}(i',j')/\sigma))n_{f,+}(j')dj'\pi_{i,i'}di'}_{\text{Expected marriage prospects}} \end{aligned} \tag{3.26}$$

$$\begin{aligned} V_f^0(j) =& v_f^0(j) + \beta\psi_f(j)\int V_f^0(j')\pi_{j,j'}dj' + \\ &+ \beta\psi_f(j)\underbrace{\iint \lambda(i',j')\theta(i',j')\ln(1+\exp(\bar{S}(i',j')/\sigma))n_{m,+}(i')di'\pi_{j,j'}dj'}_{\text{Expected marriage prospects}} \end{aligned} \tag{3.27}$$

3.10 Law of Motion.

Individual matching strategies (α, α_0) result in aggregate marriage market stocks and flows. The number of couples of type (i',j') is given by the sum of the newlyweds (i',j') and those couples that drew type (i',j') and did not divorce:

$$m(i',j') = \underbrace{\lambda(i,j)\alpha_0(i',j')n_{m,+}(i')n_{f,+}(j')}_{MF(i',j')} + \underbrace{\alpha(i',j')\iint \psi_m(i)\psi_f(j)m(i,j)\pi_{i,i'}\pi_{j,j'}didj}_{m_+(i',j')-DF(i',j')}. \tag{3.28}$$

The first term on the rhs provides a formula for the marriage flow $MF(i',j')$, while the second term implicitly provides a formula for the divorce flow $DF(i',j')$. The (unstable) measure m_+ is defined similarly to $n_{m,+}$ and $n_{f,+}$: it counts the number of couples that drew type (i',j') and still did not have time to adjust, and possibly to divorce.

3.11 Search Equilibrium.

At the steady-state search equilibrium, agents' matching strategies (α, α_0) must be consistent with the payoff structure S. Agents' reservation utilities (V_m^0, V_f^0), which enter the surplus function, depend on the supplies (n_m, n_f) of singles on the market.

Definition 1 *Assume conditions (3.1), (3.2), (3.3) and (3.4) on marginals (ℓ_m, ℓ_f) are respected. A **steady-state search equilibrium** is given by time-invariant measures of couples and singles (m, n_m, n_f), payoffs (V_m^0, V_f^0, S) and strategies α so that:*

- *accounting constraints (3.5) and (3.6) are satisfied;*

- *the law of motion* (3.28) *is respected;*
- *surplus splitting rule* (3.12) *is implemented;*
- *Bellman equations* (3.19), (3.20), (3.26) *and* (3.27) *hold;*
- *optimal matching and divorce rules are consistent with* (3.18) *and* (3.25).

These equilibrium conditions yield a fixed-point operator of type $n = T_{ext}n$ over the support $\mathcal{I} \cup \mathcal{J}$. This fixed-point operator is described in appendix A. Recently, Manea (2017) generalized the original proof of existence by Shimer and Smith (2000). It may be possible to extend this proof to a framework with random match quality, although this has not been done in the literature yet. However, proofs are based on Kakutani's theorem, thus providing little guidance on how to find the equilibrium (or, potentially, the equilibria).

4 Identification.

In this section, I discuss the identification of two key objects in the model, the meeting function M and the inclusive surplus function $\bar{S}$ starting from a dataset $(\hat{n}_m, \hat{n}_f, \hat{m}, \widehat{MF}, \widehat{DF})$. Identification is discussed in the case where the wage process is fully exogenous, so that transition matrices π can be estimated outside of the model. In addition, note that when the surplus is known and the per-period indirect utilities of singles $(v^0_m(i), v^0_f(j))$ are known (or normalized to some constant), it is possible to recover the underlying household production function for married couples starting from $\hat{\bar{S}}$ and adding information on labor supply $\hat{\ell}$ to the dataset.

4.1 Surplus.

The identification of match surplus starting from matched data has been exhaustively discussed by Choo and Siow (2006) and Galichon and Salanié (2015). Similar identification principles apply to search-and-matching models. However, the econometrician needs to address an additional question: are matches common (rare) because of a high (low) match surplus or because people meet with high (low) frequency? In this section, assume that the meeting function M and the parameters of the distributions of taste shocks $(\sigma, \kappa, \sigma_0)$ are known to the econometrician. If this is the case, then it is possible to pin down matching strategies (α, α_0) with matched data $(\hat{n}_m, \hat{n}_f, \hat{m})$, starting from the law of motion (3.28) and the following relationship between α and α_0:

$$\alpha_0(i,j) = \left[1 + \left(\kappa \frac{1-\alpha(i,j)}{\alpha(i,j)}\right)^{\sigma/\sigma_0}\right]^{-1}. \tag{4.1}$$

Solving for α in the law of motion after substituting for α_0 yields nonparametric estimators $(\hat{\alpha}, \hat{\alpha}_0)$ of matching strategies. The latter only exist for a given couple type (i,j) if the following condition is respected:

$$\hat{m}(i,j) < \lambda(i,j)\hat{n}_m(i')\hat{n}_f(j) + \hat{m}_+(i,j). \tag{4.2}$$

Interestingly, condition (4.2) implies $|\mathcal{I}| \times |\mathcal{J}|$ bounds on λ, and can thus be used to test whether the prior on the specification chosen for λ is to reject. In practice, condition (4.2) suggests that $\lambda(i,j)$ must ensure that there are enough meetings between singles (i,j) to explain the difference $\hat{m}(i,j) - \hat{m}_+(i,j)$. Underassessing the number of meetings (i,j) will lead $(\hat{\alpha}, \hat{\alpha}_0)$ to exceed their upper bounds.
Starting from $\hat{\alpha}$, it is straightforward to back out the inclusive surplus $\bar{S}$ from (3.18), and the surplus S from (3.15) using data on labor supply $\hat{\ell}$. Attempting to implement this nonparametric approach results in the model to be just-identified: there are indeed $|\mathcal{I}| \times |\mathcal{J}| \times |\mathcal{L}|$ parameters S to be estimated with as many restrictions. Hence, the model would yield a perfect fit of the matched data. However, such perfect fit will be achieved for any prior on the meeting function M, on the parameters $(\sigma, \kappa, \sigma_0)$, and also on the scaling factor σ_ℓ: all these models will thus be indistinguishable with data $(\hat{n}_m, \hat{n}_f, \hat{m}, \hat{\ell})$ only.

4.2 Meeting Function.

Goussé et al. (2017) provide a solution to this identification puzzle and suggest bringing new data in order to disentangle the structure of meetings from the structure of the surplus. Assume now that the econometrician observes a new layer of data $(\widehat{MF}, \widehat{DF})$. Recall from the law of motion (3.28) the equations that yield the steady-state marriage and divorce flows:

$$\left[1 + \left(\kappa \frac{1-\alpha(i,j)}{\alpha(i,j)}\right)^{\sigma/\sigma_0}\right]^{-1} = \frac{MF(i,j)}{\lambda(i,j) n_{m,+}(i) n_{f,+}(j)} \tag{4.3}$$

$$1 - \alpha(i,j) = \frac{DF(i,j)}{\iint m(i',j') \pi_{i'i} \pi_{j'j} di' dj'}. \tag{4.4}$$

The goal of this section is ensuring identification for $(\kappa, \sigma_0, \lambda)$. In fact, note first that a normalization of σ is necessary as only the ratio σ/σ_0 matters in explaining flows. Relationships (4.3) and (4.4) combined imply further $|\mathcal{I}| \times |\mathcal{J}|$ restrictions that can be used to achieve full identification of the model.
Consider two cases: in a first scenario, if κ and σ_0 are known or normalized to some constants, then such restrictions yield a nonparametric estimator of $\lambda(i,j)$. It follows that the nonparametric estimators of both the meeting and the surplus function would ensure a perfect fit of both the stocks *and* the flows observed in the data. Alternatively, if λ is given a parametric specification with no more than $|\mathcal{I}| \times |\mathcal{J}| - 2$ parameters, it is possible to estimate both the parameters of λ, κ and σ_0.

5 Empirical Specification.

In this section, I provide an outline of the specification used to obtain the results in section 7. Many of the choices made until now reflect the need to practice with the estimation algorithm and understand to which extent the model is indeed empirically tractable.

5.1 Age and Education.

Time-invariant human capital endowment is assumed to correspond to education. I assume $h_i, h_j \in \{1, 2\}$, where 2 stands for college degree. Note that time-invariant types could be extended to include other traits, such as ethnicity or fixed effects for labor market skills. A more challenging alternative would be to characterize h_i as a time-invariant "marriage market capital", which would most likely require to infer the distribution from marriage market behavior, and hence jointly with other parameters of the model.

Both men and women are assumed to enter the marriage market when aged 22. A period corresponds to two years, and all agents quit the market after 15 periods, when aged 50. Preliminary results (and commonsense) suggest that it is key to relax this assumption as it generates a large number of "widow(er)s". A more reasonable assumption is to assume that, once reached a certain threshold age $\bar{a}_g \leq \bar{\bar{a}}_g$, people live on although they stop searching if singles. Their younger partners can choose either to stay married with them or to quit them to find someone younger. This idea will be detailed when discussed the meeting function in section 5.6.

5.2 Wage Process.

Wage rates are assumed to be distributed log-normally conditionally on age and education. Hence, the marginal distributions (ℓ_m, ℓ_f) are such that

$$\ln w_i \sim \mathcal{N}\left(\mu_m(h_i, a_i), \sigma_m^2(h_i, a_i)\right) \tag{5.1}$$

$$\ln w_j \sim \mathcal{N}\left(\mu_f(h_j, a_j), \sigma_f^2(h_j, a_j)\right) \tag{5.2}$$

I follow Bonhomme and Robin (2009) and map the marginal distributions of $\ln w_i$ and $\ln w_{i'}$ into the joint distribution of $(\ln w_i, \ln w_{i'})$ by using a copula. The joint c.d.f. of the current log-wage and the future log-wage is given by

$$C_m\left(\Phi\left(\frac{\ln w_i - \mu_m(h_i, a_i)}{\sigma_m(h_i, a_i)}\right), \Phi\left(\frac{\ln w_{i'} - \mu_m(h_{i'}, a_{i'})}{\sigma_m(h_{i'}, a_{i'})}\right) |h_i, a_i\right) \tag{5.3}$$

$$C_f\left(\Phi\left(\frac{\ln w_j - \mu_m(h_j, a_j)}{\sigma_f(h_j, a_j)}\right), \Phi\left(\frac{\ln w_{j'} - \mu_f(h_{j'}, a_{j'})}{\sigma_f(h_{j'}, a_{j'})}\right) |h_j, a_j\right) \tag{5.4}$$

for men and women respectively. I choose a Gaussian copula for both C_m and C_f, which implies that I need to estimate two vector of parameters $(\rho_m(h_i, a_i), \rho_f(h_j, a_j))$ representing the wage rank correlation across two consecutive periods.

5.3 Labor Supply.

In this version of the paper, I assume all married women work full time ($l_f = 1$). This has an important implication: the wage process can be estimated exogenously. However, it does not provide a way to deal with individuals that did not earn any labor income in a given year. For these individuals, I input wage rates after estimating the wage process as described in the previous subsection.

5.4 Utility Functions.

Preferences of single and married agents are described by the following utility functions:

$$u_m^0(q_m^0) = \ln q_m^0 \tag{5.5}$$
$$u_f^0(q_f^0) = \ln q_f^0 \tag{5.6}$$
$$u_f(q_f, Q, l_f; i, j) = \tilde{\gamma}_0 + \ln q_f + \tilde{\gamma}_1 \ln Q + \tilde{\gamma}_2(l_f) + \tilde{\gamma}_3(l_f) \tag{5.7}$$
$$u_m(q_m, Q, l_f; i, j) = \tilde{\gamma}_0 + \ln q_m + \tilde{\gamma}_1 \ln Q + \tilde{\gamma}_2(l_f). \tag{5.8}$$

The *conditional indirect utility functions* are given by

$$\phi_f(t_m, t_f, l_f; i, j) = \ln(w_j - t_f) + \tilde{\gamma}_1 \ln(t_m + t_f) + \tilde{\gamma}_2(l_f) + \tilde{\gamma}_3(l_f) \tag{5.9}$$
$$\phi_m(t_m, t_f, l_f; i, j) = \ln(w_i - t_m) + \tilde{\gamma}_1 \ln(t_m + t_f) + \tilde{\gamma}_2(l_f). \tag{5.10}$$

In the Cobb-Douglas specification (5.7) and (5.8), $\tilde{\gamma}_1$ captures the couple's preferences over public good Q with respect to private consumption. Note that, while I allow $\tilde{\gamma}_1$ to depend on the spouses' traits (i, j), the couple shares the same taste for the public good. On the contrary, while $\tilde{\gamma}_2(l_f)$ captures the possible public gains from a stay-at-home wife, $\tilde{\gamma}_3(l_f)$ accounts for the wife's private taste for the current amount of hours worked l_f. Similarly to $\tilde{\gamma}_1$, $\tilde{\gamma}_2$ is allowed to depend on the spouses' traits (i, j), while $\tilde{\gamma}_3$ depends on the wife's traits j. Both $\tilde{\gamma}_2$ and $\tilde{\gamma}_3$ are normalized to zero when $l_f = 1$. Hence, in this version of the paper, I do not estimate $\tilde{\gamma}_2$ and $\tilde{\gamma}_3$ since all individuals are assumed to work.

5.5 Closed Form for Surplus.

Up to this point, I worked under the convenient restriction

$$V_m(i, j) - V_m^0(i) = V_f(i, j) - V_f^0(j) \tag{5.11}$$

which results in greater computational simplicity due to the possibility of recovering a closed form solution for surplus $S(i, j)$. In addition, note that this restriction is not necessarily rejected by the data: Goussé et al. (2017) estimated a household Nash bargaining parameter in a quasilinear specification and found it to be very close to .5, which implies that surplus is equally split between spouses.
The log-sum of (3.19) and (3.20) net of the respective reservation values yields:

$$\begin{aligned} S(i,j) =& h(i,j) - 2\ln\Big\{\exp\Big(V_m^0(i) - \beta\psi_m(i)\int V_m^0(i')\pi_{i,i'}di'\Big) + \exp\Big(V_f^0(j) - \beta\psi_f(j)\int V_f^0(j')\pi_{j,j'}\Big)\Big\} + \\ &+ \beta\psi_m(i)\psi_f(j)\iint \ln(1 + S(i',j'))\pi_{i,i'}\pi_{j,j'}di'dj'. \end{aligned} \tag{5.12}$$

The function $h(i, j)$ has the following functional form:

$$h(i, j) = \gamma_0(i, j) + 2(1 + \gamma_1(i, j)) \ln(w_i + w_j) \tag{5.13}$$

where $\gamma_1(i, j) = \tilde{\gamma}_1$, while $\gamma_0(i, j)$ is a transformation of $\tilde{\gamma}_0$. In the current version of the paper, I assume

$$\gamma_1(i,j) = \gamma_1\{h_i = h_j = 1\} + \gamma_2\{h_i = 1, h_j = 2\} + \gamma_3\{h_i = 2, h_j = 1\} + \gamma_4\{h_i = h_j = 2\} \tag{5.14}$$

$$\begin{aligned}\gamma_0(i,j) =& \gamma_5\{h_i = 1, h_j = 2\} + \gamma_6\{h_i = 2, h_j = 1\} + \gamma_7\{h_i = h_j = 2\} + \gamma_8 a_i + \gamma_9 a_i^2 + \\ &+ \gamma_{10} a_j + \gamma_{11} a_j^2.\end{aligned} \tag{5.15}$$

This parameterization is already quite rich, but could be extended in a few ways. First, there could be terms of type $h_i a_j^k$ accounting for the interaction of wage and education. Second, there could be terms of type $\{a_i > a_j\}|a_i - a_j|^k$ accounting for asymmetries in sorting on age and reflecting men's preference for younger women.

5.6 Meetings.

In this version of the paper, I employ a *random search* specification for the meeting technology M. This is equivalent to assume that $\lambda(i,j) = \lambda_0$ for each (i,j), the total number of meetings (i,j) per period being $\lambda_0 n_{m,+}(i) n_{f,+}(j)$. While this specification is arguably the most parsimonious one could think of, it would be of great interest to implement a test using restriction (4.2) and check whether it is not rejected by the data. However, implementing this test implies estimating non-parametrically the joint distribution of characteristics m, which is, as discussed at the very beginning of section 7, extremely unpractical.

Yet, theoretical restriction (4.2) suggests that the random search assumption might be inadequate to explain why young cohorts who just entered the market marry at such high rates. Under random search, there might simply be not enough meetings among young people to reproduce the trends observed in the data. In addition, when trying to match theoretical moments to their empirical counterparts, this type of misspecification would result in biased estimates of α. In particular, α would be upward biased for couples with partners close in age, and downward biased for those with partners distant in age.

If $\lambda(i,j)$ is allowed to depend on agents' ages, the puzzle can be solved. Consider the following specification

$$\lambda(i,j) = \begin{cases} \frac{\lambda_0}{\chi} \exp(-\lambda_1 |a_i - a_j|) & \text{if } a_i \leq \bar{a}_m, a_j \leq \bar{a}_f \\ 0 & \text{otherwise.} \end{cases} \tag{5.16}$$

Note that, as anticipated in section 5.1, specification (5.16) implies that individuals older than $\bar{a}_g$ do not get married any longer. Parameter λ_0 is still a scale that determines the total number of meetings: using (3.24), it is possible to back out the value of the normalizing constant χ, as shown by Jaffe and Weber (2017):

$$\iint \exp(-\lambda_1 |a_i - a_j|) n_{m,+}(i) n_{f,+}(j) di dj = \chi N_m N_f. \tag{5.17}$$

Although this version of the paper does not make use of this specification, both theoretical considerations and preliminary results seem to suggest that, in this life-cycle model, moving away from the baseline random search assumption is beneficial for the model's fit.

6 Estimation.

In this section, I first list the parameters that are calibrated or estimated outside of the model. Then, I propose an estimation method based on indirect inference. In the previous section, I show the existence of a nonparametric estimator based on the identifying restrictions embedded in the model. Implementing this estimator implies several difficulties. First, the model implies much heterogeneity, and, exactly as in Choo and Siow (2006), the nonparametric estimator suffers from a curse of dimensionality. In practice, estimating the multivariate distribution m is not an easy task: the corresponding empirical probabilities obtainable after discretizing the support of wage rates result in many thin cells, while smoothing through kernel estimators results in a possibly non-negligible small sample bias due to high-dimensional support. Estimating MF and DF is even more challenging due to the relatively small number of newlyweds and divorcing couples observed in the data.

6.1 Calibration.

I estimate marginals (p_m, p_f) outside of the model. 35.95% of men and 32.17% of women born between 1956 and 1965 are college graduates. The panel being approximately balanced, I impose that marginals with respect to age are uniform. Otherwise stated, vectors of survival probabilities ψ_m and ψ_f are s.t. $\psi_g(a) = 1$ if $a < \bar{\bar{a}}_g$, and $\psi_g(a) = 0$ if $a = \bar{\bar{a}}_g$. I estimate the parameters (μ_g, σ_g) of the wage distributions conditionally on age and education. The estimation of $(\pi_{i,i'}, \pi_{j,j'})$ requires observing the wage distribution in two consecutive periods (Bonhomme and Robin, 2009). Under the assumption that both C_m and C_f are normal copulas, the problem boils down to the estimation of the wage rank correlation ρ_g between two consecutive waves, always conditionally on agents' age and education.
As already anticipated in the previous section, the bargaining parameter $\theta(i,j)$ is equal to .5 for any (i,j). The discount factor β is set to 0.9216. A normalization is also required for σ: I set $\sigma = 2.5$,

6.2 Indirect Inference.

The remaining parameters to estimate can be grouped in the vector $\tau \equiv (\gamma_0, \gamma_1, \lambda, \kappa, \sigma_0)$. For a guess τ, I solve for the steady-state equilibrium and compute a vector of moments μ from the equilibrium distributions (n_m, n_f, m, MF, DF). Then, I compute the (normalized) distance $(\mu - \hat{\mu})/\hat{\mu}$. The minimum distance estimator is given by:

$$\hat{\tau}_{MD} \equiv \arg\min_{\tau} \sum_k \omega_k \frac{\mu_k(\tau) - \hat{\mu}_k}{\hat{\mu}_k}, \tag{6.1}$$

where ω is a vector of weights. For now, I set $\omega_k = 1$ for any k.
I match moments of the following type:

- the share of singles by type;
- the share of couples by human capital level;
- the aggregate marriage rate, defined as the ratio of new couples to the total population;

- the divorce rate by human capital level, where the divorce ratio is defined as the ratio of divorcing couples to married couples at the beginning of the period.

I employ parallel Markov chain Monte Carlo methods to search for the minimum of the objective function of minimization problem (6.1), as well as to recover the entire distribution of the estimator. I employ the algorithm proposed by Baragatti et al. (2013), using 4 parallel chains that communicate with each other to exchange relevant information at every iteration. I present results from the "coldest" chain, the one whose transition kernel has the lowest temperature. This chain has the role of exploring the support in a restricted area around the global minimum. Higher-temperature chains have the task of exploring the entire support, and make sure the algorithm does not get stuck in a local minimum.

7 Preliminary Results.

In this section, I present some preliminary estimates of the simplified version of the model outlined in section 5. The goal is mainly to set a benchmark for future work. There is a wide margin of improvement both in terms of precision and fit. However, the model is already shown to be able to reproduce some of the main life-cycle trends discussed in the introduction.

7.1 Parameter Estimates.

Tables in this section display the median value of the estimator, jointly with the 10th and the 90th percentiles of its distribution obtained with 4,000 iterations of the parallel tempering algorithm discussed in section 6.2. In table 7.1, I present results for search parameters. Estimates for λ_0 suggest the probability of a meeting is approximately 55%. The sign of parameter κ is, in principle, unrestricted, but the estimates suggest that it has positive sign, and that there is indeed a sunk cost of marriage. Interestingly, σ_0 seems to be smaller than 1, which implies that, upon a first meeting, couples value deterministic gains from marriage relatively more than idiosyncratic taste shocks with respect to couples where the spouses already know each other.

Parameter	**Median**	**10th percentile**	**90th percentile**
λ_0	1.51609	*1.11137*	*2.01436*
κ	3.62684	*1.42180*	*5.66468*
σ_0	0.81349	*0.53316*	*1.19956*

Table 7.1: Search parameters.

Table 7.2 contains findings on the relative importance of public good expenditure for married couples. Preliminary findings suggest that public good matters more for couples where both partners are college graduates, γ_4 taking the highest value in the table. Heterogamous couples display asymmetric behavior: when the wife is more educated than the husband, public good matters more.

Parameter	Median	10th percentile	90th percentile
γ_1	1.27116	*0.73658*	*1.79368*
γ_2	0.64163	*0.15418*	*1.04901*
γ_3	0.47448	*0.10722*	*1.29595*
γ_4	1.59724	*0.89136*	*1.92704*

Table 7.2: Relative importance of public good expenditure $\gamma_1(i,j)$.

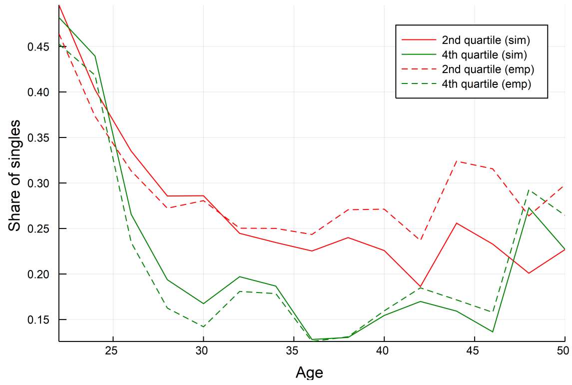
The last table contains results on the nontransferable homophily term $\gamma_0(i,j)$. Parameters γ_5, γ_6 and γ_7 are shifters depending on spouses' education, and express spouses' tastes with respect to the benchmark category, i.e., couple where no spouse holds a college degree. The degree of supermodularity on education due to this nontransferable homophily term can be quantified by computing $\gamma_7 - \gamma_5 - \gamma_6$, which, by using median values, turns out to be around 3.33. A final remark on the estimates of the coefficients of the age polynomials. It seems that men's age positively contributes to the quality of the match, and, γ_9 being positive, this contribution appears as convex in age. On the other hand, the contribution of women's age is negative approximately after age 28, due to γ_{11} being negative. However, particularly in this case, it is hard to draw any conclusion due to the low precision of the estimator at this stage.

Parameter	Median	10th percentile	90th percentile
γ_5	-2.50573	*-4.54986*	*1.63813*
γ_6	1.40787	*-4.54879*	*5.31755*
γ_7	2.23136	*-1.88825*	*5.78709*
γ_8	0.60954	*-0.47222*	*1.29492*
γ_9	0.09651	*-0.01696*	*0.17462*
γ_{10}	0.56616	*-0.09311*	*1.18402*
γ_{11}	-0.13216	*-0.18432*	*0.01028*

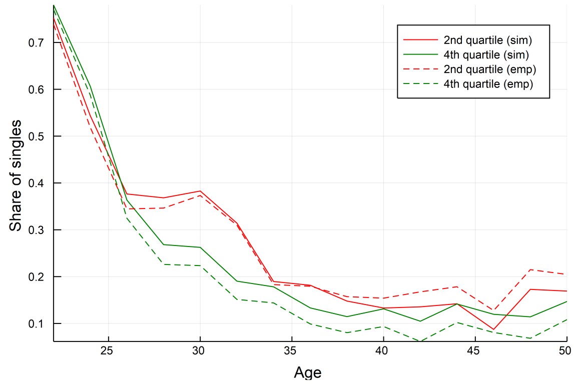
Table 7.3: Homophily component $\gamma_0(i,j)$.

7.2 Model's Fit.

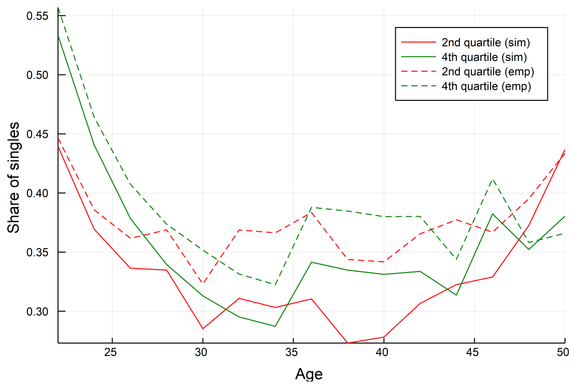
One of the key trends that the model aims to replicate is the differentials in the odds of being single by age, education, and wage. Fit for these trends is presented in figure 7.1. The fit is good for most groups, and the model succeeds in reproducing the differentials by wage very well. Differentials by education are slightly smaller in the simulation than in the data, especially for women.



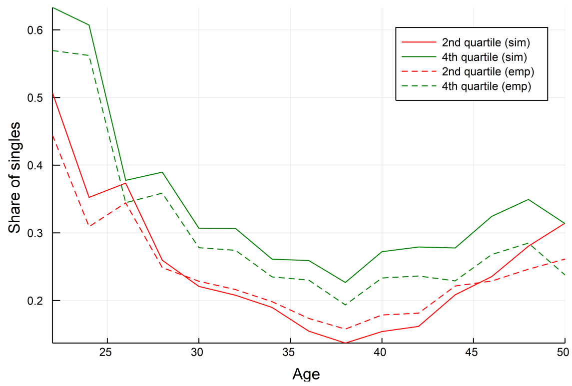
(a) Men without college degree



(b) Men with college degree



(c) Women without college degree



(d) Women with college degree

Figure 7.1: Fit for odds of being single by sex, age, education and wage quartile.

The fit for additional moments is presented in table 7.4. Sorting patterns with respect to education are reproduced decently: the model still struggles to understand why there are so many homogamous couples of college graduates, though. More importantly, while the simulation yields a slightly higher gross marriage rate, it underestimates divorce rates for each group. I suspect that this predictive failure is due to the potential misspecification of the meeting function discussed in section 5.6: since under random search successful meetings are underassessed, divorce rates need to be low in order to sustain the relatively high number of couples observed in the data.

Moment	Simulated	Empirical
% of couples, no college degree	0.564107	0.513443
% couples, wife has college degree	0.135980	0.125285
% couples, husband has college degree	0.118182	0.123996
% couples, both have college degree	0.181731	0.237276
Gross marriage rate	0.105347	0.088313
Gross divorce rate	0.019881	0.047261
Divorce rate, no college degree	0.030366	0.062908
Divorce rate, wife has college degree	0.030000	0.050197
Divorce rate, husband has college degree	0.031119	0.039165
Divorce rate, both have college degree	0.006516	0.016083

Table 7.4: Fit for other moments.

7.3 Diagnostic Exercises.

Actually, to assess the precision of the estimator, one can draw histograms for all parameters as in figure B.1 in appendix. This is also a very good diagnostic exercise to understand how the current specification is performing. For instance, consider parameter γ_4: to minimize the objective function, the algorithm pushes for very high values of γ_4, as, under this specification, the model struggles to match the high share of college-graduate couples observed in the data.
In picture B.2 in the appendix, I show the objective function behavior around the median values chosen as point-estimates to discuss. The function seems very well-behaved as slices show that it appears as locally convex around the chosen point. A last diagnostic exercise consists in checking moments' responses by moving around a single parameter and holding the others fixed. In figure B.3, I show the response of moments related to marriage and divorce flows to perturbations of selected parameters. The gross marriage rate, for instance, is increasing in λ, and thus contributes to its identification. Note that divorce rates are decreasing in κ: from figure B.3, it is clear that, all else held constant, a lower κ would improve the fit of these specific moments. However, lowering κ worsens the fit, because it leads to a worse match for the remaining moments. This type of puzzles can be solved in two ways: either by adding new parameters, or by correcting the specification.

8 Conclusion.

The relationship between marriage market behavior and inequality has already been largely discussed. In order to achieve this goal, I build a new theoretical framework for the analysis of marriage, divorce and remarriage decisions along the life-cycle. This model is a first of its kind, as it combines a dynamic matching framework with several new modeling elements taken from life-cycle models of the household. First, people match and break up while they age, and thus face changing incentives due to both their individual traits and time horizon evolving over time. Second, both economic and noneconomic gains from marriage have random components that may cause couples to break up. Third, I embed a collective model of the household that helps describe how agents make private

and public consumption choices when family formation is endogenous: in spite of its simplicity, the household model includes new features, such as discrete female labor supply and risk aversion.
The model is not only innovative from a theoretical standpoint, but also suitable for empirical analysis. In this version of the paper, I discuss the identification of the main primitives, the meeting function and the household production function, and extend key concepts already introduced by the matching literature (Choo and Siow, 2006; Galichon and Salanié, 2015; Goussé et al., 2017). I also propose an estimation method and show some preliminary results, which help set a starting point for future work. Most importantly, while this model tries to combine insights from the matching literature with others from dynamic household models, several elements are left out. In particular, in this model the only dynamic decisions are marriage and divorce. However, in a life-cycle model, these choices can be studied jointly to other long-term decisions, such as fertility and location choices, dynamic labor supply and human capital accumulation, and so on. In this regard, this paper only aims to provide a theoretical and empirical benchmark for future research.

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A Fixed-Point

The steady-state search equilibrium can be thought of as a fixed-point of an operator $n \to T_{ext}n$, with $n = (n_m, n_f)$. First, it is necessary to discretize the sets of types $|\mathcal{I}|$ and $|\mathcal{J}|$, as wage rates are continuous variables in the data. Hence, n is a vector of length $|\mathcal{I}| + |\mathcal{J}|$.

1. Start the iteration with $k = 0$ and a guess for n^k.
2. For given n^k, solve a fixed-point problem T_{int} given by equations (5.12), (3.26) and (3.27), in order to find $V^0 = (V_m^0, V_f^0)$ so that $V^0 = T_{int}V^0$.
3. Update α using (3.25).
4. Substitute the matrix α into the law of motion (3.28) and solve forwards for m.
5. Use the accounting equations (3.5) and (3.6) to compute n^{k+1}.
6. If $\Delta(n^k, n^{k+1}) < \epsilon$, keep n^{k+1}, otherwise set $n^k = n^{k+1}$ and restart from step 2.

B Diagnostic Exercises: Figures.

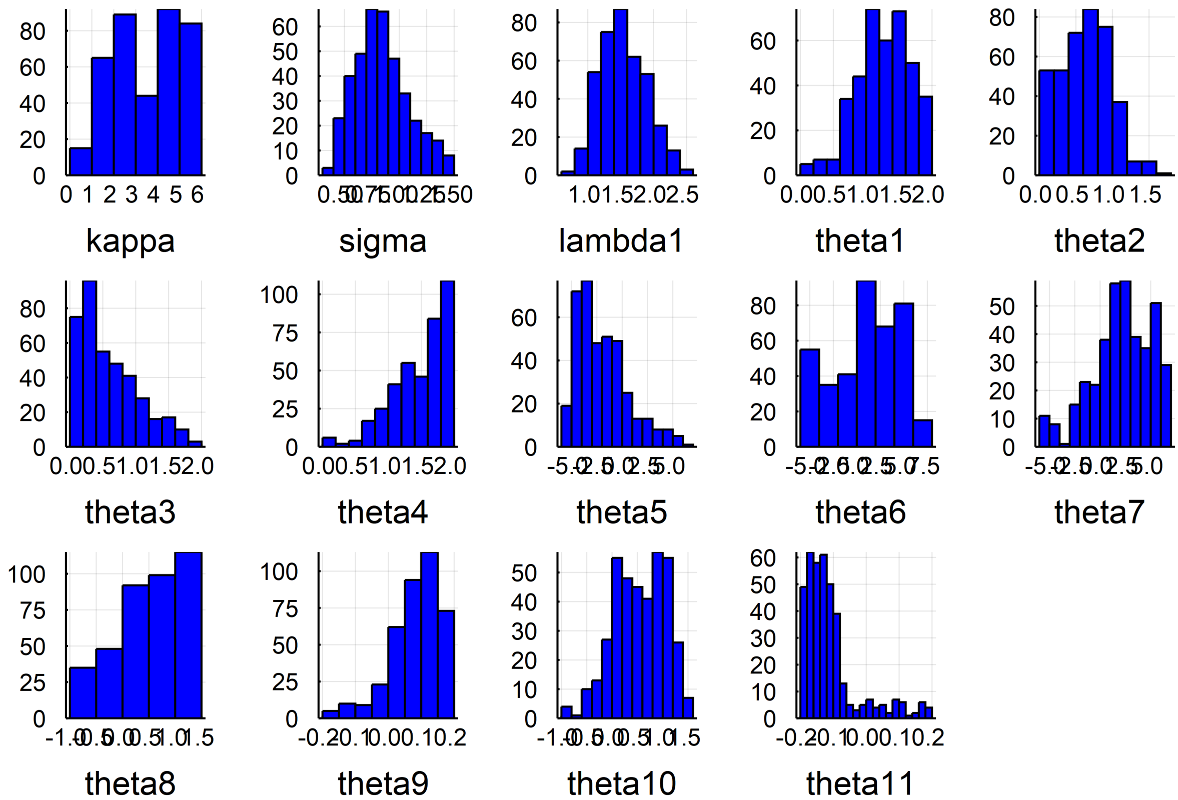

Figure B.1: Histograms of the estimator distribution obtained through parallel tempering after 4,000 iterations.

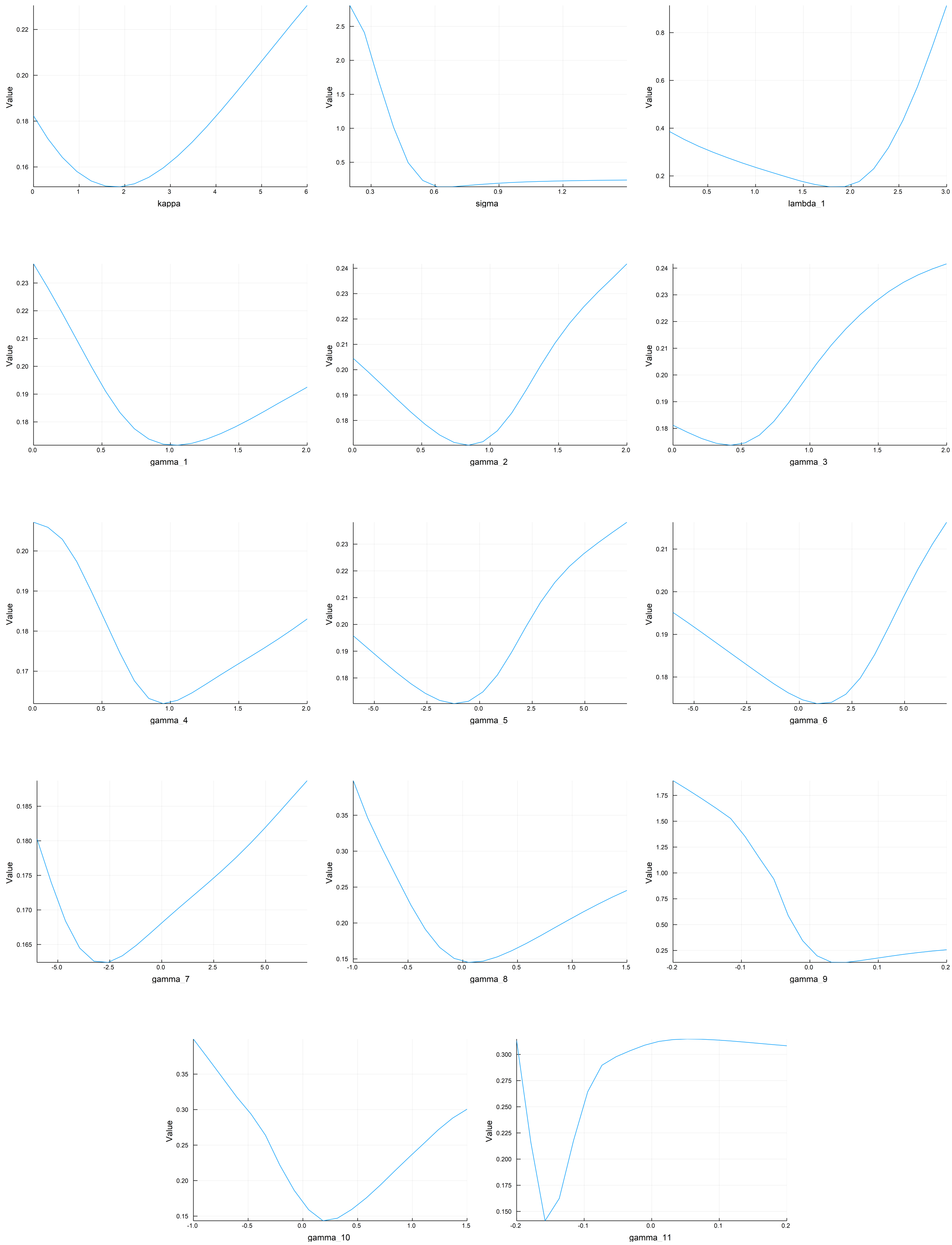

Figure B.2: Objective function of minimization problem (6.1): slices around median values of estimator.

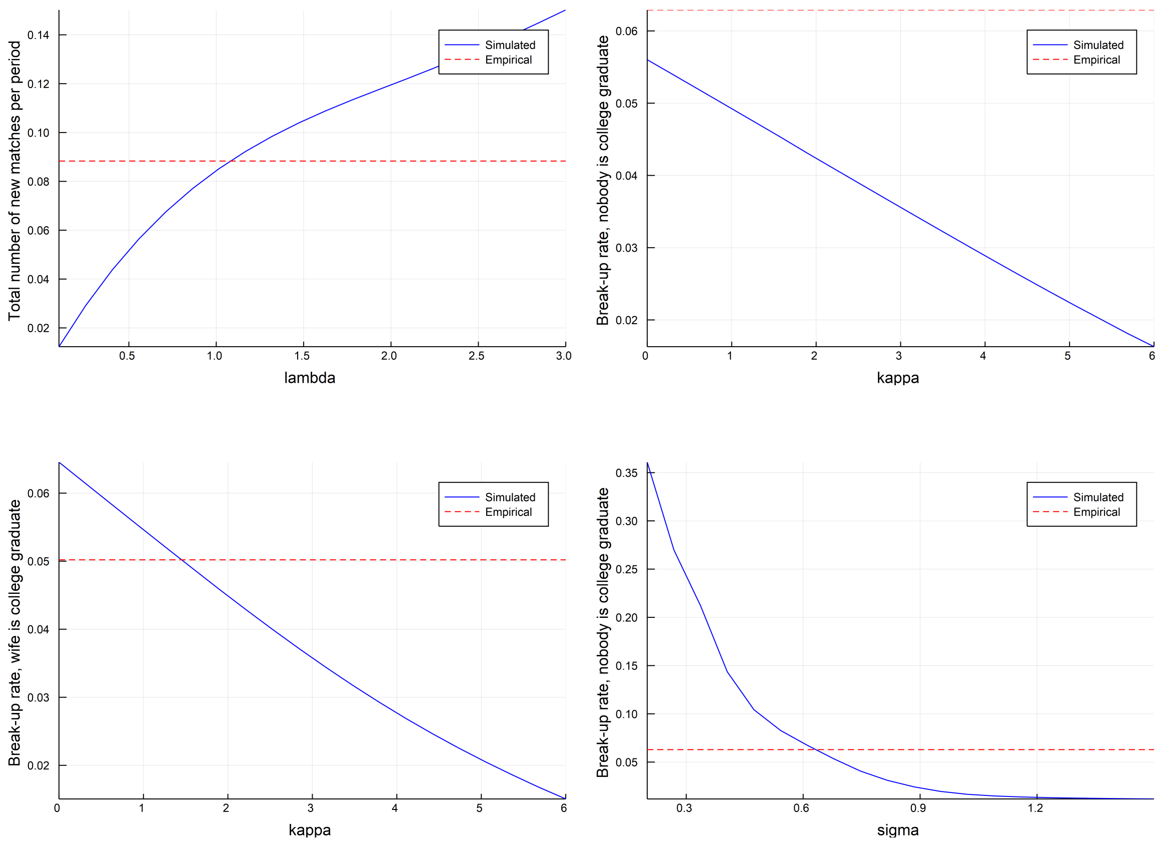

Figure B.3: Moments' response

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