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Abstract

Is lifetime inequality mainly due to differences across people established early in life or to differences in luck experienced over the working lifetime? We answer this question within a model that features idiosyncratic shocks to human capital, estimated directly from data, as well as heterogeneity in ability to learn, initial human capital, and initial wealth. We find that, as of age 23, differences in initial conditions account for more of the variation in lifetime earnings, lifetime wealth and lifetime utility than do differences in shocks received over the working lifetime.

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1 Introduction

To what degree is lifetime inequality due to differences across people established early in life as opposed to differences in luck experienced over the working lifetime? Among the individual differences established early in life, which ones are the most important?

A convincing answer to these questions is of fundamental importance. First, and most simply, an answer serves to contrast the potential importance of the myriad policies directed at modifying or at providing insurance for initial conditions (e.g. public education) against those directed at shocks over the working lifetime (e.g. unemployment insurance). Second, a discussion of lifetime inequality cannot go too far before discussing which specific type of initial condition is the most critical for determining how one fares in life. Third, a useful framework for answering these questions should also be central in the analysis of a wide range of policies considered in macroeconomics, public finance and labor economics.

We view lifetime inequality through the lens of a risky human capital model. Agents differ in terms of three initial conditions: initial human capital, learning ability and financial wealth. Initial human capital can be viewed as controlling the intercept of an agent's mean earnings profile, whereas learning ability acts to rotate this profile. Human capital and labor earnings are risky as human capital is subject to idiosyncratic shocks each period.

We ask the model to account for key features of the dynamics of the earnings distribution. To this end, we document how mean earnings and measures of earnings dispersion and skewness evolve for cohorts of U.S. males. We find that mean earnings are hump shaped and that earnings dispersion and skewness increase with age over most of the working lifetime.[1]

Our model produces a hump-shaped mean earnings profile by a standard human capital channel. Early in life earnings are low because initial human capital is low and agents allocate time to accumulating human capital. Earnings rise as human capital accumulates and as a greater fraction of time is devoted to market work. Earnings fall later in life because human capital depreciates and little time is put into producing new human capital.

Two forces within the model account for the increase in earnings dispersion. One force is that agents differ in learning ability. Agents with higher learning ability have a steeper

[1]Mincer (1974) documents related patterns in U.S. cross-section data. Deaton and Paxson (1994), Storesletten, Telmer and Yaron (2004), Heathcote, Storesletten and Violante (2005) and Huggett, Ventura and Yaron (2006) examine cohort patterns in U.S. repeated cross section or panel data.

mean earnings profiles than low ability agents, other things equal.[2] The other force is that agents differ in idiosyncratic human capital shocks received over the lifetime. These shocks, even when independent over time, produce persistent differences in earnings as they lead to persistent differences in human capital.

To identify the contribution of each of these forces, we exploit the fact that the model implies that late in life little or no new human capital is produced. As a result, moments of the change in wage rates for these agents are almost entirely determined by shocks, rather than by shocks and the endogenous response of investment in human capital to shocks and initial conditions. We estimate the shock process using precisely these moments for older males in U.S. data.[3] Given an estimate of the shock process and other model parameters, we choose the initial distribution of financial wealth, human capital and learning ability across agents to best match the earnings facts described above.[4] We find that learning ability differences produce an important part of the rise in earnings dispersion over the lifetime, given our estimate of the magnitude of human capital risk.

We use our estimates of shocks and initial conditions to quantify the importance of different proximate sources of lifetime inequality. We find that initial conditions (i.e. individual differences existing at age 23) are more important than are shocks received over the rest of the working lifetime as a source of variation in realized lifetime earnings, lifetime wealth and lifetime utility.[5] In the benchmark model, variation in initial conditions accounts for 61.5 percent of the variation in lifetime earnings and 64.0 percent of the variation in lifetime utility. When we extend the benchmark model to also include initial wealth differences as measured in U.S. data, variation in initial conditions accounts for $61.2, 62.4$ and 66.0 percent of the variation in lifetime earnings, lifetime wealth and lifetime utility, respectively.

Among initial conditions, we find that, as of age 23, variation in human capital is sub-

[2]This mechanism is supported by the literature on the shape of the mean age-earnings profiles by years of education. It is also supported by the work of Lillard and Weiss (1979), Baker (1997) and Guvenen (2007). They estimate a statistical model of earnings and find important permanent differences in individual earnings growth rates.

[3]Heckman, Lochner and Taber (1998) use a similar line of reasoning to estimate rental rates across skill groups within a model that abstracts from idiosyncratic risk.

[4]Since a measure of financial wealth is observable, we set the tri-variate initial distribution to be consistent with features of the distribution of wealth among young households.

[5]Lifetime earnings equals the present value of earnings, whereas lifetime wealth equals lifetime earnings plus initial wealth.

stantially more important than variation in either learning ability or financial wealth for how an agent fares in life after this age. This analysis is conducted for an agent with the median value of each initial condition. In the benchmark model with initial wealth differences, we find that a hypothetical one standard deviation increase in initial wealth increases expected lifetime wealth by about 5 percent. In contrast, a one standard deviation increase in learning ability or initial human capital increases expected lifetime wealth by about 8 percent and 47 percent, respectively. Intuitively, an increase in human capital affects lifetime wealth by lifting upwards an agent's mean earnings profile, whereas an increase in learning ability affects lifetime wealth by rotating this profile counterclockwise. We also calculate the permanent percentage change in consumption which is equivalent in expected utility terms to these changes in initial conditions. We find that these permanent percentage changes in consumption are roughly in line with how a change in an initial condition impacts expected lifetime wealth.

We stress an important caveat in interpreting results on the importance of variations in initial conditions. The distribution of initial conditions at a specific age is an endogenously determined distribution from the perspective of an earlier age. To better understand this point, consider a dramatic example. In the last period of the working lifetime, only variation in human capital and financial wealth is important. Variation in learning ability is of no importance for lifetime utility or lifetime wealth over the remaining lifetime. However, from the perspective of an earlier period this does not mean that variation in learning ability is unimportant. In theory, potentially all of the variation in both human capital and financial wealth in the last period of the working lifetime could be due to differences in learning ability early in life. Thus, the results that we find for variation in human capital at age 23 need to be understood as applying at that age. This paper is silent on the prior forces which shape the individual differences that we analyze at age 23.

Background

A leading view of lifetime inequality is based on the standard, incomplete-markets model in which labor earnings over the lifetime is exogenous. Storesletten et. al. (2004) analyze lifetime inequality from the perspective of such a model. Similar models have been widely used in the economic inequality and tax reform literatures.[6] Storesletten et. al. (2004)

[6]See Huggett (1996), Castañeda, Diaz-Jimenez and Rios-Rull (2003), Krueger and Perri (2006), Heathcote, Storesletten and Violante (2006) and Guvenen (2007) among many others.

estimate an earnings process from U.S. panel data to match features of earnings over the lifetime. Within their model, slightly less than half of the variation in realized lifetime utility is due to differences in initial conditions as of age 22. On the other hand, and in the context of a career-choice model, Keane and Wolpin (1997) find a more important role for initial conditions. They find that unobserved heterogeneity realized at age 16 accounts for about 90 percent of the variance in lifetime utility.

We highlight three difficulties with the incomplete-markets model with exogenous earnings, given its connection to our model and given its standing as a workhorse model in macroeconomics. First, the importance of idiosyncratic earnings risk may be overstated because all of the rise in earnings dispersion with age is attributed to shocks. In our model initial conditions account for some of the rise in dispersion. Second, although the exogenous earnings model produces the rise in U.S. within cohort consumption dispersion over the period 1980-90, the rise in consumption dispersion is substantially smaller in U.S. data over a longer time period. Our model produces less of a rise in consumption dispersion because part of the rise in earnings dispersion is due to initial conditions and agents know their initial conditions. Third, the incomplete-markets model is not useful for some purposes. Specifically, since earnings are exogenous, the model can not shed light on how policy may affect inequality in lifetime earnings or may affect welfare through earnings. Models with exogenous wage rates (e.g. Heathcote et. al. (2006)) face this criticism, but to a lesser extent, since most earnings variation is attributed to wage variation. It therefore seems worthwhile to pursue a more fundamental approach that endogenizes wage rate differences via human capital theory. In our view, a successful quantitative model of this type would bridge an important gap between the macroeconomic literature with incomplete markets and the human capital literature and would offer an important alternative workhorse model for quantitative work and policy analysis.

The paper is organized as follows. Section 2 presents the model. Section 3 documents earnings distribution facts and estimates properties of shocks. Section 4 sets model parameters. Sections 5 and 6 analyze the model and answer the two lifetime inequality questions. Section 7 concludes.

2 The Model

An agent maximizes expected lifetime utility, taking initial financial wealth k_1, initial human capital h_1 and learning ability a as given.[7] The decision problem for an agent born at time t is stated below.

$\max_{\{c_j,l_j,s_j,h_j,k_j\}_{j=1}^{J}} E[\sum_{j=1}^{J} \beta^{j-1}u(c_j)]$ subject to

(i) $c_j + k_{j+1} = k_j(1 + r_{t+j-1}) + e_j - T_{j,t+j-1}(e_j, k_j), \forall j$ and $k_{J+1} = 0$

(ii) $e_j = R_{t+j-1}h_j l_j$ if $j < J_R$, and $e_j = 0$ otherwise.

(iii) $h_{j+1} = exp(z_{j+1})H(h_j, s_j, a)$ and $l_j + s_j = 1, \forall j$

The only source of risk to an agent over the working lifetime comes from idiosyncratic shocks to an agent's human capital. Let $z^j = (z_1, ..., z_j)$ denote the j-period history of these shocks. Thus, the optimal consumption choice $c_{j,t+j-1}(x_1, z^j)$ for an age j agent at time $t + j - 1$ is risky as it depends on shocks z^j as well as initial conditions $x_1 = (h_1, k_1, a)$. The period budget constraint says that consumption c_j plus financial asset holding k_{j+1} equals earnings e_j plus the value of assets $k_j(1+r_{t+j-1})$ less net taxes $T_{j,t+j-1}$. Financial assets pay a risk-free, real return r_{t+j-1} at time $t+j-1$. Earnings e_j before a retirement age J_R equal the product of a rental rate R_{t+j-1} for human capital services, an agent's human capital h_j and the fraction l_j of available time put into market work. Earnings are zero at and after the retirement age J_R. An agent's future human capital h_{j+1} is an increasing function of an idiosyncratic shock z_{j+1}, current human capital h_j, time devoted to human capital or skill production s_j, and an agent's learning ability a. Learning ability is fixed over an agent's lifetime and is exogenous.

We now embed this decision problem within a general equilibrium framework and focus on balanced-growth equilibria. There is an aggregate production function $F(K_t, L_t A_t)$ with constant returns in aggregate capital and labor (K_t, L_t) and with labor augmenting technical change $A_{t+1} = A_t(1 + g)$. Aggregate variables are sums of the relevant individual decisions across agents. In defining aggregates, ψ is a time-invariant distribution over initial conditions

[7] The model generalizes Ben-Porath (1967) to allow for risky human capital and extends Levhari and Weiss (1974) and Eaton and Rosen (1980) to a multi-period setting. Krebs (2004) analyzes a multi-period model of human capital with idiosyncratic risk. Our work differs in its focus on lifetime inequality among other differences.

x_1 and μ_j is the fraction of age j agents in the population. Population fractions satisfy $\sum_{j=1}^{J}\mu_j = 1$ and $\mu_{j+1} = \mu_j/(1+n)$, where n is a constant population growth rate. In the analysis of equilibrium, we consider the case where initial financial assets are zero and, thus, ψ is effectively a bivariate distribution over $x_1 = (h_1, a)$.

$$K_t \equiv \textstyle\sum_{j=1}^{J}\mu_j \int E[k_{j,t}(x_1, z^j)]d\psi \text{ and } L_t \equiv \sum_{j=1}^{J}\mu_j \int E[h_{j,t}(x_1, z^j)l_{j,t}(x_1, z^j)]d\psi$$

$$C_t \equiv \textstyle\sum_{j=1}^{J}\mu_j \int E[c_{j,t}(x_1, z^j)]d\psi \text{ and } T_t \equiv \sum_{j=1}^{J}\mu_j \int E[T_{j,t}(e_{j,t}, k_{j,t})]d\psi$$

Definition: A balanced-growth equilibrium is a collection of decisions $\{\{c_{j,t}, l_{j,t}, s_{j,t}, h_{j,t}, k_{j,t}\}_{j=1}^{J}\}_{t=-\infty}^{\infty}$, *factor prices, government spending and taxes* $\{R_t, r_t, G_t, T_t\}_{t=-\infty}^{\infty}$ *and a distribution* ψ *over initial conditions such that*

(1) Agent decisions are optimal, given factor prices.

(2) Competitive Factor Prices: $R_t = A_t F_2(K_t, L_t A_t)$ *and* $r_t = F_1(K_t, L_t A_t) - \delta$

(3) Resource Feasibility: $C_t + K_{t+1}(1+n) + G_t = F(K_t, L_t A_t) + K_t(1-\delta)$

(4) Government Budget: $G_t = T_t$

(5) Balanced Growth: (i) $\{c_{j,t}, k_{j,t}\}_{j=1}^{J}$ *grow at rate* g *as a function of time, whereas* $\{l_{j,t}, s_{j,t}, h_{j,t}\}_{j=1}^{J}$ *are time invariant. (ii)* (G_t, T_t, R_t) *grow at rate* g*, whereas* r_t *is time invariant.*

Our focus on balanced-growth equilibria requires that individual decisions, aggregate variables and factor prices grow at constant rates. Balanced growth leads us to employ homothetic preferences and a constant returns technology. More specifically, we use the property that if preferences over lifetime consumption plans are homothetic and the budget set for consumption plans is homogeneous of degree 1 in rental rates, then optimal consumption plans are homogeneous of degree 1 in rental rates.[8]

[8]Let $\Gamma(x_1, \vec{R})$ denote the set of lifetime consumption plans satisfying budget conditions (i)-(iii), given initial conditions x_1 and rental rates $\vec{R} = (R_1, ..., R_J)$. $\Gamma(x_1, \vec{R})$ is homogeneous in $\vec{R}$ provided $c \in \Gamma(x_1, \vec{R}) \Rightarrow \lambda c \in \Gamma(x_1, \lambda\vec{R}), \forall \lambda > 0$. $\Gamma(x_1, \vec{R})$ has this property when taxes T_{jt} are homogeneous of degree 1 in earnings and assets and when initial assets are zero. The model tax system (see section 4) induces this property when $T_{jt}(\vec{R}, h_j, l_j, k_j)$ is homogeneous of degree 1 in $(\vec{R}, k_j)$.

The functional forms that we employ are provided below. The equilibrium concept does not restrict the functional forms for the human capital production function $H(h, s, a)$, the distribution of initial conditions ψ or the nature of idiosyncratic shocks. The human capital production function is of the Ben-Porath class which is widely used in empirical work. The distribution ψ is a bivariate lognormal distribution which allows for a skewed distribution of initial human capital. Recall that our equilibrium analysis considers the case where initial assets are set to zero. Idiosyncratic shocks are independent and identically distributed over time and follow a normal distribution.

$$u(c) = c^{(1-\rho)}/(1-\rho), F(K, LA) = K^{\gamma}(LA)^{1-\gamma} \text{ and } H(h, s, a) = h + a(hs)^{\alpha}$$

$$x = (h_1, a) \sim \psi = LN(\mu_x, \Sigma) \text{ and } z \sim N(\mu, \sigma^2)$$

We comment on four key features of the model. First, while the earnings of an agent are stochastic, the earnings distribution for a cohort of agents evolves deterministically. This occurs because the model has idiosyncratic but no aggregate risk.[9] Second, the model has two sources of growth in earnings dispersion within a cohort - agents have different learning abilities and different shock realizations. The next section characterizes empirically the rise in U.S. earnings dispersion over the working lifetime. Third, although the model has a single source of shocks, which are independently and identically distributed over time, we will show that this structure is sufficient to endogenously produce many of the statistical properties of earnings that researchers have previously estimated. Fourth, the model implies that the nature of human capital shocks can be identified from wage rate data, independently from all other model parameters. This holds, as an approximation, because the model implies that the production of human capital goes to zero towards the end of the working lifetime. The next section develops the logic of this point.

3 Empirical Analysis

We use data to address two issues. First, we characterize how mean earnings and measures of earnings dispersion and skewness evolve with age for a cohort. Second, we estimate a human capital shock process from wage rate data.

[9] More specifically, the probability that an agent receives a j-period shock history z^j is also the fraction of the agents in a cohort that receive z^j.

3.1 Age Profiles

We estimate age profiles for mean earnings and measures of earnings dispersion and skewness for ages 23 to 60. We use earnings data for males who are the head of the household from the Panel Study of Income Dynamics (PSID) 1969-2004 family files. To calculate earnings statistics at a specific age and year, we employ a centered 5-year age bin.[10] For males over age 30, we require that they work between 520 and 5820 hours per year and earn at least 1500 dollars (in 1968 prices) a year. For males age 30 and below, we require that they work between 260 and 5820 hours per year and earn at least 1000 dollars (in 1968 prices).

These selection criteria are motivated by several considerations. First, the PSID has many observations in the middle but relatively fewer at the beginning or end of the working life cycle. By focusing on ages 23-60, we have at least 100 observations in each age-year bin with which to calculate earnings statistics. Second, labor force participation falls near the traditional retirement age for reasons that are abstracted from in the model. This motivates the use of a terminal age that is below the traditional retirement age. Third, the hours and earnings restrictions are motivated by the fact that within the model the only alternative to time spent working is time spent learning. For males above 30, the minimum hours restriction amounts to a quarter of full-time work hours and the minimum earnings restriction is below the annual earnings level of a full-time worker working at the federal minimum wage.[11] For younger males, we lower both the minimum hours and earnings restrictions to capture students doing summer work or working part-time while in school.

We now document how mean earnings, two measures of earnings dispersion and a measure of earnings skewness evolve with age for cohorts. We consider two measures of dispersion: the variance of log earnings and the Gini coefficient of earnings. We measure skewness by the ratio of mean earnings to median earnings.

The methodology for extracting age effects is in two parts. First, we calculate the statistic of interest for males in age bin j at time t and label this $stat_{j,t}$. For example, for mean earnings we set $stat_{j,t} = ln(e_{jt})$, where e_{jt} is real mean earnings of all males in the age bin centered at age j in year t.[12] Second, we posit a statistical model governing the evolution of

[10] To calculate statistics for the age 23 and the age 60 bin we use earnings for males age 21-25 and 58-62.

[11] A full-time worker (working 40 hours per week and 52 weeks a year) who receives the federal minimum wage in 1968 earns $3,328$ dollars in 1968 prices.

[12] We use the Consumer Price Index to convert nominal earnings to real earnings.

the earnings statistic as indicated below. The earnings statistic is viewed as being generated by several factors that we label cohort (c), age (j), and time (t) effects. We wish to estimate the age effects β_j^{stat}. We employ a statistical model as our economic model is not sufficiently rich to capture all aspects of time variation in the data.

$$stat_{j,t} = \alpha_c^{stat} + \beta_j^{stat} + \gamma_t^{stat} + \epsilon_{j,t}^{stat}$$

The linear relationship between time t, age j, and birth cohort $c = t - j$ limits the applicability of this regression specification. Specifically, without further restrictions the regressors in this system are co-linear and these effects cannot be estimated. This identification problem is well known.[13] Any trend in the data can be arbitrarily reinterpreted as due to year (time) effects or alternatively as due to age or cohort effects.

Given this problem, we provide two alternative measures of age effects. These correspond to the cohort effects view where we set $\gamma_t^{stat} = 0, \ \forall t$ and the time effects view where we set $\alpha_c^{stat} = 0, \ \forall c$. We use ordinary least squares to estimate the coefficients.[14]

In Figure 1(a) we graph the age effects of the levels of mean earnings implied by each regression. Figure 1 highlights the familiar hump-shaped profile for mean earnings. Mean earnings almost doubles from the early 20's to the peak earnings age. Figure 1 is constructed using the coefficients $exp(\beta_j)$ from the regression based upon mean earnings. The age effects in Figure 1(a) are first scaled so that mean earnings at age 38 in both views pass through the mean value across panel years at this age and are then both scaled so the time effects view is normalized to equal 100 at age 60.

Figure 1(b)-(d) graphs the age effects for measures of earnings dispersion and skewness. Our measures of dispersion are the variance of log earnings and the Gini coefficient, whereas skewness is measured by the ratio of mean earnings to median earnings. Each profile is normalized to run through the mean value of each statistic across panel years at age 38. Figures 1(b)-(d) show that measures of dispersion and skewness increase with age in both the time and cohort effects views. The cohort effect view implies a rise in the variance of

[13] See Weiss and Lillard (1978) and Deaton and Paxson (1994) among others.

[14] A third approach, discussed in Huggett et. al. (2006), allows for age, cohort and time effects but with the restriction that time effects are mean zero and are orthogonal to a time trend. That is $(1/T)\sum_{t=1}^{T}\gamma_t^{stat} = 0$ and $(1/T)\sum_{t=1}^{T} t\gamma_t^{stat} = 0$. Thus, trends over time are attributed to cohort and age effects rather than time effects. The results of this approach for means, dispersion and skewness are effectively the same as those for cohort effects.

log earnings of about 0.4 from age 23 to 60 whereas the time effects view implies a smaller rise of about 0.2. Thus, there is an important difference in the rise in dispersion coming from these two views. The same qualitative pattern holds for the Gini coefficient measure of dispersion. Figure 1(d) shows that the rise in earnings skewness with age is also greater for the cohort effects view than for the time effects view.

We will ask the economic model to match both views of the evolution of the earnings distribution. Given the lack of a consensus in the literature, we are agnostic about which view should be stressed. To conserve space, the paper highlights the results of matching the time effects view in the main text but summarizes results for the cohort effects view in section 6.3.[15] It turns out that both views give similar answers to the two lifetime inequality questions that we pose.

3.2 Human Capital Shocks

The model implies that an agent's wage rate, defined within the model as earnings per unit of work time, equals the product of the rental rate and an agent's human capital. Here it is important to recall that within the model work time and learning time are distinct activities. The model also implies that late in the working lifetime human capital investments are approximately zero. This occurs as the number of working periods over which an agent can reap the returns to these investments falls as the agent approaches retirement. The upshot is that when there is no human capital investment over a period of time, then the change in an agent's wage rate is in theory entirely determined by rental rates and the human capital shock process and not by any other model parameters.

In what follows, assume that in periods t through $t+n$ an individual devotes zero time to learning. The first equation below states that the wage w_{t+n} is determined by the rental rate R_{t+n}, shocks $(z_{t+1}, ..., z_{t+n})$ and human capital h_t. Here it is understood that $h_{t+1} = exp(z_{t+1})H(h_t, s_t, a) = exp(z_{t+1})[h_t + f(h_t, s_t, a)]$ and that there is zero human capital production in periods when there is no investment (i.e. $f(h, s, a) = 0$ when $s = 0$). The second equation takes logs of the first equation, where a hat denotes the log of a variable. The third equation states that measured n-period log wage differences (denoted $y_{t,n}$) are true log wage differences plus measurement error differences $\epsilon_{t+n} - \epsilon_t$. The third equation highlights the point that log wage differences are due solely to rental rate differences and

[15]Heathcote et. al. (2005) present an argument for stressing the importance of time effects.

shocks.

$$w_{t+n} = R_{t+n}h_{t+n} = R_{t+n}exp(z_{t+n})H(h_{t+n-1}, 0, a) = R_{t+n}\prod_{i=1}^{n} exp(z_{t+i})h_t$$

$$\hat{w}_{t+n} = \ln w_{t+n} = \hat{R}_{t+n} + \sum_{i=1}^{n} z_{t+i} + \hat{h}_t$$

$$y_{t,n} = \hat{w}_{t+n} - \hat{w}_t + \epsilon_{t+n} - \epsilon_t = \hat{R}_{t+n} - \hat{R}_t + \sum_{i=1}^{n} z_{t+i} + \epsilon_{t+n} - \epsilon_t$$

Our strategy for estimating the nature of human capital shocks is based on the log-wage-difference equation. Thus, it is important to be able to measure the wage concept used in the theory and to have individuals for which the assumption of no time spent accumulating human capital is a reasonable approximation. The wage concept in the theoretical model is earnings per unit of work time. Thus, two critical assumptions are that (1) measured work time is only work time and not a combination of work and learning time - distinct activities in human capital theory and in our model - and (2) no time on the job or off the job is spent learning and, thus, producing new human capital. We focus on older workers to address both of these issues. Young workers are likely, in our view, to be problematic on both issues.

We use the log-wage-difference equation and make some specific assumptions. We assume that both human capital shocks z_t and measurement errors ϵ_t are independent and identically distributed over time and people. Furthermore, we assume that $z_t \sim N(\mu, \sigma^2)$ and $Var(\epsilon_t) = \sigma_\epsilon^2$. These assumptions imply the three cross-sectional moment conditions below.

$$E[y_{t,n}] = \hat{R}_{t+n} - \hat{R}_t + n\mu$$

$$Var(y_{t,n}) = n\sigma^2 + 2\sigma_\epsilon^2$$

$$Cov(y_{t,n}, y_{t,m}) = m\sigma^2 + \sigma_\epsilon^2 \text{ for } m < n$$

We calculate real wages in PSID data as total male labor earnings divided by total hours for male head of household, using the Consumer Price Index to convert nominal wages to real wages. We follow males for four years and thus calculate three log wage differences (i.e. $y_{t,n}$ for $n = 1, 2, 3$). In utilizing the wage data we impose the same selection restriction as in the construction of the age-earnings profiles but also exclude observations for which earnings growth is above (below) 20 (1/20) to trim potential extreme measurement errors. In estimation we use all cross-sectional variances and all cross-sectional covariances aggregated across panel years.[16] For each year we generate the sample analog to the moments: $\mu_{t,n} \equiv \frac{1}{N_t}\sum_{i=1}^{N_t} y_{t,n}^i$ and $\frac{1}{N_t}\sum_{i=1}^{N_t}(y_{t,n}^i - \mu_{t,n})^2$ and $\frac{1}{N_t}\sum_{i=1}^{N_t}(y_{t,n}^i - \mu_{t,n})(y_{t,m}^i - \mu_{t,m})$. We stack the moments across the panel years and use a 2-step General Method of Moments estimation with an identity matrix as the initial weighting matrix.

Table 1 provides the estimation results. The top panel of Table 1 considers the sample of males age 55-65 and a comparison sample of males age 50-60. The point estimate for the age 55-65 sample is $\sigma = .111$ so that a one standard deviation shock moves wages by about 11 percent. This is the shock estimate that we employ in our analysis of lifetime inequality. When we analyze the age 50-60 sample, we find that the point estimate is $\sigma = .117$. This slightly younger sample may violate assumptions (1) and (2) to a greater degree but may also display less selection out of the work force in response to shocks as compared to the 55-65 age group.

The remainder of Table 1 examines sensitivity in two directions. First, the middle panel of Table 1 examines sensitivity to alternative minimum annual earnings levels stated in 1968 dollars. We find that the point estimate of σ increases for males age 55-65 as the minimum earnings level in the sample is lowered. As a reference point, we note that a full-time worker (working 40 hours per week and 52 weeks) who receives the federal minimum wage earns $3,328$ dollars in 1968 prices. Second, the bottom panel of Table 1 considers estimates based on different subperiods. The point estimates for 55-65 age group are about the same for both subperiods. The 50-60 age group has a smaller point estimate in the 1969-1981 subperiod as compared to the 1982-2004 subperiod. It is well known that cross-sectional measures of earnings and wage inequality increased over the period 1982-2004.

4 Setting Model Parameters

This section sets model parameters. The parameters are listed in Table 2 and are set in two steps. The first collection of model parameters is set without solving the model. The

[16] The PSID data is not available for the years 1997, 1999, 2001, and 2003. Thus, we can not calculate the sample analog to the covariance $Cov(y_{t,n}, y_{t,m})$ for $t \geq 1996$ and $m \neq n$, given that the max n value we consider is $n = 3$. Thus, in estimation we use all variances and covariances that can be calculated in the data, given our choice of $n = 3$

remaining model parameters are set so that the equilibrium properties of the model best match the earnings distribution facts documented in the previous section while matching some steady-state quantities.

The first step is to set parameters governing demographics, preferences, technology, the tax system and shocks.

Demographics:

Demographic parameters (J, J_R, n) are set using a model period of one year. An agent lives from a real-life age of 23 to a real-life age of 75 so that the lifetime is $J = 53$ periods. The agent receives retirement benefits at age $J_R = 39$ or a real-life age of 61. The population growth rate is set to $n = .012$. This is the geometric average over 1959-2007 from the Economic Report of the President (2008, Table B35).

Preferences:

The value of the parameter governing risk aversion and intertemporal substitution is set to $\rho = 2$. This value lies in the middle of the range of estimates based upon micro-level data which are surveyed by Browning, Hansen and Heckman (1999, Table 3.1) and is the value used by Storesletten et. al. (2004) in their analysis of lifetime inequality.

Technology:

We set the parameter $\gamma = .322$ governing the capital elasticity of the Cobb-Douglas production function (i.e. $F(K, LA) = K^{\gamma}(LA)^{1-\gamma}$) to match capital and labor's share of output.[17] The depreciation rate on physical capital $\delta = .067$ is set so that the return to capital equals the U.S. data value, given γ.[18] The growth rate of labor-augmenting

[17] Labor's share is the 1959-2006 average based on Economic Report of the President (2008, Table B26 and B28) and Bureau of Economic Analysis (2008, Table 1.1.2) and is calculated as the compensation of employees divided by national income plus depreciation less proprietor's income and indirect taxes.

[18] We use two restrictions: $K/Y = K/F(K, LA)$ and $r = F_1(K, LA) - \delta$. The first pins down K/LA given γ. The second pins down δ, given γ and K/LA. We later choose the discount factor β to produce the equilibrium quantity K/LA as described in the Appendix. Thus, we choose two model parameters (δ, β) to satisfy two restrictions. The return to capital $r = .042$ is the average of the annual real return to stock and long-term bonds over the period 1946-2001 from Siegel (2002, Table 1-1 and 1-2). The capital-output ratio averages $K/Y = 2.947$ over the period 1959-2000. The capital measure includes fixed private capital, durables, inventories and land. The data for capital and land are from Bureau of Economic Analysis (Fixed Asset Account Tables) and Bureau of Labor Statistics (Multifactor Productivity Program Data).

technological change $g = .0019$ is set to equal the average growth rate of mean male real earnings in PSID data over the period 1968-2001.

Tax System:

The tax system T_j includes a social security and an income tax: $T_j = T_j^{ss} + T_j^{inc}$. Social security has a proportional earnings tax of 10.6 percent, the old-age and survivors insurance tax rate in the U.S., before the retirement age. The social security system pays a common retirement transfer after the retirement age set equal 40 percent of mean economy-wide earnings in the last period of an agent's working lifetime - denoted $\bar{e}$. We set this quantity using the mean earnings profile in Figure 1. The income tax in the model captures the pattern of effective average federal income tax rates as a function of income as documented in Congressional Budget Office (2004, Table 3A and 4A). These average tax rates rise with income in cross section. The average tax rate schedule applies to an agent's income. Details for how this income tax function is implemented are in Huggett and Parra (2010). The income tax rates in the model are indexed over time to the growth in the rental rate of human capital. This tax system produces a budget set which is homogeneous in rental rates as discussed in section 2.

Shocks:

The parameters of the shock process are (μ, σ). The standard deviation of human capital shocks is set to $\sigma = 0.111$ based on the estimate from Table 1. We set $\mu = -.029$, governing the mean human capital shock, so that the model matches the average rate of decline of mean earnings for the cohorts of older workers in U.S. data documented in Figure 1. The ratio of mean earnings between adjacent model periods equals $(1+g)e^{\mu+\sigma^2/2}$ when agents make no human capital investments. Thus, μ is set, given the value g and σ, so that mean earnings in the model late in life fall at the rates documented in Figure 1. The quantity $E[exp(z)] = e^{\mu+\sigma^2/2}$ can be viewed as one minus the mean rate of depreciation of human capital. The values in Table 2 imply a mean depreciation rate of approximately two percent per year.[19]

[19] We acknowledge that while the theory asserts that the total time allocated to work and learning is the same at each age, our PSID sample displays a fall in work hours towards the end of the working lifetime. The fall in mean log PSID work hours for males age 50-60 is approximately 1 percent per year. This suggests that our implied mean depreciation rate may be somewhat too large. We also examined whether the age patterns in the variance of log earnings are primarily due to movements in the variance of log wages or the variance in PSID log work hours. In our sample, the variance of log earnings and log wages display very

The remaining model parameters are set so that the equilibrium properties of the model best match the earnings distribution facts. The Appendix describes the distance metric between data and model statistics. The parameters selected are those governing the distribution of initial conditions $\psi = LN(\mu_x, \Sigma)$, the elasticity of the human capital production function α and the agent's discount factor β. We do this in two stages. Given any trial guess of (μ_x, Σ, α), we set the remaining parameter β so that the model produces the equilibrium real return to capital (i.e. $r = .042$) used earlier to set technology parameters. The elasticity parameter is then $\alpha = .70$ and initial conditions are described by $(\mu_h, \mu_a, \sigma_h^2, \sigma_a^2, \sigma_{ha}) = (4.66, -1.12, .213, .012, .041)$.[20]

We have examined the fit of the model at pre-specified values of the parameter α, while choosing the parameters of the initial distribution to best match the earnings distribution facts. The distance between model and data statistics displays a U-shaped pattern in the parameter α, where the bottom of the U is the value in Table 2. For values of α above the value in Table 2 the model produces too large of a rise in dispersion and skewness compared to the patterns in Figure 1. The parameter α has been estimated in the human capital literature. The estimates, surveyed by Browning et. al. (1999, Table 2.3- 2.4), lie in the range 0.5 to just over 0.9. These estimates are based on models that abstract from idiosyncratic risk.

5 Properties of the Benchmark Model

In this section, we report on the ability of the benchmark model to produce the earnings facts documented in section 3. We also provide a number of other properties of the benchmark model. Specifically, we highlight the model implications for consumption dispersion over the lifetime and for the statistical properties of earnings and wage rates.

similar patterns. Hence, the pattern in earnings dispersion in our sample is not driven by the dispersion pattern in hours.

[20] It is important to note that the model does not trivially fit the age profiles. After estimating the process for shocks, there are 5 parameters governing initial conditions and 1 parameter governing human capital production to fit 3 age profiles, 3×38 moments.

5.1 Dynamics of the Earnings Distribution

The age profiles of mean earnings, earnings dispersion and skewness produced by the benchmark model are displayed in Figure 2. The model generates the hump-shaped earnings profile for a cohort by a standard human capital argument. Early in the working life cycle, individuals devote more time to human capital production than at later ages. These time allocation decisions lead to a net accumulation of human capital in the early part of the working life cycle. Thus, mean earnings increase with age as human capital and mean time worked increase with age.

Towards the end of the working life-cycle, mean human capital levels fall. This happens as the mean multiplicative shock to human capital is smaller than one (i.e. $E[exp(z)] = exp(\mu + \sigma^2/2) < 1$). This corresponds to the notion that on average human capital depreciates. The implication is that average earnings in Figure 2 fall late in life because growth in the rental rate of human capital is not enough to offset the mean fall in human capital.

Figure 3 graphs the age profiles of the mean fraction of time allocated to human capital production and the mean human capital levels that underlie the earnings dynamics in the model. Figure 3(a) shows that approximately 25 percent of available time is directed at human capital production at age 23 but less than 5 percent of available time is used after age 55. This result is consistent with a key assumption we employ to identify human capital shocks: human capital production is negligible towards the end of the working lifetime.

Figure 3(b) shows that the mean human capital profile is hump shaped and that it is flatter than the earnings profile. A relatively flat mean human capital profile and a declining time allocation profile to human capital production is how the model accounts for a hump-shaped earnings profile. This point is quite important. The fact that the mean human capital profile is flatter than the earnings profile means that average human capital as of age 23 is quite high. This is a key reason why we find in the next section that human capital differences are such an important source of individual differences at age 23 compared to ability differences.

Two forces account for the rise in earnings dispersion. First, since individual human capital is repeatedly hit by shocks, these shocks are a source of increasing dispersion in human capital and earnings as a cohort ages. Second, differences in learning ability across agents produce mean earnings profiles with different slopes. This follows since within an age group, agents with high learning ability choose to produce more human capital and devote more time to human capital production than their low ability counterparts. Huggett et. al. (2006, Proposition 1) establish that this holds in the absence of human capital risk. This mechanism implies that earnings of high ability individuals are relatively low early in life and relatively high late in life compared to agents with lower learning ability, holding initial human capital equal.

5.2 Earnings Dispersion: Risk Versus Ability Differences

We now try to understand the quantitative importance of risk and ability differences for producing the increase in earnings dispersion in the benchmark model. We do so by either eliminating ability differences or eliminating shocks. The analysis holds factor prices constant as risk or ability differences are varied.

We eliminate ability differences by changing the initial distribution so that all agents have the same learning ability, which we set equal to the median ability. In the process of changing learning ability, we do not alter any agent's initial human capital. Figure 4(a) shows that eliminating ability differences flattens the rise in the variance of log earnings with age. Even more striking, earnings dispersion actually falls over part of the working lifetime. This latter result is due to two opposing forces. First, human capital risk leads ex-ante identical agents to differ ex-post in human capital and earnings. Second, the model has a force which leads to decreasing dispersion in human capital and earnings with age. It is quite amazing that this force has received almost no attention in work which interprets patterns of earnings dispersion over the lifetime. Without risk and without ability differences, all agents within an age group produce the same amount of new human capital regardless of the current level of human capital – see Huggett et. al. (2006, Proposition 1). This holds for any value of the elasticity parameter $\alpha \in (0, 1)$ of the human capital production function and is independent of the utility function. This implies that as a cohort of agents ages both the distribution of human capital and earnings become more equal. Specifically, the Lorenz curves associated with these distributions shift inward towards the 45 degree line as a cohort ages. Thus, measures of earnings or human capital dispersion that respect the Lorenz order (e.g. the Gini coefficient) decrease for a cohort as the cohort ages in the model without risk and ability differences.

Figure 4(a) shows that earnings dispersion increases at the very end of the working lifetime. This occurs as human capital production at the end of life goes to zero because the time allocated to production goes to zero. This means that the opposing force leading to convergence is gradually eliminated with age.

To highlight the role of human capital risk, we eliminate idiosyncratic risk by setting $\sigma = 0$. We adjust the mean log shock μ to keep the mean shock level constant. We do not change the distribution of initial conditions. Removing idiosyncratic risk leads to a counter-clockwise rotation of the mean earnings profile and an L-shaped earnings dispersion profile. Figure 4(b) displays these results. When idiosyncratic risk is eliminated, human capital accumulation becomes more attractive for risk-averse agents. Thus, all else equal, agents spend a greater fraction of time accumulating human capital early in life. The result is a counter-clockwise movement in the mean earnings profile.[21] Eliminating risk results in substantial changes in the time allocation decisions of agents with relatively high learning ability. Absent risk, these agents allocate an even larger fraction of time into human capital accumulation. This leads to very high earnings dispersion early in life as some of these agents have very low earnings. Later in life these agents have higher earnings than agent's with lower learning ability, other things equal.

5.3 Properties of the Initial Distribution

Table 3 summarizes properties of the distribution of initial conditions. A key finding is that human capital and learning ability are positively correlated at age 23. To develop some intuition, consider two agents who differ in learning ability but have the same initial human capital. The mean earnings profile for the agent with higher learning ability is rotated counter clockwise from his lower ability counterpart due to the extra time spent in learning early in life and the greater amount of human capital built up later in life. Thus, if there were a zero correlation in learning ability and human capital at age 23, then the model would produce a U-shaped earnings dispersion profile. The rise in dispersion later in life would be due in part to high ability agents overtaking the earnings of low learning ability agents. Given that Figure 1 does not support a strong U-shaped dispersion profile over ages 23-60 in U.S. data, the model accounts for this fact by making learning ability and human capital positively correlated at age 23. Thus, if high learning ability agents also have high initial human capital, then this produces an upward shift of an agent's mean earnings profile to

[21]Levhari and Weiss (1974) examine this issue in a two-period model. They show that time input into human capital production is smaller with human capital risk than without when agents are risk averse.

eliminate the strong U-shaped dispersion profile that otherwise would occur.

Table 3 also summarizes the distribution of initial conditions in the model with initial wealth differences. We choose this distribution to be in essence a tri-variate lognormal distribution. The parameters related to financial wealth are set to match features of financial wealth holding of young households in U.S. data as is explained in section 6.1. The remaining parameters of this distribution are selected to match the earnings facts in Figure 1. Table 3 shows that the distribution of human capital and learning ability in the model with initial wealth differences has similar properties compared to the model without initial wealth differences. This foreshadows later results where we find that the two models have similar implications for lifetime inequality.

5.4 Statistical Models of Earnings

We now examine what an empirical researcher might conclude about the nature of earnings risk in the benchmark model from the vantage point of a standard statistical model of earnings. A common view in the literature is that log earnings is the sum of a predictable component capturing age and time effects among other things and an idiosyncratic component with mean zero. The former is a function of observables X_t^i. The latter is the sum of a fixed effect α^i, a growth rate effect $\beta^i j$, a persistent shock $z_{j,t}^i$ and a purely temporary shock $\epsilon_{j,t}^i$, where (i, j, t) index agents, age and time. That is log earnings are assumed to follow,

$$
\begin{aligned}
\log e_{j,t}^i &= g(\theta, X_t^i) + \alpha^i + \beta^i j + z_{j,t}^i + \epsilon_{j,t}^i \\
z_{j,t}^i &= \rho z_{j-1,t-1}^i + \eta_{j,t}^i, \qquad z_{1,t}^i = 0
\end{aligned}
$$

where ρ is the autocorrelation of the persistent component and $(\sigma_\alpha^2, \sigma_\beta^2, \sigma_\eta^2, \sigma_\epsilon^2, \sigma_{\alpha\beta})$ are the respective variances and covariances. The variables $(\alpha^i, \eta_{j,t}^i, \epsilon_{j,t}^i)$ are uncorrelated as are the variables $(\beta^i, \eta_{j,t}^i, \epsilon_{j,t}^i)$.

This type of model has been extensively examined in the literature. The literature can be separated into a strand (see MaCurdy (1982), Hubbard, Skinner and Zeldes (1995), Storesletten, Telmer and Yaron (2004) among others) that imposes $\beta^i = 0$ and a strand (see Lillard and Weiss (1979), Baker (1997) and Guvenen (2007) among others) that allows for heterogeneity in this coefficient. Following Guvenen (2007), we will call the former models

RIP models (restricted income profiles) and the latter HIP models (heterogeneous income profiles). These two strands of the literature come to different conclusions about the degree of persistence of shocks. The RIP literature finds that ρ is close to 1, whereas the HIP literature finds that ρ is substantially below 1. Meghir and Pistaferri (2010) review this literature.

Table 4 presents results from repeatedly estimating the parameters of RIP and HIP models, using earnings data drawn from our benchmark model. Specifically, we simulate the model 500 times with each simulation having 200 observations per age group. We add zero mean and normally distributed measurement error to each realization of the log of model earnings. The standard deviation of the measurement error is set to 0.15 following the estimate in Guvenen and Smith (2008). For each simulated panel we use variances and auto covariances to estimate model parameters.[22] Table 4 provides the means and standard deviations, in parentheses, of the parameter estimates across the 500 simulations.

We take away two main messages from the findings in Table 4. First, an empirical researcher would conclude that our human capital model produces coefficients governing persistent shocks (ρ, σ_η^2) that are similar to results found using U.S. data. This holds for the RIP and HIP model under either of the two treatments for the number of auto covariances used in estimation. For example, Guvenen (2007, Table 1) finds that in U.S. data $(\rho, \sigma_\eta^2) = (.988, .015)$ for the RIP model and $(\rho, \sigma_\eta^2) = (.821, .029)$ for the HIP model. Second, one should be cautious in making statements about the true nature of shocks based on information derived from such statistical models. Specifically, there seems to be the view in the literature that the HIP model captures some of the structure of human capital models and may therefore be useful in uncovering the underlying structure of shocks in human capital models. Shocks in our human capital model are independent and identically distributed over time but produce what an empirical researcher might describe as persistent earnings shocks. This occurs because the human capital accumulation mechanism propagates the effect of our shocks into the future.

[22]First, estimate the deterministic component to calculate estimated residuals. This amounts to calculating the sample mean of log earnings at each age. Second, calculate sample variances and covariances of the residuals. Third, the estimate is the parameter vector minimizing the equally-weighted squared distance between sample and model moments. All variance and covariance restrictions of the statistical model are used in estimation. Table 4 analyzes two cases: the first (labeled $cov = 1$) uses one auto covariance in estimation, whereas the second (labeled $cov = 5$) uses five auto covariances in estimation.

We also analyze the autocorrelation structure of the first differences of log earnings. It is well known in the labor literature that these earnings growth rates display negative first-order autocorrelation and close to zero higher-order autocorrelations (e.g. Abowd and Card (1989)). Using the simulated earnings data from our model, we find that the first-order, autocorrelation coefficient is $-.386$ and higher order coefficients are nearly zero.[23] The first-order autocorrelation estimates in Abowd and Card (1989) range from -0.23 to -0.44 across sample years.

In sum, the analysis in this subsection indicates that our benchmark human capital model, with shocks inferred from the wage rates of older workers, is broadly consistent with the dynamics of earnings and earnings growth rates as documented in the literature.

5.5 Statistical Models of Wage Rates

The benchmark model that we analyze imposes that shocks to human capital arriving each period have a standard deviation set to $\sigma = .111$. While the previous section documented that a number of the earnings properties of the resulting model are consistent with earnings properties from previous empirical studies, one might ask whether such shocks are broadly consistent with wage rate data?

Heathcote, Storesletten and Violante (2006) posit a statistical model for wage rates that is broadly similar in structure to the earnings models reviewed in the last section. They then estimate that the persistent component of wage rate shocks in their PSID sample has a standard deviation that averages $\sigma_{HSV} = .136$ across years. Their estimate is based on wage rate data for male workers age 20-59. Our estimate of human capital shocks is $\sigma = .111$. Thus, one might question whether our shocks are too small. We note that, under the theory developed here, it is not surprising that an estimate based on data for an older age group is smaller than one based on a younger age group or based on pooling together many age groups. The reason is that the variance of log wage differences for younger age groups is in theory determined both by shocks as well as by endogenous responses to shocks and initial conditions.

This logic is articulated below. The first three equations restate the human capital,

[23] Huggett, Ventura and Yaron (2006) document that the Ben-Porath model, which abstracts from shocks, generates autocorrelation coefficients for earnings growth rates that are large and positive.

wage and measured log wage differences equations from section 3.2 while allowing for non-zero human capital production. The hat notation below denotes the log of a variable. We assume, for the sake of argument, that one can measure wages as earnings per unit of work time - the theoretical wage concept. The key point is that measured n-period log wage differences (i.e. $y_{t,n}$) now have an extra human capital production term as for younger age groups human capital production is non-zero.

$$h_{t+n} = exp(z_{t+n})H(h_{t+n-1}, s_{t+n-1}, a) = exp(z_{t+n})[h_{t+n-1} + f(h_{t+n-1}, s_{t+n-1}, a)]$$

$$w_{t+n} = R_{t+n}h_{t+n} = R_{t+n}exp(z_{t+n})(1 + \frac{f(h_{t+n-1}, s_{t+n-1}, a)}{h_{t+n-1}})h_{t+n-1}$$

$$y_{t,n} = \hat{R}_{t+n} - \hat{R}_t + \sum_{i=1}^{n} z_{t+i} + \sum_{i=1}^{n} \ln(1 + \frac{f(h_{t+i-1}, s_{t+i-1}, a)}{h_{t+i-1}}) + \epsilon_{t+n} - \epsilon_t$$

The upshot is that the variance $Var(y_{t,n})$ may rise faster with the gap n between periods for younger or for pooled age groups than for older age groups simply because human capital production is a source of wage variation that differs across agents. This explanation is consistent with our analysis of PSID data. We find that applying the methodology employed in section 3.2 to wage data for males in the PSID age 23-60 over the period 1969-2004 produces a point estimate of $\sigma = .146$, which is close to $\sigma_{HSV} = .136$ but is substantially above our estimate from Table 1 of $\sigma = .111$.

5.6 Consumption Implications

A common view is that a useful model for analyzing lifetime inequality within an incomplete-markets framework should also be broadly consistent in terms of its implications for consumption inequality. We therefore compare the model's implications for the rise in consumption dispersion over the lifetime with the patterns found in U.S. data. A number of studies analyze the variance of log adult-equivalent consumption in U.S. data by regressing the variance of log adult-equivalent consumption for households in different age groups on age and time dummies or alternatively on age and cohort dummies. The coefficients on age dummies are then used to highlight how consumption dispersion varies for a cohort with age.

Figure 5 plots the variance of log adult-equivalent consumption in U.S. data from three such studies and from the model economy. Deaton and Paxson (1994) analyze U.S. Consumer Expenditure Survey (CEX) data from 1980 to 1990. Heathcote et. al. (2005), Slesnick and Ulker (2005), Aguiar and Hurst (2008) and Primiceri and van Rens (2009) examine this issue using CEX data over a longer time period. All of these later studies find that the rise in dispersion with age is smaller than the rise in Deaton and Paxson (1994). Aguiar and Hurst (2008) find that the rise is somewhat larger than that found by Heathcote et. al. (2005a), Slesnick and Ulker (2005) and Primiceri and van Rens (2009). Aguiar and Hurst (2008) note that the increase in the variance is about 12 log points when consumption is measured as total nondurable expenditures but about 5 log points when consumption is measured as core nondurable expenditures.

The exogenous earnings model analyzed in Storesletten et. al. (2004) produces the rise in consumption dispersion documented in Deaton and Paxson (1994). This is the case when their exogenous earnings model has a social insurance system. Without social insurance, their model produces a rise in dispersion greater than the rise in Deaton and Paxson (1994). Figure 5 shows that the rise in consumption dispersion within our benchmark model is less than the rise in Deaton and Paxson (1994) and between the rise in Primiceri and van Rens (2009) and Aguiar and Hurst (2008).

The benchmark model produces less of a rise in consumption dispersion than the model analyzed by Storesletten et. al. (2004). A key reason for this is that part of the rise in earnings dispersion in our model is due to initial conditions.[24] These differences are foreseen and therefore built into consumption decisions early in life. This mechanism relies on agents knowing their initial conditions. Cunha, Heckman and Navarro (2005), Guvenen (2007) and Guvenen and Smith (2008) analyze the information individuals have about future earnings as revealed by economic choices. They conclude that much is known early in life about future earnings prospects.

[24] A separate possible reason for why our results on consumption dispersion differ from Storesletten et. al. (2004) is that they allow borrowing up to next period's expected earnings, whereas we allow for a somewhat more generous borrowing limit. However, this difference may not be very important. When they analyzed the more generous limit that we consider, they found that this had almost no impact on the rise in consumption dispersion over the lifetime within their model.

6 Lifetime Inequality

We now use the model to answer the two lifetime inequality questions posed in the introduction.

6.1 Initial Conditions Versus Shocks

We decompose the variance in the relevant variable into variation due to initial conditions versus variation due to shocks. This is done for lifetime utility, lifetime earnings and lifetime wealth.[25] Such a decomposition makes use of the fact that a random variable can be written as the sum of its conditional mean plus the variation from its conditional mean. As these two components are orthogonal, the total variance equals the sum of the variance in the conditional mean plus the variance around the conditional mean.

Table 5 presents the results of this decomposition. Lifetime inequality is analyzed as of the start of the working life cycle, which we set to a real-life age of 23. In the benchmark model, we find that 61.5 percent of the variation in lifetime earnings and 64.0 percent of the variation in lifetime utility is due to initial conditions.

The benchmark model abstracts from initial wealth differences. This is a potentially important omission as wealth inequality is substantial among the young. Nevertheless, when we extend the benchmark model to include initial wealth inequality of the nature found in U.S. data, we find that the results are similar to the findings for the benchmark model.[26] Accounting for wealth inequality, Table 5 documents that $61.2, 62.4$ and 66.0 percent of the variation in lifetime earnings, lifetime wealth and lifetime utility is due to initial conditions. These results are quite similar to those for the benchmark model. This is not too surprising as Table 3 from section 5.3 indicates that some central features of the distribution of initial human capital and learning ability are very similar across these two models.

[25] Lifetime utility and lifetime earnings along a shock history z^J from initial condition $x_1 = (h_1, k_1, a)$ are defined as $\sum_{j=1}^{J} \beta^{j-1} u(c_j(x_1, z^j))$ and $\sum_{j=1}^{J} e_j(x_1, z^j)/(1+r)^{j-1}$. Lifetime wealth is lifetime earnings plus the value of initial wealth. These decompositions are invariant to an affine transformation of the period utility function.

[26] This is a partial equilibrium exercise. We fix all model parameters, except those describing the initial distribution, at the values specified in Table 2 and fix factor prices to those in the benchmark model. A balanced-growth equilibrium with non-zero initial wealth would require some mechanism for producing the initial wealth distribution.

Our results for lifetime inequality with initial wealth differences are based on PSID net-wealth data for households with a male head age 20 to 25.[27] For each male household head age 20 to 25 in a given year, we calculate net wealth as a ratio to mean earnings of males in this age group. We then pool these ratios across years. We maintain the multi-variate log-normal structure for describing initial conditions. However, we do allow for negative wealth holding. Specifically, we approximate the empirical pooled wealth distribution with a lognormal distribution which is shifted a distance δ. We choose δ so that 95 percent of the distribution has a wealth to mean earnings ratio above $-\delta$. The distribution of the wealth to mean earnings ratio in the model is given by $e^x - \delta$, where x is distributed $N(\mu_1, \sigma_1^2)$. The parameters (μ_1, σ_1^2) are set equal to the sample mean and sample variance of the log of the sum of the wealth to earnings ratio plus δ for ratios above $-\delta$. In the PSID sample we calculate that $(\mu_1, \sigma_1^2, \delta) = (-0.277, 0.849, 0.381)$. The median, mean and standard deviation of the wealth to mean earnings ratio in the model is then $(0.377, 0.778, 1.340)$.[28] This implies that there is a substantial amount of initial wealth dispersion within the model.

The distribution of initial wealth, human capital and learning ability is selected to best match the earnings facts documented earlier. The distribution is a tri-variate lognormal, where the parameters describing the mean and variance of shifted log wealth are those calculated above in U.S. data. Thus, wealth in the model is right skewed and mean wealth is more than double median wealth.

6.2 How Important are Different Initial Conditions?

The analysis so far has not addressed how important variation in one type of initial condition is compared to variation in other types. We analyze the importance of different initial conditions at age 23 by asking an agent how much compensation is equivalent to starting at age 23 with a one standard deviation change in any initial condition, other things equal. We express this compensation, which we call an equivalent variation, in terms of the percentage change in consumption in all periods that would leave an agent with the same expected lifetime utility as an agent with a one standard deviation change in the relevant initial condition. The baseline initial condition is set so the agent starts with the median value

[27] The data is from the PSID wealth supplement for 1984, 1989, 1994, 1999, 2001 and 2003. The sample size is 1176 when pooled across these years.

[28] In the PSID the median, mean and standard deviation of the wealth to mean earnings ratio is $(0.313, 0.776, 1.432)$.

of human capital, learning ability and initial wealth. These values are specified by the parameters governing the mean terms of the lognormal distribution. We then change the baseline initial condition by one standard deviation in terms of logged variables.

In the analysis of equivalent variations, Table 6 shows that a one standard deviation movement in log human capital is substantially more important than a one standard deviation movement in either log learning ability or log initial wealth. A one standard deviation increase in initial human capital is equivalent to approximately a 39 percent increase in consumption. In contrast, a one standard deviation increase in learning ability and initial wealth is equivalent to approximately a 6 and a 7 percent increase in consumption, respectively. Thus, we find that an increase in human capital leads to the largest impact and an increase in learning ability or initial financial wealth have substantially smaller impacts at age 23.

We also analyze the importance of different initial conditions by determining how changes in initial conditions affect an agent's budget constraint. More specifically, we determine the percent by which an agent's expected lifetime wealth changes in response to a one standard deviation change in an initial condition. In interpreting the results, it is useful to keep two points in mind. First, an increase in human capital acts as a vertical shift of the expected earnings profile, whereas an increase in learning ability rotates this profile counter clockwise. Second, the impact of additional initial financial wealth is both through a direct impact on lifetime resources as well as the indirect impact through earnings arising from different time allocation decisions.

Broadly speaking, Table 6 shows that the impact on expected lifetime wealth of changes in initial conditions is roughly in line with the calculation of equivalent variations. Table 6 states that a one standard deviation increase in initial human capital increases expected lifetime wealth by about 47 percent. Given that this is a fairly large effect, it seems useful to examine the general plausibility of this result. We first note that this change increases initial human capital by about 57 percent, based on the information in Table 2. So the impact in Table 6 is somewhat less than the percentage change in human capital. The second thing to note is that, absent risk, the present value of future earnings is linear in human capital and learning ability contributes an additive effect to the present value of future earnings. Both assertions are established for the deterministic model in the proof of Proposition 1 in Huggett et. al. (2006). Thus, the deterministic model implies an effect on the present value of earnings nearly equal to the percentage increase in human capital provided that the additive effect due to learning ability is small at age 23 for this agent compared to the human capital component.

We stress that the results in Table 6 highlight the impact of these hypothetical changes at age 23. They do *not* convey information about the impact of such changes at earlier or at later ages. We acknowledge that the importance of variation in learning ability and financial resources may be greater at earlier ages. Variation in learning ability or financial resources at an earlier age might have a greater impact provided that such variations produce important changes in human capital or financial wealth at age 23. While we could in principle use the model to describe impacts of such changes at later ages, we think that it is especially interesting to carry out an analysis at earlier ages, given that we find that the majority of the variance in lifetime inequality is due to differences existing at age 23. We leave such an analysis for future research, as a coherent interpretation of the initial conditions at earlier ages will require a richer model that incorporates the role of one's family. This type of analysis will also benefit from data that speaks to what happens before age 23.

6.3 Sensitivity

We examine the sensitivity of our lifetime inequality findings along several dimensions.

Cohort Effects

The cohort effects view of the earnings distribution dynamics, analyzed in section 3, produces a larger increase in earnings dispersion and a steeper mean earnings profile over the working lifetime than the time effects view. The benchmark model addressed the time effects view. We now choose the distribution of initial conditions and the elasticity parameter of the human capital production function to match the earnings facts under the cohort effects view. We find that the distribution of initial conditions at age 23 has a lower mean human capital, less human capital dispersion, and a greater amount of dispersion in learning ability compared to the time effects view. These changes help account for the steeper increase in mean earnings and earnings dispersion under the cohort view. Initial conditions account for 60.3 and 64.9 percent of the variation in lifetime earnings and lifetime utility under the cohort effects view, compared to 61.5 and 64.0 percent under the time effects view. Thus, the cohort effects view leads to effectively the same conclusion concerning the importance of initial conditions in lifetime inequality. The key difference from the time effects view is that the importance of variation in learning ability increases and the importance of variation in human capital decreases. An increase in learning ability and initial human capital by one standard deviation are now equivalent to an 8.0 percent and a 34.1 percent increase in consumption, respectively, compared to effects of 5.7 and 39.3 percent for the analysis of the time effects case.

Human Capital Shocks

A critical parameter is the standard deviation σ of human capital shocks. We now examine a low shock case $\sigma = .104$ and a high shock case $\sigma = .118$ by decreasing or increasing our point estimate by the estimated standard error in Table 1. In each case the parameter controlling the mean shock is adjusted to match the fall in mean earnings at the end of the working lifetime. Intuitively, when the driving shocks are smaller, then initial conditions must play a larger role in accounting for the increase in earnings dispersion over the lifetime and, thus, account for more of the variance in lifetime earnings and utility. This is precisely what occurs. In the low shock case initial conditions account for fractions .656 and .695 of the variance in lifetime earnings and lifetime utility. The corresponding results for the high shock case are .570 and .620. Thus, for both cases, initial conditions continue to account for the majority of the variation in lifetime inequality.[29]

Elasticity Parameter

We allow the elasticity parameter α of the human capital production function to vary over the interval $[.5, .9]$. For each value, we then set the distribution of initial conditions to best match the earnings facts. Figure 6 describes lifetime inequality as the elasticity parameter is varied. Figure 6 shows that the fraction of the variance in lifetime earnings that is due to initial conditions tends to fall as the elasticity parameter increases. This finding is related to some results in Kuruscu (2006). He analyzes the importance of training in producing differences in lifetime earnings in models without idiosyncratic risk. An interpretation of one aspect of his work is that as the elasticity parameter increases, then the marginal benefit

[29]These results fix the elasticity of the human capital production function to the value in Table 2.

and marginal cost curves from producing extra units of human capital move closer together.[30] Lifetime earnings for agent's with quite different learning abilities but the same initial human capital will not differ strongly when this holds.

Moments to Match

We change the data moments that are used to select initial conditions. We now set initial conditions to match both the mean earnings profile and the age profiles of the 90-10 and 75-25 quantile ratios of the earnings distributions. The quantile ratios are not sensitive to earnings observations in the extreme tails of the distribution. With these new moments, initial conditions now account for 53.8 and 57.3 percent of the variance in lifetime earnings and lifetime utility. Thus, by deemphasizing the tails of the earnings distribution the importance of initial conditions falls by more than a handful of percentage points.

7 Concluding Remarks

This paper analyzes the proximate sources of lifetime inequality. We find that differences in initial conditions as of a real-life age of 23 account for more of the variation in realized lifetime earnings, lifetime wealth and lifetime utility than do shocks over the working lifetime. Among initial conditions evaluated at age 23, a one standard deviation increase in human capital is substantially more important as of age 23 than either a one standard deviation increase in learning ability or initial wealth for how an agent fares over the remaining lifetime. While our framework is silent on the forces that shape individuals from birth to age 23, we think that the results of the paper help to motivate work which shines light on these forces.

The conclusions stated above come from a specific model and reflect the choice of a specific age to evaluate lifetime inequality. Below we discuss three issues that are useful for providing perspective on these choices and the conclusions that depend on them:

Issue 1: We address lifetime inequality as of age 23 - an age when many people will have finished formal schooling. This choice brings up several issues. First, analyzing lifetime in-

[30] In the Ben-Porath model, the marginal benefit of extra units of human capital is constant at any age, whereas the marginal cost of producing an additional unit is $MC_j(q;a) = \frac{R_j}{\alpha a}(\frac{q}{a})^{\frac{1}{\alpha}-1}$ when total production is q, a is learning ability, α is the elasticity parameter and R_j is the rental rate.

equality at later ages would likely produce an even greater importance for "initial conditions" established at later ages within our model. Second, analyzing lifetime inequality at earlier ages might lead one to conjecture that the relative importance of learning ability compared to initial human capital might increase. This might hold if learning ability is crystallized well before age 23 and produces some of the human capital differences as of age 23. We think that this is an interesting conjecture. However, pushing back the age at which lifetime inequality is evaluated will raise the issue of the importance of one's family more directly than is pursued here. The importance of one's family, or more broadly one's environment, up to age 23 is not modeled in our work but is implicitly captured through their impact on initial conditions as of age 23: human capital, learning ability and financial wealth.

Issue 2: One can ask what key features of the data lead us to conclude that variation in human capital must be so important as of age 23 compared to learning ability or to initial wealth, given our model. Four features of the data are key: (i) the magnitude of the persistent component of wage variation at the end of the working lifetime, (ii) the steepness of the mean earnings profile, (iii) the amount of earnings dispersion early in the working lifetime together with the nature of the rise in dispersion at later ages and (iv) the distribution of financial wealth among young households.

These features of the data shape our answer. First, absent persistent wage variation among older workers, learning ability differences would have to account for all of the rise in earnings dispersion over the lifetime. Thus, a greater magnitude of such wage variation implies a smaller role for ability differences in accounting for rising earnings dispersion over the lifetime. Second, the shape of the mean human capital profile must be flatter than the shape of the mean earnings profile implying that mean human capital is quite high early in life. This limits the importance of learning ability differences as of age 23. Third, the dispersion in earnings early in life is large and this needs to be accounted for largely by differences in human capital. The lack of a strong U-shaped earnings dispersion profile in the data dictates large differences in human capital early in life rather than solely large differences in learning ability and dictates a positive correlation between human capital and learning ability at age 23. Fourth, given our estimate of shocks, a larger rise in earnings dispersion over the lifetime dictates a larger role for learning ability differences. Thus, learning ability differences play a larger role under the cohort effects view of the dynamics of the earnings distribution. Fifth, we find that a one standard deviation increase in initial financial wealth moves wealth by 1.43 times the mean earnings of young males. Such a change in initial wealth leads to only a small direct impact on expected lifetime wealth, given a working lifetime of approximately 40 years.

Issue 3: Our model does not capture all the possible shocks impacting individuals after age 23. Instead, the model has a single source of shocks impacting human capital. The model successfully captures the permanent and persistent idiosyncratic earnings shocks highlighted by statistical models of earnings. It does not capture purely temporary idiosyncratic earnings shocks, any aggregate shocks or interactions between aggregate and idiosyncratic shocks. We doubt that adding a source of temporary earnings shocks to our model will substantially change our conclusions about lifetime inequality. Theoretical work by Yaari (1976) and simulations of related models suggest that, with a long lifetime and a low interest rate, shocks that impact earnings in a purely temporary way are reasonably well insured. It remains to be determined if a richer modeling of multiple sources of idiosyncratic shocks or the interaction of idiosyncratic and aggregate shocks will substantially change our conclusions. We conjecture that moments of the changes in wages of older workers will continue to be valuable as part of a procedure to identify shocks in multiple-shock models nesting our model.

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A Appendix: Computation

A.1 Algorithm

We compute a balanced-growth equilibrium to the benchmark model. The algorithm finds a discount factor β so that the computed equilibrium has the factor prices specified below, given all other model parameters.

Algorithm:

1. Set $(R_j, r) = ((1.0019)^{j-1}, .042)$. Find $(K_1/L_1, A_1)$ satisfying equilibrium condition 2: $R_1 = A_1 F_2(K_1, L_1 A_1)$ and $r = F_1(K_1, L_1 A_1) - \delta$.
2. Compute solutions $(c_j, h_j, l_j, s_j, k_j)$ to the agent's problem for the cohort of agents born at time 1, given a guess of the agent's discount factor β.
3. Compute the implied capital-labor ratio K_1'/L_1' and net taxes T:
   $K_1' \equiv \sum_j \mu_j \int E[\frac{k_j(x_1, z^j)}{(1+g)^{j-1}}] d\psi$ and $L_1' \equiv \sum_j \mu_j \int E[h_j(x_1, z^j) l_j(x_1, z^j)] d\psi$
   $T \equiv \sum_j \mu_j \int E[\frac{T_j(e_j(x_1, z^j), k_j(x_1, z^j))}{(1+g)^{j-1}}] d\psi$
4. If $K_1'/L_1' = K_1/L_1$ and $T > 0$, then stop. Otherwise, update β and repeat steps 2-3.

A.2 Solving the Agent's Problem

We compute solutions to the agent's problem using dynamic programming. The dynamic programming problem is given below, where the state is $x = (h, k, a)$. The model implies that the period borrowing limits should depend upon age, human capital, learning ability and the distribution of shocks. We impose ability-specific limits $\underline{k}(a)$ and relax these limits until they are not binding. We also directly penalize choices leading to negative consumption later in life. This is a device for effectively imposing the endogenous limits implied by the model.

$$V_j(x) = \max_{(c,k',l,s)} u(c) + \beta E[V_{j+1}(h', k', a)]$$

subject to $c + k' \leq R_j h l + k(1+r) - T_j(R_j h l, k), \quad h' = exp(z')H(h, s, a), \quad l + s = 1, \quad k' \geq \underline{k}(a)$

We compute solutions by backwards recursion. We use a rectangular grid on the state variables (h, k) which is learning-ability specific. For each grid point and age j, we numerically solve the maximization problem on the right-hand-side of the Bellman's equation. To evaluate the objective, we employ a bi-linear interpolation of V_{j+1} across grid points. To compute expectations, we follow Tauchen (1986) and discretize the shock into 5 equally-spaced values over the interval $[\mu - 2\sigma, \mu + 2\sigma]$. Proceeding in this way gives a computed value function $V_j(x)$ and decision rules $(c_j(x), k_j(x), l_j(x), s_j(x))$ at grid points.

A.3 Computing Capital-Labor Ratios

To calculate implied capital-labor ratios, we put a grid on initial conditions (h_1, k_1, a). We draw grid point $x_1 = (h_1, k_1, a)$ with a probability proportional to the density of the distribution ψ at x_1. For any draw of an initial condition, we also draw a lifetime shock history z^J. We calculate realizations $(k_j(x_1, z^J), h_j(x_1, z^J), l_j(x_1, z^J))$, using computed decision rules. Capital-labor ratios are computed from age group sample averages using $20,000$ draws of initial conditions and lifetime histories. These draws are fixed both across iterations in the algorithm that computes an equilibrium and across the search over model parameters which we discuss next. To calculate aggregate capital and taxes, we divide the mean wealth and mean taxes of a cohort over the life cycle by $(1+g)^{j-1}$ to capture the mean wealth and taxes of age j agents in cross section.

A.4 Selecting Model Parameters

We set the parameter α and the parameters governing the distribution ψ to minimize the squared distance of log model moments from log data moments. To do so, we compute balanced-growth equilibria for given (α, ψ) and simulate to find the corresponding model moments. The objective of the minimization problem is

$$\sum_{j=1}^{J_R - 1} [(\log(m_{1j}/d_{1j}))^2 + (\log(m_{2j}/d_{2j}))^2 + (\log(m_{3j}/d_{3j}))^2],$$

where (m_{1j}, m_{2j}, m_{3j}) denote mean earnings, var (log earnings) and earnings skewness at age j in the model and where (d_{1j}, d_{2j}, d_{3j}) are the corresponding data moments. The simplex minimization routine AMOEBA, from Press et. al. (1992), is used to solve this minimization problem.

Table 1: Estimation of Human Capital Shocks

Age Range	Period	e_{min}	e_{max}	N	σ	S.E.(σ)	σ_ϵ	S.E.(σ_ϵ)
55 - 65	1969-2004	2,000	1.8 M	103	0.111	0.007	0.137	0.005
50 - 60	1969-2004	2,000	1.8 M	199	0.117	0.006	0.142	0.004
55 - 65	1969-2004	3,000	1.8 M	98	0.104	0.006	0.130	0.004
55 - 65	1969-2004	2,000	1.8 M	103	0.111	0.007	0.137	0.005
55 - 65	1969-2004	1,500	1.8 M	106	0.119	0.007	0.137	0.004
55 - 65	1969-2004	1,000	1.8 M	110	0.132	0.007	0.134	0.005
55 - 65	1969-1981	2,000	1.8 M	104	0.108	0.008	0.130	0.006
50 - 60	1969-1981	2,000	1.8 M	210	0.107	0.007	0.137	0.005
55 - 65	1982-2004	2,000	1.8 M	102	0.107	0.011	0.159	0.009
50 - 60	1982-2004	2,000	1.8 M	193	0.142	0.009	0.146	0.006

Note: The entries provide the estimates for σ and σ_ϵ for various samples. The first column provides the minimum and maximum age in the sample, whereas the second specifies which PSID years are included. The columns labeled e_{min} and e_{max} refer to the minimum and maximum earnings levels in 1968 dollars, where M denotes a million. The column labeled N refers to the median number of observation across panel years. Columns labeled $S.E.$ refer to standard errors.

Table 2: Parameter Values - Benchmark Model

Category	Symbol	Parameter Value
Demographics	(J, J_R, n)	$(J, J_R, n) = (53, 39, .012)$
Preferences	$\beta, u(c) = c^{(1-\rho)}/(1-\rho)$	$(\beta, \rho) = (.981, 2)$
Technology	(γ, δ, g)	$(\gamma, \delta, g) = (.322, .067, .0019)$
Tax System	$T_j = T_j^{ss} + T_j^{inc}$	$T_j^{ss}(e_j) = .106e_j$ for $j < J_R$ $T_j^{ss}(e_j) = -.4\bar{e}$ otherwise T_j^{inc} - see text
Human Capital Shocks	$z \sim N(\mu, \sigma^2)$	$(\mu, \sigma) = (-.029, 0.111)$
Human Capital Technology	$h' = exp(z')H(h, s, a)$ $H(h, s, a) = h + a(hs)^\alpha$	$\alpha = .70$
Initial Conditions	$\psi = LN(\mu_x, \Sigma)$	$\mu_x = (\mu_h, \mu_a) = (4.66, -1.12)$ $(\sigma_h^2, \sigma_a^2, \sigma_{ha}) = (.213, .012, .041)$

Table 3: Properties of the Initial Distribution

Statistic	Benchmark Model	Benchmark Model with Initial Wealth Differences
Mean Learning Ability (a)	0.329	0.328
Coefficient of Variation (a)	0.112	0.124
Mean Initial Human Capital (h_1)	116.9	117.5
Coefficient of Variation (h_1)	0.487	0.476
Correlation (a, h_1)	0.746	0.655

Note: Entries show properties of the distribution of initial conditions for the parameters that best match the profiles of mean earnings, earnings dispersion and skewness.

Table 4: Statistical Models of Earnings -Benchmark Model Data

Statistical Model	ρ	σ^2_α	σ^2_η	σ^2_ϵ	σ^2_β	$\sigma_{\alpha\beta}$
RIP Model						
cov=1	.964	.283	.013	.025	-	-
	(.015)	(.021)	(.005)	(.006)	-	-
cov=5	.963	.271	.014	.026	-	-
	(.013)	(.021)	(.004)	(.009)	-	-
HIP Model						
cov=1	.860	.264	.032	.006	.00006	.0003
	(.010)	(.024)	(.007)	(.006)	(.00006)	(.0010)
cov=5	.862	.268	.029	.010	.00006	.0005
	(.007)	(.023)	(.005)	(.009)	(.00006)	(.0011)

Note: Means and standard deviations of model parameters are based on drawing 500 samples, where each sample has 200 agents at each age. cov=# indicates the number of covariance terms per agent used in estimation. Measurement error, distributed $N(0, .15^2)$, is added to each value of log earnings. Estimation uses all variance and covariance restrictions.

Table 5: Sources of Lifetime Inequality

Statistic	Benchmark Model	Benchmark Model with Initial Wealth Differences
Fraction of Variance in Lifetime Utility Due to Initial Conditions	.640	.661
Fraction of Variance in Lifetime Earnings Due to Initial Conditions	.615	.613
Fraction of Variance in Lifetime Wealth Due to Initial Conditions	.615	.626

Note: Entries show the fraction of the variance in the statistic accounted for by initial conditions (initial human capital, learning ability and initial wealth). Wealth differences are measured directly from PSID data as explained in the text.

Table 6: Changes in Initial Conditions

Variable	Change in Variable	Equivalent Variation (%)	Change in Lifetime Wealth (%)
Human Capital	+ 1 st. deviation	39.3	47.5
	− 1 st. deviation	-28.3	-31.7
Learning Ability	+ 1 st. deviation	5.7	8.1
	− 1 st. deviation	-2.6	-3.9
Initial Wealth	+ 1 st. deviation	7.1	5.0
	− 1 st. deviation	-1.6	-1.3

Note: The table states equivalent variations and the percentage change in the expected lifetime wealth associated with changes in each initial condition. The baseline initial condition is set equal to the mean log values of initial human capital, learning ability and wealth. Changes in initial conditions are also in log units.