

**Human Capital and Economic Opportunity:
A Global Working Group**

Working Paper Series

Working Paper No. 2012-014

**The Interplay Between Student Loans and Credit
Cards: Implications for Default**

**Felicia Ionescu
Marius Ionescu**

July, 2012

**Human Capital and Economic Opportunity Working Group
Economic Research Center
University of Chicago
1126 E. 59th Street
Chicago IL 60637
humcap@uchicago.edu**

The Interplay Between Student Loans and Credit Cards: Implications for Default*

Felicia Ionescu[†] Marius Ionescu[‡]
Colgate University *Colgate University*

July 13, 2012

Abstract

We analyze, theoretically and quantitatively, the interactions between two different forms of unsecured credit and their implications for default behavior of young U.S. households. One type of credit mimics credit cards in the U.S. and the default option resembles a bankruptcy filing under Chapter 7 and the other type of credit mimics student loans in the U.S. and the default option resembles Chapter 13 of the U.S. Bankruptcy Code. In the credit card market financial intermediary offers a menu of interest rates based on individual default risk, whereas in the student loan market the government sets a fix interest rate. We prove the existence of a steady-state equilibrium and characterize the circumstances under which a household defaults on each of these loans. We demonstrate that the institutional differences between the two markets make borrowers prefer default on student loans rather than on credit card debt. Our quantitative analysis shows that the increase in college debt together with the changes in the credit card market fully explain the increase in the default rate for student loans in recent years. While having credit card debt increases student loan default, loose credit card markets help borrowers with large student loans smooth out consumption and reduce student loan default. We find that the recent 2010 reform on income-based repayment on student loans is justified on welfare grounds, and in particular, in an economy with tight credit card markets.

JEL Codes: D91; I22; G19;

Keywords: Default, Bankruptcy, Student Loans, Credit Cards

*The authors thank Jeff Baldani, Andy Glover, Juan-Carlos Hatchondo, Takao Kato, Dirk Krueger, Wenli Li, Leo Martinez, Makoto Nakajima, Pierre-Daniel Sarte, and Nicole Simpson for comments. A special thank to Kartik Athreya and Satyajit Chatterjee for helpful suggestions. Comments from participants at the Society for Economic Dynamics, Midwest Macroeconomics Meeting, Society of Advancements in Economic Theory, the NASM, and seminar participants at Colgate University and FRB of Richmond are also gratefully acknowledged.

[†] Department of Economics, 13 Oak Drive, Hamilton NY, 13346; (315)228-7955, fionescu@colgate.edu.

[‡] Department of Mathematics, 13 Oak Drive, Hamilton NY, 13346; (315)228-7955, mionescu@colgate.edu.

1 Introduction

As the cost of financing higher education falls increasingly on students and families, student loan debt is rising at alarming rates. College debt has steadily increased in the last two decades and has reached records high in the past several years (with a cumulative growth rate of 282 percent from 1990 to 2008). In fact, as of June 2010, total student loan debt passed total credit card debt for the first time (see Figure 1 in the Appendix).¹ Currently 70 percent of individuals who enroll in college take out student loans (College Board (2009)) and the graduates of 2011 are the most indebted in history, with an average debt load of \$27,300. At the same time, the two-year basis cohort default rate (CDR) for student loans has steadily declined from 22.4 percent in 1990 to 4.6 percent in 2005 and has increased ever since reaching records high in the last decade (at 8.8 percent in 2009).²

The increase in college debt alone cannot explain the recent increases in student loan default rates of young U.S. households. A second market is needed to understand this behavior: currently the majority of individuals with college debt (62 percent) also have credit card debt, according to our findings from the Survey of Consumer Finances (SCF). Credit card usage is rampant among college students, with approximately 84 percent of the student population having at least one credit card in 2008. As students proceed through college, they use their credit cards more heavily, with 30 percent of undergraduate credit cardholders charging tuition on their credit card and 92 percent of them charging textbooks, school supplies, or other direct education expenses (Sallie Mae (2009)). While both of these loans represent important components of young households' portfolios in the U.S., the financial arrangements in the two markets are very different, and in particular with respect to the roles played by bankruptcy arrangements and default pricing. Furthermore, credit terms on credit card accounts have worsen in the recent years adversely affecting households's capability to diversify risk. For young borrowers this is particularly problematic, because even modest balances have more of an impact than the same balance belonging to a consumer who has a much older or robust credit history.³

We propose a theory about the interactions between student loans and credit cards in the

¹According to the Federal Reserve releases, U.S. households owed \$826.5 billion in revolving credit (98 percent of revolving credit is credit card debt) and they owed \$829.785 billion in student loans — both federal and private — in 2010. Partially, this is due to large increases in college cost by 40 percent in the past decade and partially due to paying down credit card debt.

²The 2-year CDR is computed as the percentage of borrowers who enter repayment in a fiscal year and default by the end of the next fiscal year. Trends in the 2-year CDR are presented in Figure 2 in the Appendix.

³This is because of the principle of revolving utilization as well as the weight given in credit scoring to the age of a consumer's credit file. These borrowers have younger credit reports and fewer accounts, which implies that they are likely be scored in a "thin file" or "young file" score card.

U.S. and their impact on default incentives of young U.S. households. As we argue in this paper, the interaction between different bankruptcy arrangements induces significant trade-offs in default incentives in the two markets. Understanding these trade-offs is particularly important in the light of the recent trends in borrowing and default behavior. Data show that young U.S. households have the second highest rate of bankruptcy (just after those aged 35 to 44) and the rate among 25- to 34-year-olds increased between 1991 and 2001; this fact indicates that the current generation is more likely to file for bankruptcy as young adults than were young boomers at the same age.⁴ Furthermore, student loans have a higher default rate than credit cards or any other loan, including car loans and home loans.⁵

These trends are alarming considering the large risks that young borrowers face: the college dropout rate has increased dramatically in the past decade (from 38 percent to 50 percent for the cohorts that enrolled in college in 1995 and 2003 respectively).⁶ Furthermore, the unemployment rate among young workers with college education has jumped up significantly during the Great recession: 8 percent of young college graduates and 14.1 percent of young workers with some college are unemployed in 2010 (Bureau of Labor Statistics). In addition, in order to begin repaying their college debts, many resort to underemployment far outside their fields of study, a move that sets them back financially for years.

The combination of high income risks, high indebtedness and worse financial terms implies that borrowers are more likely to default on at least one of their loans. A couple of questions arise immediately: First, which default option do young borrowers find more attractive and why? In particular, is the current environment conducive to higher default incentives in the student loan market? Secondly, how much of the increase in default on student loans is explained by trends in the student loan market and how much by the interaction between the two markets?

In order to address the proposed issues we develop a general equilibrium economy that mimics features of student and credit card loans. Infinitely lived agents differ in college debt and income levels. Agents face uncertainty in income and may save/borrow and, as in practice, borrowing terms are individual specific. Central to the model is the decision of young college educated individuals to repay or default on their credit card and student loans. Consequences to default for student and credit card loans differ in several important ways: for student loans they include a

⁴Source: <http://www.creditcards.com/>

⁵According to a survey conducted by the FRB New York, the national student loan delinquency rate 60+ days in 2010 is 10.4 percent compared to only 5.6 percent for the mortgage delinquency rate 90+ days, 1.9 percent for bank card delinquency rate and 1.3 percent for auto loans delinquency rate. Based on an analysis of the Presidents FY2011 budget, in FY2009 the total defaulted loans outstanding are around \$45 billion.

⁶We define the dropout rate as the fraction of students who enroll in college and do not obtain a bachelor degree 6 years after they enroll. Numbers are based on the BPS 1995 and 2003 data.

wage garnishment and for credit cards they induce exclusion from borrowing for several periods. More importantly, credit card loans can be discharged in bankruptcy (under Chapter 7), whereas student loans cannot be discharged (borrowers need to reorganize and repay under Chapter 13 in the Bankruptcy code). Borrowing and default behavior in both markets determine the individual default risk. This risk, in turn, determines the loan terms agents face on their credit card accounts, including loan prices. In contrast, the interest rate in the student loan market does not account for the risk that some borrowers may default.

In the theoretical part of the paper, we first characterize the default behavior and show how it varies across households characteristics and behavior in both markets. Our main theoretical contribution consists in proving the existence of *cross-market effects* and their implications for default behavior. This contribution is two fold:

1) Our theory delivers that in equilibrium credit card loan prices differ across loan sizes, i.e. the interest rate increases in the size of the loan.⁷ In addition, we show that in equilibrium borrowers with high risk of default receive higher rates on their credit card loans (for the same loan size). In our model the default risk conditional on the size of the loan is given by the amount of debt and the default status in the student loan market. Our result arises from the fact that the probability of default on any credit card loan decreases in the amount of debt owed in the student loan market. Also this probability is higher for an individual with a default flag in the student loan market relative to an individual without a default flag. This set of results is new in the literature and provide a rationale for pricing credit card loans by using a default risk that takes into account participation in other credit markets.

2) In any steady-state equilibrium, we find a combination of student loan and credit card debt levels for which the agent defaults on at least one type of her loans. Moreover, we find that for even larger levels of student loans or credit card debt default occurs for student loans. This result innovates by showing that while a high college debt is necessary to induce default on student loans, this effect is amplified by indebtedness in the credit card market. This arises from the differences in bankruptcy arrangements in the two markets: the financially constrained borrower finds it optimally to default on student loans (even though she cannot discharge her debt) in order to be able to access the credit card market. Since there is no effect on her credit card market participation from defaulting on student loans (except for higher costs of loans) the borrower with high college debt prefers the default penalty in the student loan market over restricted credit card market participation.

In the quantitative part of our paper, we parametrize the model to match statistics regarding

⁷This result is not new. It was first demonstrated in Chatterjee, Corbae, Nakajima, and Rios-Rull (2007).

college debt, credit card debt, and income of young borrowers with student loans aged 20-30 as delivered by the SCF 2004 as well as the 2-year CDR on student loans. Our results are consistent with the observed behavior in several ways: First, the incentive to default on student loans increases in college debt and in college debt burden (debt-to-income ratio); default on student loans is more likely to occur for individuals with low levels of earnings and high levels of college debt. These results are in line with empirical evidence in Dynarsky (1994) and Ionescu (2008). Second, the incentive to default on credit card debt increases in credit card debt, result which is consistent with findings in Athreya, Tam, and Young (2009) and Chatterjee, Corbae, Nakajima, and Rios-Rull (2007). Lastly, our results deliver that the credit card default probability decreases in income. This finding is consistent with empirical evidence in Musto and Souleles (2006).

Our findings reveal large gaps in credit card rates across individuals with different levels of college debt and default status in the student loan market. For instance, the interest premium paid by individuals who have high levels of student loans (of 1.34 percent) is more than five times the premium paid by those with low levels of student loans (of 0.25 percent). This set of findings strengthens our theory and emphasizes the quantitative importance of correctly pricing credit card debt based on behavior in other credit markets. Results also show that individuals with no credit card debt have lower default rates on student loans than individuals with credit card debt, even if these credit card debt levels are small. In fact, individuals with low levels of credit card debt and low levels of student loan debt do not default on credit card, but they do default on their student loans. For them, the benefit of discharging their credit card debt is small compared to the large cost associated with default. Individuals with large levels of credit card and student loan debt are more likely to default on student loans. Our results suggest that having debt in the credit card market increases the incentive to default on student loans. However, individuals with large levels of student loan debt use the credit card market to reduce their default. On the one hand, participating in the credit card market pushes borrowers in more default on their student loans and on the other hand, taking on credit card debt helps student loan borrowers smooth consumption and pay their college debt, in particular when their college debt burdens are large.

We use our theory to answer the proposed quantitative question of how much of the recent increase in default rates for student loans (from 4.5 percent in 2004 to 6.7 percent in 2007) is due to an increase in college debt and how much is explained by changes in the credit card market. We find that the increase in college debt and credit card debt burdens together can fully account for the observed trends in student loan default. In fact, the changes in the credit card market during this period help keep the default rate low. In the absence of these changes, default on student loans increases to 7.1 percent. We conclude that while indebtedness in the credit card market increases default on student loans, borrowers with large levels of student loans can effectively use the credit

card market to lower their default on student loans.

We explore the policy implications of our model and study the impact of the recent proposal on income-based repayments on student loans, as amended by The Health Care and Education Reconciliation Act of 2010.⁸ We find that the proposal is justified on welfare grounds, and in particular in an economy where credit card markets are tight. Our findings are particularly important in the current market conditions when, due to a significant increase in college costs, students borrow more than ever in both the student loan and the credit card markets, and, at the same time, they face more severe terms on their credit card accounts. We propose an income-based repayment plan that allows for more time to repay before loan forgiveness occurs and show that it improves welfare relative to the recently adopted reform. Our proposal reduces the per period payment and, at the same time, induces less dischargeability, and thus lower taxes in the economy.

1.1 Related literature

Our paper is related to two strands of existing literature: credit card debt default and student loans default. The first strand includes important contributions by Athreya, Tam, and Young (2009), Chatterjee, Corbae, Nakajima, and Rios-Rull (2007), Chatterjee, Corbae, and Rios-Rull (2010), and Livshits, MacGee, and Tertilt (2007). The two first studies explicitly model a menu of credit levels and interest rates offered by credit suppliers with the focus on default under Chapter 7 within the credit card market. Chatterjee, Corbae, and Rios-Rull (2010) provide a theory that explores the importance of credit scores for consumer credit based on a limited information environment. Livshits, MacGee, and Tertilt (2007) quantitatively compare liquidation in the U.S. to reorganization in Germany in a life-cycle model with incomplete markets, earnings and expense uncertainty.

In the student loan literature there are several papers closely related to the current study including research by Ionescu (2010), Ionescu and Simpson (2010) and Lochner and Monge (2010). These papers incorporate the option to default on student loans when analyzing various government policies. Out of these studies, the only one that accounts for the role of individual default risk in pricing loans is Ionescu and Simpson (2010) who recognize the importance of this risk in the context of the private market of student loans. The model, however, is silent with respect to the role of credit risk for credit cards or for the allocation of consumer credit, the study being restricted to the analysis of the student loan market. Ionescu (2010) models both dischargeability and non-dischargeability of loans for young U.S. households in the U.S., but only in the context

⁸This policy assumes payments of 15 percent of discretionary income and loan forgiveness after 25 years. Details are presented in Section 4.4.

of the student loan market. Furthermore, as in Livshits, MacGee, and Tertilt (2007), Ionescu (2010) studies various bankruptcy rules in distinct environments that mimic different periods in the student loan program (in Livshits, MacGee, and Tertilt (2007) the study is for different countries) rather than modeling them as alternative insurance mechanisms available to borrowers.⁹

Our paper builds on this body of work and improves on the modeling of insurance options available to borrowers with student loans and credit card debt. To our knowledge, we are the first to embed the trade-off between different bankruptcy arrangements in different types of credit markets into a quantitative dynamic theory. On a methodological level, our paper is related to Chatterjee, Corbae, Nakajima, and Rios-Rull (2007). As in their paper, we model a menu of prices for credit card loans based on the individual risk of default. In their paper the probability of default is linked to the size of the loan and medical services. We take a step further in this direction and model the default probability based on the size of the credit card loan, the amount owed on student loans, and on the default status on student loans. All of these three components determine credit card loan pricing in equilibrium. In this direction our study is related to recent work by Chatterjee, Corbae, and Rios-Rull (2010) who show that credit scores (which incorporate borrowing behavior and past repayment behavior in different types of markets) represent a proxy for the probability of individual default and thus provide a rationale behind using credit scores in pricing loans. Furthermore, in addition to endogenizing interest rates on credit card debt, we also allow them to respond to changes in default incentives created by alternative bankruptcy arrangements in the two markets.

To this end, the novelty of our work consists in providing a theory about interactions between credit markets with different financial arrangements and their role for default on student loans. Previous research analyzed these two markets separately with the main focus being on credit card debt. Our paper attempts to bridge this gap. Our results are not specific to the interpretation for student loans and credit cards and speak to consumer default in any environments that feature differences in financial market arrangements and thus induce a trade-off in default incentives for consumers that participate in these markets. In this respect our paper is related to Chatterjee, Corbae, and Rios-Rull (2008) who provide a theory of unsecured credit based on the interaction between unsecured credit and insurance markets. In related empirical work, Edelberg (2006) studies the evolution of credit card and student loan markets and finds that there has been an increase in the cross-sectional variance of interest rates charged to consumers which is largely due to movements in credit card loans: the premium spread for credit card loans more than doubled, but education loan and other consumer loan premiums are statistically unchanged.

⁹The modeling of alternative bankruptcy rules and induced trade-offs in default decisions poses obvious technical challenges which will be addressed in this paper.

The paper is organized as follows. In Section 2, we describe several important facts about student loans and credit card terms and the legal and financial environment in the two markets. We develop the model and present the theoretical results in Section 3. We calibrate the economy to match important features of the markets for student and credit card loans and present quantitative results in Section 4. Section 5 concludes.

2 Background

2.1 Legal environment

The financial and legal environment surrounding the U.S. credit cards and student loan markets is characterized by the following features.

1) Student loans are not secured by any tangible asset, so there might be some similarities with credit card markets, but unlike credit card loans, guaranteed student loans are uniquely risky, since the eligibility conditions are very different. Loans are based on financial need, not on credit ratings, and are subsidized by the government. Agents are eligible to borrow up to the full college cost minus expected family contributions.

2) In contrast, for credit card loans, lenders use credit scores (FICO) as a proxy for the risk of default. These scores include information about repayment and borrowing behavior on all types of loans that borrowers have, in addition to credit card debt.

3) For credit cards, lenders impose interest rates that vary significantly across individuals with different risks of default. In general, in unsecured credit markets the feedback of any bankruptcy law into the interest rate is exactly how the default is paid for.

4) The interest rate on student loans, however, does not incorporate the risk that some borrowers might exercise the option to default. The interest rate is fixed by the government. Several default penalties implemented in the student loan program such as wage garnishments upon default might bear part of the default risk.¹⁰

5) Default on credit card debt triggers limited market participation. At the same time, default in the student loan market has no effect on credit card market participation.

6) Finally, individuals can file for bankruptcy for credit cards under Chapter 7 of the U.S. Bankruptcy Code, which implies permanent discharge of net debt (liabilities minus assets above exemption levels). In contrast, individuals can file for bankruptcy for student loans only under

¹⁰This penalty can be as high as 15 percent of defaulter's wages. In addition, consequences include seizure of federal tax refunds, possible hold on transcripts and ineligibility for future student loans.

Chapter 13, which does not allow for discharge and implies a fixed term repayment schedule.¹¹

2.2 Data facts

Findings documented in this section are based (for the most part) on the SCF data for young borrowers aged 20-30 years old who have some college education (with or without having a college degree) and who took out student loans to finance their college education. They are no longer enrolled in college and they need to repay their student loans. We construct these samples using the SCF 2004 and the SCF 2007. The sample sizes are 486 and 445, respectively. Summary statistics for these samples are provided in the Appendix.

1) According to the statistics from the Department of Education, the national two-year basis CDR on student loans increased from 4.5 percent in 2004 to 6.7 percent in 2007 (see Figure 2 in the Appendix).

2) College debt borrowed by young US households increased by almost 35 percent during this period of time. However, income of young individuals has not kept pace: it only increased by 28.7 percent from 2004 to 2007. Consequently, total college debt burdens (debt-to-income ratios) increased substantially. The per-period debt burden increased from 0.0486 to 0.055.

3) Young borrowers with student loans use credit cards at very high rates: around 78 percent of young U.S. households in both years in the SCF have at least a credit card and 79 percent of those who are credit card users have positive balances.

4) Credit card debt borrowed by young US households increased by 24 percent during this period of time.

5) Terms on credit card accounts of young borrowers have worsen: the interest rate that borrowers received on their credit card accounts increased from 12.07 percent in 2004 to 13.34 percent in 2007. In general, terms deteriorate after 2005. In the past several years, credit card providers have levied some of the largest increases in interest rates, fees and minimum payments.¹²

¹¹Borrowers are considered in default on student loans if they do not make any payments within 270 days in the case of a loan repayable in monthly installments or 330 days in the case of a loan repayable in less frequent installments. Loan forgiveness is very limited. It is granted only in the case constant payments are made for 25 years or in the case where repayment causes undue hardship. As a practical matter, it is very difficult to demonstrate undue hardship unless the defaulter is physically unable to work. Partial dischargeability occurs in less than 1 percent of the default cases.

¹²For instance, JPMorgan Chase, the biggest credit card provider raised the minimum payment on outstanding balances from 2 percent to 5 percent for some customers, raised its balance-transfer fee from 3 percent to 5 percent – the highest rate among the large consumer banks and also changed its United Mileage Plus Visa Signature card from a single 13.24 percent rate to a range of 13.24 percent to 19.24 percent, meaning most cardholders are likely to qualify for those costlier rates (June 30 Bloomberg article). Citigroup has reportedly raised rates on outstanding balances nearly 3 percentage points to an average of 24 percent for 13 million to 15 million cardholders (July 1 2009 Financial Times article).

Also, retailers have become stingier with credit.¹³ Our findings from the SCF 2010 showed that the average amount lent by credit card issuers to young borrowers declined by 31.5 percent from 2007 to 2010.

6) High college debt burdens increase the likelihood of default for student loans (see Dynarsky (1994) and Ionescu (2008)).

7) High credit card burdens increase the likelihood of default on credit card debt (see Athreya, Tam, and Young (2009) and Chatterjee, Corbae, Nakajima, and Rios-Rull (2007)).

8) Low income increases the likelihood of default on credit card debt (see Sullivan, Warren, and Westbrook (2001)).

3 Model

3.1 Legal environment

Consumers who participate in the student loan and credit cards markets, namely, young college educated individuals with student loans, are small, risk-averse, price takers. They differ in levels of college debt, d and income, y . They are endowed with a line of credit, which they may use for transactions and consumption smoothing. They choose to repay or default on their student loans as well as on their credit card debt. Default in each of these two markets has different consequences, which we explain below.

3.1.1 Credit cards

Bankruptcy for credit cards in the model resembles Chapter 7 “total liquidation” bankruptcy. The model captures the fact that credit card issuers use consumer characteristics to assess the likelihood that any single borrower will default. Loan prices and credit limits imposed by credit card issuers are set to account for the individual default risk and are tailored to each credit account.

Consider a household that starts the period with some credit card debt, b_t . Depending on the household decision to declare bankruptcy as well as on the household borrowing behavior, the following things happen:

1. If a household files for bankruptcy, $\lambda_b = 1$, then the household unsecured debt is discharged and liabilities are set to 0.

¹³For instance, American Express has taken the most heat over slashing credit limits. Nearly half of its portfolio underwent a major overhaul that included cutting limits by a half or more. Chase decreased credit lines or closed accounts in 2008 totaling \$129 billion. Most credit card issuers enforced fees, and reduced credit lines on their in-store cards (examples include Home Depot, Target).

2. The household cannot save during the period when default occurs, which is a simple way of modeling that the U.S. bankruptcy law does not permit those invoking bankruptcy to simultaneously accumulate assets.
3. The household begins next period with a record of default on credit cards. Let $f_t \in F = \{0, 1\}$ denote the default flag for a household in period t , where $f_t = 1$ indicates in period t a record of default and $f_t = 0$ denotes the absence of such record. Thus a household who defaults on credit in period t starts period $t + 1$ with $f_{t+1} = 1$.
4. A household who starts the period with a default flag cannot borrow and the default flag can be erased with a probability p_f .
5. In contrast, a household who starts the period with $f_t = 0$ is allowed to borrow and save according to individual credit terms: credit rates assigned to household by credit lenders vary with individual characteristics. This feature is important to allow for capturing default risk pricing in equilibrium.

This formulation captures the idea that there is restricted market participation for borrowers who have defaulted in the credit card market relative to borrowers who have not. It also implies more stringent credit terms for consumers who take on more credit card debt, i.e. precisely the type of borrowers who are more constrained in their capability to repay their loans. In addition, creditors take into account borrowing behavior in the other type of market, i.e. the student loan amount owed, d_t as well as the default status for student loans, h_t . These features are consistent with the fact that credit card issuers reward good repayment behavior and penalize bad repayment behavior taking into account this behavior in all markets that borrowers participate in. Finally, we assume that defaulters on credit cards are not completely in autarky, which is consistent with evidence. In U.S. consumer credit markets, households retain a storage technology after bankruptcy, namely, the ability to save. We assume without loss of generality that defaulters cannot borrow. In practice, borrowers who have defaulted in the past several years are still able to obtain credit at worse terms. In our model, allowing them a small negative amount or 0 does not have an effect on the results.

3.1.2 Student loans

Bankruptcy for student loans in the model resembles Chapter 13 “reorganization” bankruptcy, which requires reorganization and repayment of defaulted loans. Under the current Federal Loan Program students who participate cannot discharge on their student loans. Consequently, default on student loans in the model at period t (denoted by $\lambda_d = 1$) simply means a delay in repayment that triggers the following consequences:

1. There is no debt repayment in period t . However, the college debt is not discharged. The defaulter must repay the amount owed for payment in period $t + 1$.
2. The defaulter is not allowed to borrow or save in period t , which is in line with the fact that credit bureaus are notified when default occurs and thus access to the credit card market is restricted. Also, as in the case of the credit card market, this feature captures the fact that the U.S. bankruptcy law does not permit those invoking bankruptcy to simultaneously accumulate assets.
3. A fraction γ of the defaulter's wages is garnished starting in period $t + 1$. Once the defaulter rehabilitates her student loan, the wage garnishment is interrupted. This penalty captures the default risk for student loans in the model.
4. The household begins next period with a record of default on student loans. Let $h_t \in H = \{0, 1\}$ denote the default flag for a household in period t , where $h_t = 1$ indicates a record of default and $h_t = 0$ denotes the absence of such record. Thus a household who defaults in period t starts period $t + 1$ with $h_{t+1} = 1$.
5. A household that begins period t with a record of default must pay the debt owed in period t , d_t . The default flag is erased with probability p_h .¹⁴
6. There are no consequences on credit card market participation during the periods after default on student loan occurs. However, there are consequences on the pricing of credit card loans from defaulting on student loans, as mentioned above. This assumption is justified by the fact that in practice student loan default is reported to credit bureaus and so creditors can observe the default status immediate after default occurs and adjust terms on loans. However, immediate repayment and rehabilitation of the defaulted loan will result in deleting the default status reported by the loan holder to the national credit bureaus. In practice, the majority of defaulters rehabilitate their loans. Therefore they are still able to access the credit card market (on worse terms as explained above).

¹⁴The household cannot default the following period after default occurs. As mentioned before, less than 1 percent of borrowers repeat default given that the U.S. government seizes tax refunds in the case when the defaulter does not rehabilitate her loan soon after default occurs. This penalty is severe enough to induce immediate repayment after default.

3.2 Preferences and endowments

At any point in time the economy is composed of a continuum of infinitely lived households with unit mass.¹⁵ Agents differ in student loan payment levels, $d \in D = \{d_{\min}, \dots, d_{\max}\}$ and income levels, $y \in Y = [y_{\min}, y_{\max}]$. There is a constant probability $(1 - \rho)$ that households will die at the end of each period. Households that do not survive are replaced by newborns who have not defaulted on student loans ($h = 0$), or on their credit cards ($f = 0$), have zero assets ($b = 0$) and with labor income and college debt drawn independently from the probability measure space $(Y \times D, \mathcal{B}(Y \times D), \psi)$ where $\mathcal{B}(\cdot)$ denotes the Borel sigma algebra and $\psi = \psi_y \times \psi_d$ denotes the joint probability measure. Surviving households independently draw their labor income at time t from a stochastic process. The amount that the household needs to pay on her student loan is the same.¹⁶ Household characteristics are then defined on the measurable space $(Y \times D, \mathcal{B}(Y \times D))$. The transition function is given by $\Phi(y_{t+1})\delta_d(d_{t+1})$, where $\Phi(y_t)$ is an i.i.d. process and δ_d is the probability measure supported at d .

The preferences of the households are given by the expected value of a discounted lifetime utility which consists of:

$$E_0 \sum_{t=0}^{\infty} (\rho\beta)^t U(c_t) \quad (1)$$

where c_t represents the consumption of the agent during period t , $\beta \in (0, 1)$ is the discount factor, and $\rho \in (0, 1)$ is the survival probability.

Assumption 1. *The utility function $U(\cdot)$ is increasing, concave and twice differentiable. It also satisfies Inada condition: $\lim_{c \rightarrow 0^+} U(c) = -\infty$ and $\lim_{c \rightarrow 0^+} U'(c) = \infty$.*

3.3 Markets Arrangements

There are several similarities as well as important differences between the credit card market and the market for student loans.

3.3.1 Credit cards

The market for privately issued unsecured credit in the U.S. is characterized by a large, competitive market place where price-taking lenders issue credit through the purchase of securities backed by

¹⁵The use of infinitely lived households is justified by the fact that we focus on the cohort default rate for young borrowers, which means that age distributions are not crucial for analyzing default rates in the current study. The use of a continuum of households is natural, given the size of the credit market.

¹⁶Government student loan payments are fixed and computed based on a fixed interest rate and duration of the loan.

repayments from those who borrow. These transactions are intermediated principally by credit card issuers. Given a default option and consequences on the credit record from default behavior, the market arrangement departs from the conventional modeling of borrowing and lending. As in Chatterjee, Corbae, Nakajima, and Rios-Rull (2007) our model handles the competitive pricing of default risk, a risk that varies with household characteristics.¹⁷ In this dimension, our model departs from Chatterjee, Corbae, Nakajima, and Rios-Rull (2007) in several important ways: the default risk is based on the borrowing behavior in both markets, i.e. it depends on the size of the loan on credit cards, b_t as well as on the amount of student loans owed, d_t . In addition, it depends on the default status on student loans, h_t . Competitive default pricing is achieved through letting prices vary with all these three elements. This modeling feature is novel in the literature and is meant to capture the fact that in practice the price of the loan depends on past repayment and borrowing behavior in all the markets that borrowers participate in. Unsecured credit card lenders use this behavior (which in practice is captured in a credit score) as a signal for household credit risks and thus their probability of default. They tailor loan prices to individual default risk, not only to individual loan sizes. Obviously in the case of a default flag on credit cards, no loan is provided.

A household can borrow or save by purchasing a single one-period pure discount bond with a face value in a finite set $B \subset \mathbb{R}$. The set $B = \{b_{\min}, \dots, b_{\max}\}$ contains 0 and positive and negative elements. Let N_B be the cardinality of this set. Individuals with $f_t = 1$ (which is a result of defaulting on credit cards in one of the previous periods) are limited in their market participation, $b_{t+1} \geq 0$.¹⁸

A purchase of a discount bond in period t with a nonnegative face value b_{t+1} means that the household has entered into a contract where it will receive $b_{t+1} \geq 0$ units of the consumption good in period $t + 1$. The purchase of a discount bond with a negative face value b_{t+1} means that the household receives $q_{d_t, h_t, b_{t+1}}(-b_{t+1})$ units of the period- t consumption good and promises to deliver, conditional on not declaring bankruptcy, $-b_{t+1} > 0$ units of the consumption good in period $t + 1$; if it declares bankruptcy, the household delivers nothing. The total number of credit indexes is $N_B \times N_D \times N_H$. Let the entire set of $N_B \times N_D \times N_H$ prices in period t be denoted by the vector $q_t \in \mathbb{R}^{N_B \times N_D \times N_H}$. We restrict q_t to lie in a compact set $Q \equiv [0, q_{\max}]^{N_B \times N_D \times N_H}$ where

¹⁷Chatterjee, Corbae, Nakajima, and Rios-Rull (2007) handles the competitive pricing of default risk by expanding the “asset space” and treating unsecured loans of different sizes for different types of households (of different characteristics) as distinct financial assets.

¹⁸Note that households are liquidity constrained in the model. The existence of such constraints in credit card markets has been documented by Gross and Souleles (2002). Overall credit availability has not decreased along with bankruptcy rates over the past several years before the crisis and so aggregate response of credit supply to changing default has not been that large (Athreya (2002)).

$$0 < q_{\max} < 1.$$

3.3.2 Student loans

Student loans represent a different form of unsecured credit. First, loans are primarily provided by the government (either direct or indirect and guaranteed through the FSLP), and do not share the features of a competitive market.¹⁹ Unlike for credit cards, the interest rate on student loans, r_g is fixed by the government and does not reflect the risk of default in the student loan market.²⁰ However, the penalties for default capture some of this risk. In particular, the wage garnishment is adjusted to cover default. More generally, loan terms are based on financial need, not on default risk. Secondly, taking out student loans is a decision made during college years. Once they are out of college, households need to repay their loans in equal rounds over a determined period of time subject to the fixed interest rate. We model college loan bound households that are out of school and need to repay d per period; there is no borrowing decision for student loans.²¹ Thirdly, defaulters cannot discharge their debt. Recall that in the case the household has a default flag ($h = 1$), a wage garnishment is imposed and she keeps repaying the amount owed during the following periods after default occurs.

We define the state space of credit characteristics of the households by $\mathcal{S} = B \times F \times H$ to represent the asset position, the credit card, and student loan default flags. Let $N_S = N_B \times 2 \times 2$ be the cardinality of this set.

To this end, an important note is that the assumption that all debt that young borrowers access is unsecured is made for a specific purpose and is not restrictive. The model is designed to represent the section of households who have student loans and credit card debt. As argued, these borrowers rely on credit cards to smooth consumption and have little or no collateral debt.

3.4 Decision problems

The timing of events in any period is: (i) idiosyncratic shocks, y_t are drawn for survivors and newborns and college debt is drawn for newborns; (ii) households default/repay on both credit

¹⁹Recently, students have started to use pure private student loans, not guaranteed by the government. This new market is a hybrid between government loans and credit cards featuring characteristics of both markets. However, this new market is still small and concerns about the national default rates are specific to student loans in the government program, default rates for pure private loans being of much lower magnitudes (for details see Ionescu and Simpson (2010)). Therefore we focus on government student loans in the current study.

²⁰After the Higher Education Reconciliation Act of 2005 was passed, the interest rate has been set to 6.8 percent. Before 2005 the rate was based on the 91-day Treasury-bill rate.

²¹While returning to school and borrowing another round of loans is a possibility, this decision is beyond the scope of the paper.

card and student loans and borrowing/savings decisions are made; also consumption takes place and default flags for the next period are determined. We focus on steady state equilibria where $q_t = q$.

3.4.1 Households

We present the households' decision problem in a recursive formulation where any period t variable x_t is denoted by x and its period $t + 1$ value by x' .

Each period, given their college debt, d , current income, y , and beginning-of-period assets, b , households must choose consumption, c , and asset holdings, b' , to carry forward into the next period. In addition, agents may decide to repay/default on their student loans, $\lambda_d \in \{0, 1\}$ and on credit card loans, $\lambda_b \in \{0, 1\}$. As described before, these decisions have different consequences: while default on student loans implies a wage garnishment γ and no effect on market participation (however it may deteriorate terms on credit card accounts), default on credit card payments triggers exclusion from borrowing for several periods and has no effect on income.

The household's current budget correspondence, $B_{b,f,h}(d, y; q)$, depends on the exogenously given income, y , college debt, d , beginning of period asset position, b , credit card default record, f , student loan default record, h , and the prices in the credit card market, q . It consists of elements of the form $(c, b', h', f', \lambda_d, \lambda_b) \in (0, \infty) \times B \times H \times F \times \{0, 1\} \times \{0, 1\}$ such that

$$c + q_{d,h,b'} b' \leq y(1 - g) - t + b(1 - \lambda_b) - d(1 - \lambda_d),$$

and such that the following cases hold:

1. If a household with income y and college debt d has a good student loan record, $h = 0$, and a good credit card record, $f = 0$, then we have the following: $\lambda_d \in \{0, 1\}$ and $\lambda_b \in \{0, 1\}$ if $b < 0$ and $\lambda_b = 0$ if $b \geq 0$. In the case where $\lambda_d = 1$ or $\lambda_b = 1$ then $b' = 0$ and in the case where $\lambda_d = \lambda_b = 0$ then $b' \in B$. Also $g = 0$, $h' = \lambda_d$, $f' = \lambda_d$. The household can choose to pay off both loans ($\lambda_b = \lambda_d = 0$), in which case the household can borrow freely on the credit card market. If the household chooses to exercise its default option on either one of the loans ($\lambda_d = 1$ or $\lambda_b = 1$), then the household cannot borrow or accumulate assets. Since $h = 0$ there is no income garnishment ($g = 0$).

2. If a household with income y and college debt d has a good student loan record, $h = 0$, and a bad credit card record, $f = 1$, then $\lambda_b = 0$, $\lambda_d \in \{0, 1\}$, $b' \geq 0$, $g = 0$, $h' = \lambda_d$, $f' = 1$. In this case, there is no repayment on credit card debt; the household chooses to pay or default on the student loan debt. The household cannot borrow and the credit card record will stay 1.

3. If a household with income y and college debt d has a bad student loan record, $h = 1$, and

a good credit card record, $f = 0$, then $\lambda_b \in \{0, 1\}$ if $b < 0$ and $\lambda_b = 0$ if $b \geq 0$, $\lambda_d = 0$, $g = \gamma$, $f' = \lambda_b$, and $h' = 1$. The household pays back the credit card debt (if net liabilities, $b < 0$) or defaults, pays the student loan and its income is garnished by a factor of γ . The student record will stay 1. As in case 1, $b' \in B$ if $\lambda_b = 0$ and $b' = 0$ if $\lambda_b = 1$.

4. If a household with income y and college debt d has a bad student loan record, $h = 1$, and a bad credit card record, $f = 1$, then $\lambda_d = \lambda_b = 0$, $b' \geq 0$, $g = \gamma$, $f' = 1$, $h' = 1$. The household cannot borrow in the credit card market, pays the student loan, and her income is garnished.

There are several important observations: 1) we account for the fact that the budget constraint may be empty; in particular if the household is deep in debt, earnings are low, new loans are expensive, then the household may not be able to afford non-negative consumption. The implication of this is that involuntary default may occur; and 2) Repeated default on student loans occurs on a limited basis (i.e. when $B_{b,f,1}(d, y; q) = \emptyset$) and it is followed by partial dischargeability, assumption which is in line with the data. All households pay taxes t .

Assumption 2. *We assume that consuming y_{min} today and starting with zero assets, $b = 0$ and a bad credit card record, $f = 1$ and student loan default record, $h = 1$ with garnished wage (i.e. the worst utility with a feasible action) gives a better utility than consuming zero today and starting next period with maximum savings, b_{max} and a good credit card record, $f = 0$ and student loan default record, $h = 0$ (i.e. the best utility with an unfeasible action).*

Let $v(d, y; q)(b, f, h)$ or $v_{b,f,h}(d, y; q)$ denote the expected lifetime utility of a household that starts with student loan debt d , earnings y , has asset b , credit card default record f , and student loan default record h , and faces prices q . Then v is in the set \mathcal{V} of all continuous functions $v : D \times Y \times Q \rightarrow \mathbb{R}^{Ns}$. The household's optimization problem can be described in terms of an operator $(Tv)(d, y; q)(b, f, h)$ which yields the maximum lifetime utility achievable if the household's future lifetime utility is assessed according to a given function $v(d, y; q)(b, f, h)$.

Definition 1. For $v \in \mathcal{V}$, let $(Tv)(d, y; q)(b, f, h)$ be defined as follows:

1. For $h = 0$ and $f = 0$

$$(Tv)(d, y; q)(b, f, h) = \max_{(c, b', h', f', \lambda_d, \lambda_b) \in B_{b,f,h}(d, y; q)} U(c) - \tau_b \lambda_b + \beta \rho \int v_{b', f', h'}(d, y'; q) \Phi(dy')$$

where τ_d is the utility cost that the household incurs in case of default in the credit card market.

2. For $h = 0$ and $f = 1$ (in which case $\lambda_b = 0$ and $f' = 1$ with probability $1 - p_f$ and $f' = 0$

with probability p_f)

$$(Tv)(d, y; q)(b, f, h) = \max_{B_{b,f,h}(d,y;q)} \left\{ U(c) + (1 - p_f)\beta\rho \int v_{b',1,h'}(d, y'; q)\Phi(dy') \right. \\ \left. + p_f\beta\rho \int v_{b',0,h'}(d, y'; q)\Phi(dy') \right\}.$$

3. For $h = 1$ and $f = 0$ (in which case $\lambda_d = 0$ and $h' = 1$ with probability $1 - p_h$ and $h' = 0$ with probability p_h)

$$(Tv)(d, y; q)(b, f, h) = \max \left\{ \max_{B_{b,f,h}(d,y;q)} \left\{ U(c) - \tau_b\lambda_b + (1 - p_h)\beta\rho \int v_{b',f',1}(d, y'; q)\Phi(dy') \right. \right. \\ \left. \left. + p_h\beta\rho \int v_{b',f',0}(d, y'; q)\Phi(dy') \right\}, \right. \\ \left. U(y) - \tau_b + \beta\rho \int v_{0,1,1}(d, y'; q)\Phi(dy') \right\}.$$

4. For $h = 1$ and $f = 1$

$$(Tv)(d, y; q)(b, f, h) = \max \left\{ \max_{B_{b,f,h}(d,y;q)} \left\{ U(c) + (1 - p_f)(1 - p_h)\beta\rho \int v_{b',1,1}(d, y'; q)\Phi(dy') \right. \right. \\ \left. \left. + (1 - p_f)p_h\beta\rho \int v_{b',1,0}(d, y'; q)\Phi(dy') \right. \right. \\ \left. \left. + p_f(1 - p_h)\beta\rho \int v_{b',0,1}(d, y'; q)\Phi(dy') \right. \right. \\ \left. \left. p_fp_h\beta\rho \int v_{b',0,0}(d, y'; q)\Phi(dy') \right\}, \right. \\ \left. U(y) + \beta\rho \int v_{0,1,1}(d, y'; q)\Phi(dy') \right\}.$$

The first part of this definition says that a household with good student loan and credit card default records may choose to default on either type of loan, on both or on none of them. For all these cases to be feasible we need to have that the budget sets conditional on not defaulting on student loans or on credit card debt are non-empty. In the case at least one of these sets is empty, then automatically the attached option is not available. In the case where both default and no default options deliver the same utility the household may choose either. Finally, recall that in the case the household chooses to repay on her student loans or on her credit card debt she may also choose borrowing and savings and in the case she decides to default on either of these loans there is no choice on assets position.

The second part of the definition says that if the household with a good student loan default record and with a default flag on credit cards will only have the choice to default/repay on student loans since she does not have any credit card debt. Recall that as long as the household carries the default flag in the credit card market she cannot borrow.

The last two parts represent cases for a household with a bad student loan default record. In these last cases, default on student loans is not an option. In part three the household has the choice to default on credit card. As before, this is an option only if the associated budget set is non-empty. In the case all of these sets are empty then involuntarily default occurs. We assume that when involuntarily default happens it will occur on both markets (this is captured in the second term of the maximization problem).²²

In part four, however, there is no choice to default given that $f = 1$ and $h = 1$. Thus, the household simply solves a consumption/savings decision if the budget set conditional on not defaulting on either loan is non-empty. Otherwise, we assume that involuntarily default occurs. In this case, this happens only in the student loan market since there is no credit card debt.

There are two additional observations: First, in all the cases where default occurs on credit card debt, the household incurs a utility cost, which is denoted by τ_b . Consistent with modeling of consumer default in the literature, these utility costs are meant to capture stigma following default as well as attorney and collection fees associated with default.²³ Second, involuntarily default happens when borrowers with very low income realizations and high indebtedness have no choice but defaulting. Note that this case occurs repeatedly in the student loan market, i.e. for a household with default flag, $h = 1$. Under these circumstances we assume that the household may discharge her student loan and there is no wage garnishment. This feature captures the fact that in practice a small proportion of households partially discharge their student loan debt.

We next proceed as follows: we provide a first set of results which contains the existence and uniqueness of the household's problem and the existence of the invariant distribution. The second set of results contains the characterization of both default decisions in terms of households characteristics and market arrangements. The last set of results contains the existence of the equilibrium and the characterization of prices. We prove the existence of cross-market effects and characterize how financial arrangements in one market affect default behavior in the other market. All the proves are provided in the Appendix.

²²This assumption is made such that default is not biased towards one of the two markets.

²³See Athreya, Tam, and Young (2009), Chatterjee, Corbae, Nakajima, and Rios-Rull (2007), and Livshits, MacGee, and Tertilt (2007).

Existence and uniqueness of a recursive solution to the household's problem

Theorem 1. *There exists a unique $v^* \in \mathcal{V}$ such that $v^* = Tv^*$ and*

1. v^* is increasing in y and b .
2. Default decreases v^* .
3. The optimal policy correspondence implied by Tv^* is compact-valued, upper hemi-continuous.
4. Default is strictly preferable to zero consumption and optimal consumption is always positive.

Since Tv^* is a compact-valued upper-hemicontinuous correspondence, Theorem 7.6 in Stokey, Lucas, and Prescott (Measurable Selection Theorem) implies that there are measurable policy functions, $c^*(d, y, ; q)(b, f, h)$, $b^*(d, y; q)(b, f, h)$, $\lambda_b^*(d, y; q)(b, f, h)$ and $\lambda_d^*(d, y; q)(b, f, h)$. These measurable functions determine a transition matrix for f and f' , namely $F_{y,d,b,h,q}^* : F \times F \rightarrow [0, 1]$:

$$F_{y,d,b,h,q}^*(f, f' = 1) = \begin{cases} 1 & \text{if } \lambda_b^* = 1 \\ 1 - p_f & \text{if } \lambda_b^* = 0 \text{ and } f = 1 \\ 0 & \text{otherwise} \end{cases}$$

$$F_{y,d,b,h,q}^*(f, f' = 0) = \begin{cases} 0 & \text{if } \lambda_b^* = 1 \\ p_f & \text{if } \lambda_b^* = 0 \text{ and } f = 1 \\ 1 & \text{otherwise} \end{cases}$$

Also the policy functions determine a transition matrix for the student loan default record, $H_{y,d,b,f,q}^* : H \times H \rightarrow [0, 1]$ which gives the student loan record for the next period, h' :

$$H_{y,d,b,f,q}^*(h, h' = 1) = \begin{cases} 1 & \text{if } \lambda_d^* = 1 \text{ and } h = 0 \\ 1 - p_h & \text{if } h = 1 \\ 0 & \text{otherwise,} \end{cases}$$

$$H_{y,d,b,f,q}^*(h, h' = 0) = \begin{cases} 0 & \text{if } \lambda_d^* = 1 \text{ and } h = 0 \\ p_h & \text{if } h = 1 \\ 1 & \text{otherwise.} \end{cases}$$

Existence of invariant distribution

Let $X = Y \times D \times B \times F \times H$ be the space of household characteristics. In the following we write $F_q^*(y, d, b, h, f, f') := F_{y,d,b,h,q}^*(f, f')$ and $H_q^*(y, d, b, f, h, h') := H_{y,d,b,f,q}^*(h, h')$. Then the transition function for the surviving households' state variable $TS_q^* : X \times \mathcal{B}(X) \rightarrow [0, 1]$ is given by

$$TS_q^*(y, d, b, f, h, Z) = \int_{Z_y \times Z_d \times Z_b \times Z_f \times Z_h} \mathbf{1}_{\{b^* \in Z_b\}} F_q^*(y, d, b, h, f, df') H_q^*(y, d, b, f, h, dh') \Phi(dy') \delta_d(d')$$

where $Z = Z_y \times Z_d \times Z_b \times Z_f \times Z_h$ and $\mathbf{1}$ is the indicator function. The households that die are replaced with newborns. The transition function for the newborn's initial conditions, $TN_q^* : X \times \mathcal{B}(X) \rightarrow [0, 1]$ is given by

$$TN_q^*(y, d, b, f, h, Z) = \int_{Z_y \times Z_d} \mathbf{1}_{\{(b', h', f') = (0, 0, 0)\}} \Psi(dy', dd')$$

Combining the two transitions we can define the transition function for the economy, $T_q^* : X \times \mathcal{B}(X) \rightarrow [0, 1]$ by

$$T_q^*(y, d, b, f, h, Z) = \rho TS_q^*(y, d, b, f, h, Z) + (1 - \rho) TN_q^*(y, d, b, f, h, Z)$$

Given the transition function T_q^* , we can describe the evolution of the distribution of households μ across their state variables (y, d, b, f, h) for any given prices q . Specifically, let $\mathcal{M}(x)$ be the space of probability measures on X . Define the operator $\Gamma_q : \mathcal{M}(x) \rightarrow \mathcal{M}(x)$:

$$(\Gamma_q \mu)(Z) = \int T_q^*((y, d, b, f, h), Z) d\mu(y, d, b, f, h).$$

Theorem 2. *For any $q \in Q$ and any measurable selection from the optimal policy correspondence there exists a unique $\mu_q \in \mathcal{M}(x)$ such that $\Gamma_q \mu_q = \mu_q$.*

3.4.2 Characterization of the default decisions

We first determine the set for which default occurs for student loans (including involuntarily default with partial dischargeability), the set for which default occurs for credit card debt, as well as the set for which default occurs for both of these two loans. Let $D_{b,f,1}^{SL}(q)$ be the set for which involuntarily default on student loans and partial dischargeability occurs. This set is defined as combinations of earnings, y , and student loan amount, d , for which $B_{b,f,1}(d, y; q) = \emptyset$ in the case $h = 1$. For $h = 0$ let $D_{b,f,0}^{SL}(d; q)$ be the set of earnings for which the value from defaulting on student loans exceeds

the value of not defaulting on student loans. Similarly, let $D_{b,0,h}^{CC}(d; q)$ be the set of earnings for which the value from defaulting on credit card debt exceeds the value of not defaulting on credit card debt in the case $f = 0$. Finally, let $D_{b,0,0}^{Both}(d; q)$ be the set of earnings for which default on both types of loans occurs with $h = 0$ and $f = 0$. Note that the last two sets are defined only in the case $f = 0$ since for $f = 1$ there is no credit card debt to default on.

Theorem 3 characterizes the sets when default on student loans occurs (voluntarily or non-voluntarily). Theorem 4 characterizes the sets when default occurs on credit card debt and Theorem 5 presents the set for which default occurs for both types of loans.

Theorem 3. *Let $q \in Q$, $b \in B$. If $h = 1$ and the set $D_{b,f,1}^{SL}(q)$ is nonempty, then $D_{b,f,1}^{SL}(q)$ is closed and convex. In particular the sets $D_{b,f,1}^{SL}(d; q)$ are closed intervals for all d . If $h = 0$ and the set $D_{b,f,0}^{SL}(d; q)$ is nonempty, then $D_{b,f,0}^{SL}(d; q)$ is a closed interval for all d .*

Theorem 4. *Let $q \in Q$, $(b, 0, h) \in \mathcal{S}$. If $D_{b,0,h}^{CC}(d; q)$ is nonempty then it is a closed interval for all d .*

Theorem 5. *Let $q \in Q$, $(b, 0, 0) \in \mathcal{S}$. If the set $D_{b,0,0}^{Both}(d; q)$ is nonempty then it is a closed interval for all d .*

Next we determine how the set of default on credit card debt varies with the credit card debt, the student loan debt and the default status on student loans of the individual. Specifically, Theorem 6 shows that the set of default on credit card debt expands with the amount of debt for credit cards. This result was first demonstrated in Chatterjee, Corbae, Nakajima, and Rios-Rull (2007).

Theorem 6. *For any price $q \in Q$, $d \in D$, $f \in F$, and $h \in H$, the sets $D_{b,f,h}^{CC}(d; q)$ expand when b decreases.*

In addition, we show two new results in the literature: 1) the set of default on credit card only shrinks when the student loan amount increases and the set of default on both credit card and student loans expands when the student loan amount increases. These findings imply that individuals with lower levels of student loans are more likely to default only on credit card debt and individuals with higher levels of student loans are more likely to default on both credit card and college debt (Theorem 7); and 2) the set of default on credit card is larger when $h = 1$ relative to the case where $h = 0$. This result implies that individuals with a default record on student loans are more likely to default on their credit card debt (Theorem 8).

Theorem 7. *For any price $q \in Q$, $b \in B$, $f \in F$, and $h \in H$, the sets $D_{b,f,h}^{CC}(d; q)$ shrink and $D_{b,f,h}^{Both}(d; q)$ expand when d increases.*

Theorem 8. For any price $q \in Q$, $b \in B$, $d \in D$, and $f \in F$, the set $D_{b,f,0}^{CC}(d; q) \subset D_{b,f,1}^{CC}(d; q)$.

This last set of theorems show the importance of accounting for borrowing and default behavior in the student loan market when determining the risk of default on credit card debt. These elements will be considered in the decision of the financial intermediary, which we explain next.

3.4.3 Financial intermediaries

The (representative) financial intermediary has access to an international credit market where it can borrow or lend at the risk-free interest rate $r \geq 0$. The intermediary operates in a competitive market and takes prices as given and chooses the number of loans $\xi_{d_t, h_t, b_{t+1}}$ for all type (d_t, h_t, b_{t+1}) contracts for each t to maximize the present discounted value of current and future cash flows $\sum_{t=0}^{\infty} (1+r)^{-t} \pi_t$, given that $\xi_{d_{-1}, h_{-1}, b_0} = 0$. The period t cash flow is given by

$$\pi_t = \rho \sum_{d_{t-1}, h_{t-1}} \sum_{b_t \in B} (1 - p_{d_{t-1}, h_{t-1}, b_t}^b) \xi_{d_{t-1}, h_{t-1}, b_t} (-b_t) - \sum_{d_t, h_t} \sum_{b_{t+1} \in B} \xi_{d_t, h_t, b_{t+1}} (-b_{t+1}) q_{d_t, h_t, b_{t+1}} \quad (2)$$

where $p_{d_t, h_t, b_{t+1}}^b$ is the probability that a contract of type (d_t, h_t, b_{t+1}) where $b_{t+1} < 0$ experiences default; if $b_{t+1} \geq 0$, automatically $p_{d_t, h_t, b_{t+1}}^b = 0$. These calculations take into account the survival probability ρ .

If a solution to the financial intermediary's problem exists, then optimization implies $q_{d_t, h_t, b_{t+1}} \leq \frac{\rho}{(1+r)} (1 - p_{d_t, h_t, b_{t+1}}^b)$ if $b_{t+1} < 0$ and $q_{d_t, h_t, b_{t+1}} \geq \frac{\rho}{(1+r)}$ if $b_{t+1} \geq 0$. If any optimal $\xi_{d_t, h_t, b_{t+1}}$ is nonzero then the associate conditions hold with equality.

3.4.4 Government

The only purpose for the government in this model is to operate the student loan program. The government needs to collect all student loans. The cost to the government is the total amount of college loans plus the interest rate subsidized in college. Denote by L this loan price. We compute the per period payment on student loans, d as the coupon payment of a student loan with its face value equals to its price (a debt instrument priced at par) and infinite maturity (console). Thus the coupon rate equals its yield rate, r_g . In practice, this represents the government interest rate on student loans. When no default occurs the present value of coupon payments from all borrowers (revenue) is equal to the price of all the loans made (cost), i.e. the government balances its budget.

However, since default is a possibility, government's budget constraint may not hold. In this case the government revenue from a household in state b with credit card default status f , income y and college debt d is given by $(1 - p_d^d)d$ where p_d^d is the probability that a contract of type d experiences default for student loans. The government will choose taxes, t to recover the losses

incurred when default for student loans arises. The budget constraint is then given by

$$\int d\psi_d(dd) = \int [(1 - p_d^d)d\psi_d(dd) + \int td\mu.$$

Taxes are lump-sum and equally distributed in the economy. They are chosen such that the budget constraint balances. We turn now to the definition of equilibrium and characterize the equilibrium in the economy.

3.5 Steady-state equilibrium

In this section we define a steady state equilibrium, prove its existence and characterize the properties of the price schedule for individuals with different default risks.

Definition 2. A steady-state competitive equilibrium is a set of non-negative price vector $q^* = (q_{d,h,b'}^*)$, non-negative credit card loan default frequency vector $p^{b*} = (p_{d,h,b'}^{b*})$, a non-negative student loan default frequency p_d^* , taxes, t^* , a vector of non-trivial credit card loan measures, $\xi^* = (\xi_{d,h,b'}^*)$, decision rules $b'^*(y, d, f, b, h, q^*)$, $\lambda_b^*(y, d, f, b, h, q^*)$, $\lambda_d^*(y, d, f, b, h, q^*)$, $c^*(y, d, f, b, h, q^*)$, and a probability measure μ^* such that:

1. $b'^*(y, d, f, b, h, q)$, $\lambda_b^*(y, d, f, b, h, q)$, $\lambda_d^*(y, d, f, b, h, q)$, $c^*(y, d, f, b, h, q)$ solve the household's optimization problem;
2. t^* solves the government's budget constraint;
3. $p_d^{d*} = \int \lambda_d^*(y, d, f, b, h)d\mu^*(dy, d, df, db, dh)$ (government consistency);
4. ξ^* solves the intermediary's optimization problem;
5. $p_{d,h,b'}^{b*} = \int \lambda_b^*(y', d, 0, b', h'^*)\Phi(dy')H^*(h, dh')$ for $b' < 0$ and $p_{d,h,b'}^{b*} = 0$ for $b' \geq 0$ (intermediary consistency);
6. $\xi_{d,h,b'}^* = \int \mathbf{1}_{\{b'^*(y,d,f,b,h,q^*)=b'\}}\mu^*(dy, d, df, db, h)$ (market clearing conditions (for each type (d, h, b')));
7. $\mu^* = \mu_{q^*}$ where $\mu_{q^*} = \Gamma_{q^*}\mu_{q^*}$ (μ^* is an invariant probability measure).

The computation of equilibrium in incomplete markets models has been made standard by a series of papers including (Aiyagari, 1994) and (Huggett, 1993) and have been extensively used in recent papers with the one by (Chatterjee, Corbae, Nakajima, and Rios-Rull, 2007) being the most related to the current study. The dimensionality of the state vector, the non-trivial market clearing

conditions which include a menu of loan prices, the condition for the government balancing budget as well as the interaction between the two types of credit make computation more involved than previous work.

3.5.1 Existence of equilibrium and characterization

Theorem 9. Existence *A steady-state competitive equilibrium exists.*

In equilibrium the credit card loan price vector has the property that all possible face-value loans (household deposits) bear the risk-free rate and negative face-value loans (household borrowings) bear a rate that reflects the risk-free rate and a premium that accounts for the default probability. This probability depends on the characteristics in the student loan markets, such as loan amount and default status as well as on the size of the credit card loan. This result is delivered by the free entry condition of the financial intermediary which implies that cross-subsidization across loans made to individuals of different characteristics in the student loan market is not possible. Each (d, h) market clears in equilibrium and it is not possible for intermediary to charge more than the cost of funds for individuals with very low risk in order to offset losses on loans made to high risk individuals. Positive profits in some contracts would offset the losses in others, and so intermediaries could enter the market for those profitable loans. We turn now to characterizing the equilibrium price schedule.

Theorem 10. Characterization of equilibrium prices *In any steady-state equilibrium the following is true:*

1. For any $b' \geq 0$, $q_{d,h,b'}^* = \rho/(1+r)$ for all $d \in D$ and $h \in H$.
2. If the grids of D and B are sufficiently fine and $h = 0$ then there are $\underline{d} > 0$ and $\underline{b}' < 0$ such that $q_{d,h,b'}^* = \rho/(1+r)$ for all $d < \underline{d}$ and $b' > \underline{b}'$.
3. If the set of income levels for which the household is indifferent between defaulting on credit card debt and any other available option is of measure zero, then $d_1 < d_2$ implies $q_{d_1,h,b'}^* > q_{d_2,h,b'}^*$ for any $h \in H$ and $b' \in B$.
4. If the set of income levels for which the household is indifferent between defaulting on credit card debt and any other available option is of measure zero, then $q_{d,h=1,b'}^* < q_{d,h=0,b'}^*$ for any $d \in D$ and $b' \in B$.

Theorem 10 demonstrates that firms charge the risk-free interest rate on deposits (property 1) and to small loan sizes made to individuals with no default record on student loans and small

enough levels of student loans (property 2). Property 3 shows that individuals with lower levels of student loans are assigned higher loan prices. The last property shows that individuals with a default record on student loans pay higher prices than individuals with no default record for any loan size, b' and for any amount of student loans they owe, d .

3.5.2 The interplay between the two markets

Since the novel feature in this paper is the interaction between different types of unsecured credit markets and its effects on default decisions, we show how the default decision varies not only with the loan amount in the respective market, but also with the loan amount in the other market. We already established that the default probability on credit card loans increases in the amount of student loans. In this section we demonstrate that a borrower with high enough loans to repay will prefer defaulting on her student loans rather than on her credit card debt. Theorem 11 shows that we can find a combination of credit card debt and college debt which induces a borrower to default. Furthermore, if the amounts owed to student loans and to credit card accounts are higher than the two values in this combination then the borrower will choose to default on student loans rather than on credit card debt.

Theorem 11. *If the grid of D is fine enough, then we can find $d_1 \in D$ and $b_1 \in B$ such that the agent defaults. Moreover, we can find $d_2 \geq d_1$ and $b_2 \leq b_1$ such that the agent defaults on student loans.*

The intuition behind this result is that with high enough debt levels, consumption is very small in the case the agent does not default at all. Consequently she finds it optimal to default. In the case where the student loan amount and credit card debt are large, defaulting on student loans is optimal since the option of defaulting only on credit card debt triggers limited market participation. Defaulting on credit card debt is too costly compared with the benefit of discharging one's debt. Therefore when borrowers find themselves in financial hardship and have to default they will always choose to default on student loans. They delay their repayments on student loans at the expense of having their wage garnished in the future. But this penalty is less severe compared to being excluded from borrowing for several periods. These are precisely the types of borrowers who are most in need of using the credit card market to help them smooth out consumption.

To conclude, our theory produces several facts consistent with the reality (presented in Section 2): First, the incentive to default on student loans increases in college debt burden (debt-to-income ratio), i.e. default on student loans is more likely to occur for individuals with low levels of earnings and high levels of college debt. Second, the incentive to default on credit card debt increases in credit card debt, which is consistent with findings in Chatterjee et. al. (2007).

Our theory innovates by showing that a household with a high amount of student loans or with a record of default on student loans is more likely to default on credit card debt. This result emphasizes the importance of accounting for other markets in which the individual participates in when studying default on credit card debt. Finally, we show that while a high college debt burden is necessary to induce default on student loans, this effect is amplified by indebtedness in the credit card market. The financial arrangements in the two markets, and in particular the differences in bankruptcy rules and default consequences between the two types of credit certainly play an important role in shifting default incentives.

4 Quantitative analysis

4.1 Mapping the model to the data

There are four sets of parameters that we calibrate: 1) standard parameters, such as the discount factor and the coefficient of risk aversion; 2) parameters for the initial distribution of college debt and income; 3) parameters specific to student loan markets such as default consequences and the interest rate on student loans; and 4) parameters specific to credit card markets. Our approach includes a combination of setting some parameters to values that are standard in the literature, calibrating some parameters directly to data, and jointly estimating the parameters that we do not observe in the data by matching moments for several observable implications of the model.

We calibrate the model to 2004 and use the Survey of Consumer Finances in 2004 for moments in the distribution of income, student loan, and credit card debt. The sample consists of households aged 20-30 years old with college education and college debt. The age group is chosen such that to include college dropouts and recent graduates. All individuals are out of college. The sample size is 486. Some summary statistics are provided in the Appendix. All numbers in the paper are provided in 2004 dollars.

The model period is one year and the coefficient of risk aversion chosen ($\sigma = 2$) and the discount factor ($\beta = 0.96$) are standard in the literature. We set the interest rate on student loans $r_g = 0.068$ as in the data. The annual risk-free rate is set equal to $r_f = 0.04$, which is the average return on capital reported by McGrattan and Prescott (2000). Table 1 presents the basic parameters of the model. We estimate the survival probability $\rho = 0.975$ to match average years of life to 40.²⁴ The probabilities to keep default flags in the two markets are set to $1 - p_f = 0.9$ and $1 - p_h = 0.5$ to match average years of punishments in the credit card and in the student loan markets (of ten and two years, respectively). The first choice is consistent with estimates in the literature (see

²⁴Since our agents are 26 years old, this matches a lifetime expectancy of 68 years old.

Table 1: Parameter Values

Parameter	Name	Value	Target/Source
Targets determined independently			
σ	Coef of risk aversion	2	standard
β	Discount factor	0.96	standard
r_g	Interest on student loans	0.068	Dept. of Education
$1 - p_f$	Probability to keep CC default flag	0.9	Avg years of punishment=10
$1 - p_h$	Probability to keep SL default flag	0.5	Avg years of punishment=2
ρ	Survival probability	0.975	Avg years of life=40
r_f	Risk-free rate	0.04	McGrattan and Prescott (2000)
Targets determined jointly			
γ	Wage garnishment	0.031	Default rate on SL in 2004
τ_p	Utility loss from CC default	9.4	Avg CC debt in 2004

Chatterjee, Corbae, Nakajima, and Rios-Rull (2007) and Livshits, MacGee, and Tertilt (2007)) and the second choice is consistent with regulations from the Department of Education.

We use the joint distribution of college debt and income for young households as delivered by the SCF 2004. We assume a log normal distribution with parameters $(\mu_y, \sigma_y, \mu_d, \sigma_d, \rho_{yd}) = (-1.473; 0.704; -4.53; 0.758; -0.106)$ on $[0, 1] \times [0, 0.12]$.²⁵ We pick the grid for assets consistent with the distribution of credit card debt in the SCF 2004, for which the moments are (\$2,906; \$4,330). We jointly estimate the wage garnishment (γ) and the utility loss from defaulting on credit card loans (τ_p) to match the default rate for student loans of 4.5 percent (according to the Department of Education annual releases) and the average level of credit card debt in our sample from SCF 2004. We get a default rate on student loans of 4.5 percent and an average level of credit card debt of \$3,018.²⁶

4.2 Results: benchmark economy

4.2.1 Model versus data

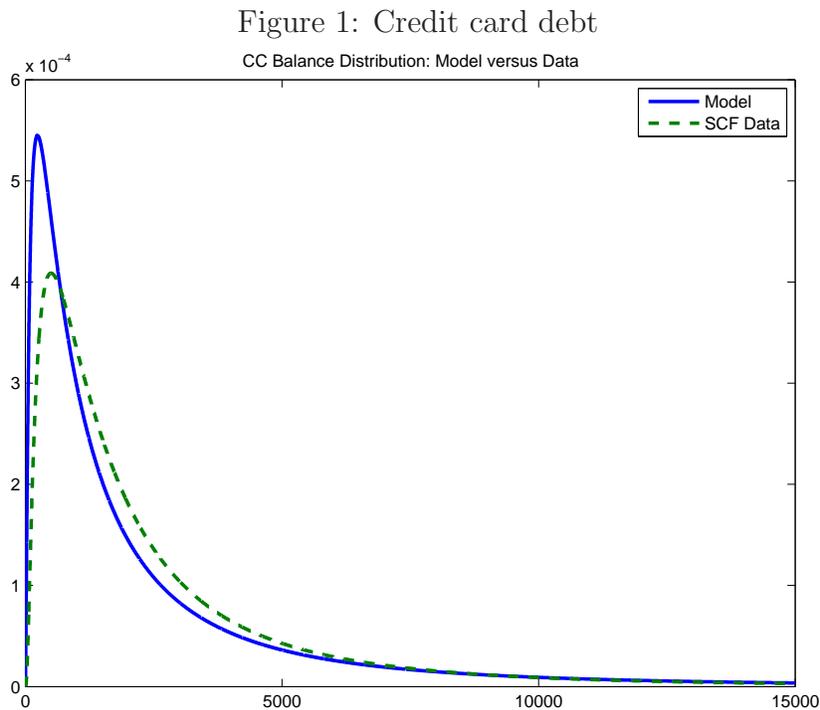
The model does a good job at matching debt burdens in the two markets for borrowers in the SCF 2004 as evident from Table 2. The model predicts that 24 percent have negative assets (without including student loans). The data counterpart is 32 percent.²⁷ Also, the model replicates quite

²⁵We normalize \$163,598=1. This represents the mean of income + 3 times the standard deviation of income.

²⁶Our estimate is in line with the data where the garnishment can be anywhere from 0 to 15 percent. Also, as in practice, wage garnishments do not apply if income levels are below a minimum threshold below which the borrower experiences financial hardship.

²⁷This measure is computed using total unsecured debt (but excluding student loans) minus financial assets defined as the sum of checking and savings accounts, money market deposit accounts, money market mutual funds,

well the distribution of credit card debt, as evident from Figure 4.2.1.



The default rate on credit card debt is 0.54 percent, which is in the range used in the literature (see Athreya, Tam, and Young (2009)). Also, all defaulters on credit card debt also default on student loans. The model delivers an interest rate on credit card loans of 4.65 percent on average. These last two model predictions cannot be tested in the data. For instance, the interest rate in the model is lower compared to the credit card rate in the data. However, the interest rate in the model represents the effective rate at which borrowers pay whereas in the data borrowers pay the high rate only in the case they roll over their debt. Lastly, taxes to cover defaulters in the economy are insignificant ($5.1170e-04$ percent of average income in the economy).

Table 2: Data versus model

	Data	Model
Percentage with negative assets	32%	24%
Credit card default	0.8	0.54
Credit card debt-to-income ratio	0.061	0.064
Per period college debt-to-income ratio	0.049	0.049

value of certificates of deposit, value of savings and bonds.

4.2.2 Default behavior

We study default behavior in the two markets across individual characteristics (student loan amount, d , credit card debt, b , and income, y). Table 3 shows these findings across individuals with high levels of d , b , y (defined as the top 50 percentile) versus individuals with low levels of d , b , y (defined as the bottom 50 percentile).

Default rates for student loans are larger for individuals with high amounts owed to the student loan program relative to those with low amounts of student loans. The gap between the default rates for the two groups is significant. Similarly, the default rates for individuals with low income levels are larger relative to those with high levels of income and the difference between the two groups is significant. Overall, the default probability for student loans is higher for individuals with relatively high debt burdens in the student loan market, fact consistent with the data (presented in Section 2). At the same time, individuals with credit card debt have higher default rates for student loans (4.81 percent) relative to individuals with no credit card debt (4.39 percent). But, individuals with high levels of credit card debt have lower default rates on student loans relative to individuals with low levels of credit card debt. The model delivers that individuals with relatively low levels of credit card debt are those who have high levels of student loan debt as Table 3 shows. They represent a higher risk for the credit card market and therefore they will receive worse terms (lower prices and therefore higher interest rates) on their credit card debt. Consequently they will borrow less in the credit card market. In fact, the model delivers that individuals in the top percentile of student loan debt borrow \$5,824, on average, compared to individuals in the bottom 50 percentile of student loans, who borrow \$6,403, on average.

Default rates on credit card debt are higher for individuals with high levels of *both* types of loans. Individuals with high levels of credit card debt are more likely to default on their credit card debt even though this triggers limited market participation, given that they can discharge their debt. In addition, having high levels of student loans make borrowers more likely to default on credit card loans as well. Recall that our theory predicts that high levels of student loans decrease the incentive to default only on credit card debt but increase the incentive to default on both types of loans. Quantitatively, the second effect dominates. In our model the majority of defaulters on credit card loans default on both their college and their credit card debt. The incentive to default on credit card is higher for individuals with a bad default record for student loans than for individuals without a default record for student loans. This happens for two reasons: first, defaulters on student loans do not have the option to default on their student loans, so if they must default they do so in the credit card market; and secondly, in addition to being asked to repay on their student loans, individuals with a default record on student loans also have a part

of their earnings garnished.

Table 3: Default rates across individual characteristics

	Characteristic	“Low”	“High”
Default SL	SL debt	0.26%	8.78%
	CC debt	5.51%	4.1%
	Income	7.3%	1.93%
Default CC	SL debt	0.06%	0.18%
	CC debt	0.07%	0.12%
	Income	0.03%	0.0%

Lastly, the likelihood of default on credit card debt is higher for individuals with low income and it is lower for individuals with high income. This finding is consistent with empirical evidence. For instance, Musto and Souleles (2006) show that low-income borrowers tend to have low credit scores and therefore high default probabilities. However, for the most part, the theoretic literature on unsecured default cannot capture this feature of the data. The intuition is that agents with relatively low income levels stand to lose more from defaulting on credit cards relative to individuals with high income levels, for whom the penalties associated with default are less costly in relative terms. Therefore, models of default easily deliver the counterfactual prediction that low income borrowers have lower probabilities of default. In our model, however, individuals also possess other types of loans, fact which pushes them into default. As we mentioned, the model delivers that default rates on credit card increase in college debt levels. In addition, college debt is negatively correlated with income. These facts deliver the default probability to decrease in income in our model. This finding shows the importance of accounting for other types of loans when analyzing default behavior, feature that is absent in previous models of consumer default.

4.2.3 Loan pricing

Consistent with our results on the individual probability of default for credit cards, the model delivers a pricing scheme of credit card loans based on individual default risk as proxied by the size of the loan, the amount owed in the student loan market, and the default status in the student loan market. Recall that our theoretical results show that the interest rate on credit card debt increases in both amounts of loans and it is higher for individuals with a default flag on student loans. In our quantitative analysis we find that:

First, agents with high levels of credit card debt (top 50 percentile) have a credit card rate of 5.4 percent and agents with low credit card debt (bottom 50 percentile) have a credit card rate of 4.09 percent. Individuals with low levels of credit card debt have a very low probability of default,

as shown in Table 3 and therefore pay a small premium over the risk-free rate, whereas individuals with high levels of credit card debt pay a premium of 1.4 percent to account for their higher likelihood of defaulting on their credit card debt. Second, agents with high levels of student loans receive a credit card rate of 5.34 percent and agents with low levels of student loans receive a credit card rate of 4.25 percent. The wedge in the interest rates accounts for the gap in the probabilities of default between these two groups presented in Table 3. Finally, defaulters on student loans have a credit card rate of 4.77 percent and nondefaulters on student loans have a credit card rate of 4.63 percent. These last two results show that the amount of student loan debt and the default status on student loans represent important components of the pricing of credit card loans. For instance, the interest premium paid by individuals who have high levels of student loans (of 1.34 percent) is more than five times the premium paid by those with low levels of student loans (of 0.25 percent). This is a direct consequence of the result that the likelihood of default for individuals with relatively high levels of college debt triples compared to the likelihood of default for those with relatively low levels of credit card debt.

These three findings represent the quantitative counterpart of our theoretical results in Theorem 10. In addition, our quantitative analysis predicts that agents with low income receive higher rates (4.73 percent), on average, than agents with high income, who receive an average interest rate of 4.03 percent. This is a direct implication of the differences in default rates across income groups presented in Table 3.

4.2.4 The interplay between the two markets

We turn now to the interaction between the two markets and its effect on default behavior, the main theoretical result of the paper. Recall from Theorem 11 that in any steady-state equilibrium we can find a combination of student loans and credit card debt such that individuals default. Furthermore, if loan amounts in both markets are larger than these two levels of debt, then default occurs for student loans. Our quantitative analysis in this subsection complements this theoretical result.

First, recall that in our model everyone who defaults on credit card debt also defaults on student loan debt. There is no borrower who strictly prefers defaulting on credit card debt to defaulting on student loans. Table 4 shows our findings regarding default behavior across groups of college and credit card debt. We divide individuals in two groups based on the amount owed to the student loan program, d (low and high defined as before) and in three groups based on the credit card debt, b : one group with positive assets and two groups with negative assets (low and high defined as before).

Table 4: Default rates across debt levels in the two markets

	$b \geq 0$	" $b < 0$ Low"	" $b < 0$ High"
d "Low"			
Default SL	0.2%	0.21%	0.6%
Default CC	-	0	0.47%
d "High"			
Default SL	8.01%	12.9%	12.1%
Default CC	-	0.35%	2.66%

Our results reveal the trade-off in default behavior captured in the model: First individuals with no credit card debt have lower default rates on student loans than individuals with credit card debt regardless of the amount owed in the student loan market; Secondly, conditional on having low levels of student loan debt, individuals with low levels of credit card debt do not default on their credit cards, but rather default on their student loans (if they must default). The benefit of discharging their credit card debt upon default is too small compared to the large cost of being excluded from borrowing. At the same time, the penalties associated with default in the student loan market are not contingent on their credit card debt. Similarly, conditional on having high levels of student loan debt, individuals with high levels of credit card debt have a higher likelihood of defaulting on their credit card debt. Thirdly, individuals with high levels of college debt have a higher likelihood of defaulting on their student loans relative to individuals with low levels of college debt. The gap, however, between default rates by student loan amounts is higher for individuals with relatively low levels of credit card debt. In other words, conditional on having low levels of student loan debt, having low amounts of credit card debt or not having any credit card debt does not matter too much, but large levels of credit card debt induce more default on student loans. At the same time, conditional on having high levels of student loan debt, individuals with large levels of credit card debt have a lower default rate relative to those with low levels of credit card debt.

We conclude that having debt in the credit card market amplifies the incentive to default on student loans. However, individuals with large levels of student loan debt use the credit card market to reduce their default. On the one hand, participating in the credit card market pushes borrowers in more default on their student loans and on the other hand, taking on credit card debt helps student loan borrowers smooth consumption and pay their college debt, in particular when their college debt burdens are large. Therefore, various conditions and terms in the credit market may affect the default behavior in the student loan market. We analyze this issue in the next section.

4.3 Experiments: effects of college versus credit card debt on default for student loans

Recall that the national default rate for student loans increases from 4.5 percent in 2004 to 6.7 percent in 2007. At the same time, the college debt increased, on average, by 35 percent during this period. We conjecture that the increase in the college debt alone cannot explain the recent trends in student loan defaults. This claim is based on the following observations: 1) Default rates for student loans declined from 1990 to 2004 and then increased after 2005; 2) During this entire period student loan amounts increased steadily; and 3) Young U.S, households with student loan debt increased their participation in the credit card market in the past decade and the terms on credit card accounts have changed in the past years.

In this section we study how much of the increase in default rates is due to an increase in college debt and how much do the changes in the credit card market account for the observed increase in student loan default during 2004-2007? To answer these questions, we run the following three experiments: 1) We first isolate the effect of the changes in college debt burdens during this period on default rates for student loans; 2) In addition, we consider the effects of the changed in credit card levels during this period on default rates for student loans; 3) Lastly, we consider an economy where college debt borrowers do not have access to the credit card market. Table 4.3 presents a summary of our findings from these three experiments compared to the baseline economy.

Table 5: Summary of results: experiments versus baseline

	Baseline	Exp 1 (d,y from 2007)	Exp 2 (d,y,b from 2007)
SL default	4.5%*	7.13%	6.74%
CC default	0.54%	1.44%	0.78%
Perc with neg assets	24%	20.4%	24.6%
CC balance	3,018*	2,700	3,938
CC interest rate	4.65%	6.21%	5.1%
CC burden	0.064*	0.048	0.07
SL burden	0.0486*	0.055	0.055

Note: * means targeted in the calibration procedure.

In our first experiment we account for the change in the distribution of college debt and income from SCF 2004 to SCF 2007 but keep the credit card market as in the baseline economy.²⁸ This change implies that college debt increases on average by 35 percent and income increases on average by 28 percent. Overall, the college debt burden increases from 0.049 to 0.055. As in the baseline

²⁸Our sample in SCF 2007 is constructed similarly to the one in SCF 2004. The sample size is 445.

Table 6: Default rates across debt levels in the two markets: experiment 1

	$b \geq 0$	" $b < 0$ Low"	" $b < 0$ High"
<i>d</i> "Low"			
Default SL	2.48%	2.55%	3.13%
Default CC	-	0	1.21%
<i>d</i> "High"			
Default SL	11.07%	15.05%	19.78%
Default CC	-	0.62%	7.2%

economy, we use a lognormal distribution with parameters now given by $(\mu_y, \sigma_y, \mu_d, \sigma_d, \rho_{yd}) = (-1.388; 0.907; -4.227; 0.74; -0.106)$ on $[0, 1] \times [0, 0.12]$.

This experiment delivers an increase in the default rate on student loans from 4.5 percent in the baseline economy to 7.13 percent. Consequently, taxes in the economy increase compared to the baseline economy, but are still small ($4.4318e - 04$ of average income in the economy). This experiment delivers that the default rate on credit cards increases from 0.53 percent in the baseline economy to 1.44 percent. Having more student loan debt induces higher default on credit card debt. With more risk in the credit card market, the average interest rate on credit card loans in the economy increases from 4.65 percent in the baseline economy to 6.21 percent.

In this experiment, like in the benchmark economy, having credit card debt induces more default on student loans regardless of the level of college debt. However, opposite from the benchmark, taking more credit card debt induces more default on student loans even for individuals with relatively high levels of student loans. Moreover, the differences between default rates on student loans for individuals with relatively low levels of credit card debt and for those with high levels of credit card debt are quite large. These findings are presented in Table 6. This change in default behavior is the result of a relatively tighter credit card market, which does not keep pace with expansion of student loans (like in the data). The model delivers that the percentage of individuals who have negative assets (excluding student loans) declines from 24 percent in the baseline to 20.4 percent. Also, the average credit card debt in this economy is \$2,700, which represent a reduction of 10.5 percent from the baseline economy. Individuals with large levels of student loans cannot effectively use the credit card market to smooth out and therefore are forced to default more on their student loans. In fact, the default rates for those with large levels of debt in both market is almost 20 percent. In addition, these individuals also default at very high rates in the credit card market (7.2 percent).

In the second experiment, in addition to the changes in the income and college debt distribution, we account for the changes in credit card debt levels from SCF 2004 to SCF 2007. This change

implies and increase in credit card debt of 24.1 percent on average. The credit card debt burden decreases on average from 0.061 to 0.0585. In order to capture this change in behavior, we recalibrate the economy to match credit card debt levels that young U.S. households have in SCF 2007. This calibration delivers a utility cost of 13.8 (compared to 9.4 in the baseline economy).

We obtain a default rate on student loans of 6.74 percent. With more borrowing in the credit card market, the incentive to default on student loans is lower. More credit helps reduce the incentive to default on student loans, especially for individuals with relatively high levels of student loans. There are two effects: on the one hand, some of the borrowers may rely on the credit card market to smooth out, and so having more access to the credit card market (and at better terms) may induce them to reduce default on their student loans. On the other hand, higher levels of debt on credit card accounts increase their overall debt burden and therefore may increase the incentive to default on student loans. Our results suggest that the first effect dominates. In experiment 2 the cost of default is higher (to match a relatively higher level of credit card debt). This increase in the default cost, in turn, induces less frequent default in the credit card market (almost half of the default rate in experiment 1).²⁹ Consequently, the interest rate on credit card accounts is lower, on average, compared to the interest rate in experiment 1. Under these circumstances, taking on more credit card debt help relax constraints due to high indebtedness in the student loan market and therefore reduce student loan defaults. Participation in the credit market increases even more than in the baseline economy.

Lastly, to assess the importance of the credit card market for default on student loans, we run a counterfactual experiment where there is no credit card market. Individuals can only save. This experiment delivers a default rate on student loans of 3.14 percent (compared to 4.5 percent in the baseline). However, welfare declines by 0.62 percent relative to the baseline economy.

We conclude that having credit card debt amplifies default on student loans. However, once individuals access the credit card market and can borrow large enough amounts and at relatively better terms, then they effectively use the credit card market to smooth out and reduce default on their college debt. When the credit market is tight, however, as in the recent years, default on student loans is higher (for the same levels of college debt burdens). In the latter case, borrowers receive lower prices on the same loan sizes and this higher cost of borrowing negatively affects default on student loans.

²⁹Our predictions in this experiment are consistent with empirical evidence provided by Gross and Souleles (2002) who show that the willingness to default due to changes in default costs explain a lot of the observed trends in default.

4.4 Policy implications

4.4.1 Income based repayment plan

We analyze the recent proposal on student loan repayment which has been mandated recently by The Health Care and Education Reconciliation Act of 2010. The proposal comes as a response to a petition calling on President Obama and Congress to forgive all student loans given the recent economic downturn. It assumes income-based repayment (IBR) which bases a borrower's loan payments on a percentage of their discretionary income, as opposed to what they owe. Borrowers who earn less than 150 percent of the poverty line have a loan payment of zero. That's \$14,148 (in 2004 constant dollars) for a single borrower.³⁰ Borrowers who have an income higher than this threshold pay the minimum between 10 percent of the income above this threshold and the amount owed per period (established according to the 10 year standard repayment plan). Any remaining debt after 20 years in repayment is forgiven, including both principal and interest. Borrowers whose debt exceeds their income will benefit from IBR.³¹

While everyone agrees that this policy will provide meaningful repayment relief, the timing of it and its cost have been debated. Initially only “new borrowers” (who do not have any pre-existing loans on the effective date of the change) were eligible. But petitioners have insisted on changing the eligibility requirements arguing that borrowers are suffering because of the economy, tight credit, and high unemployment rates. Indeed, as mentioned in Section 2, credit card issuers raised interest rates and fees in the recent years. Creditors have also become stingier with credit. As a result, the proposal was changed to make the lower monthly payments under the new income-based repayment plan available to borrowers starting in 2012 (instead of 2014). Recall that using the newly released SCF 2010, we document that young borrowers aged 20-30 years old with college loans receive much worse terms on their credit card accounts (higher rates and lower credit limits) relative to 2007. The credit card debt decreases significantly in 2010, by 31.5 percent relative to 2007. These facts motivate the experiments in this section.

We analyze whether the IBR plan would be indeed beneficial in times of economic downturn with tight credit or whether its implementation is warranted regardless of the economic conditions. We conduct the following two experiments: First, we consider an IBR scheme, as described in the proposal, in the economy in experiment 2 in Section 4.3 with relatively loose credit conditions (as

³⁰We use the value for a single borrower given that our model is representative for U.S. households aged 20-30 years old.

³¹This policy improves the already existing IBR, which assumed a payment of 15 percent of discretionary income and loan forgiveness after 25 years. However, the plan was available only through the Direct Loan program to Federal Family Education guaranteed loans (FFEL) and eligibility criteria were very limited with less than 5 percent of borrowers using it.

in SCF 2007) – economy L from now on; Second, we consider the IBR plan in an economy with relatively tight credit conditions (as in SCF 2010) with an average credit card debt of \$2,468 – economy T from now on. Economy T is simply economy L (with college debt and income as in SCF 2007) but re-calibrated to match the average level of credit card debt from SCF 2010. This procedure delivers a lower utility cost in case of default in the credit card market compared (8.5 in economy T versus 13.8 in economy L). We compute an aggregate weighted measure of welfare in consumption units in both of these experiments and compare them to those obtained in economies with standard repayments in economy L and in economy T , respectively. Our analysis takes into account the fact that the amount of student loans discharged is recovered through taxes. Note that our welfare calculations represent an upper bound since we ignore the fact that in reality a limited IBR plan already exists. However, we omit this feature in both economies with loose and with tight credit card conditions and so, for the purpose of our analysis, this omission is not restrictive.³²

We find that the IBR plan improves welfare significantly regardless of the economic conditions in the credit card market, but more so in the case where the credit card market is tight (economy T). We obtain a welfare increase of 7.48 percent in case IBR is applied in economy L and a welfare increase of 8.12 percent in case the IBR is applied in economy T . IBR provides repayment relief to poor individuals with large student loan amounts who find themselves in financial distress. The negative cost of this policy is not large enough to counteract this effect. Dischargeability rate on student loans is 17.7 and taxes in the economy are 2 percent of average income. Furthermore, with IBR there is enough insurance in the economy and therefore individuals do not need the default option in the credit card market. Therefore, the effect of utility cost on the frequency of default is not active. In fact, both of these experiments deliver a 0 default rate in the credit card market. As a result, the equilibrium outcome of the IBR policy is the same in both economy L and in economy T . The percentage of individuals with negative assets increases considerably (to 44.8 percent) and the average credit card debt is above \$11,000. However, since these two economies in the absence of the IBR plan feature a sharp difference in the credit card market (with lower participation and higher default on credit card debt in economy T) the increase in welfare relative to these baseline economies is larger in the case where the credit market is tight (economy T).

³²At the same time, we abstract from the fact that the policy encourages as many as 5.8 million borrowers with both federally guaranteed student loans and direct loans to move their guaranteed loans into the Direct Loan program. These “split borrowers” have to make loan payments to two different entities. Moving these loans into the Direct Loan program will save the government money, because then the government will get all of the interest from the loans, instead of just some of the interest. This secondary effect of the policy, if effective, would considerably lower that cost on tax payers.

Table 7: IBR policies in an economy with tight credit markets

	Current policy	Policy 1 (15%)	Policy 2 (30 years)
Welfare gain	+8.12%	+7.91%	+8.28%
Taxes (perc of avg income)	2.01%	1.87%	1.65%
SL dischargeability	17.7%	12.9%	15.5%
CC default	0	0	0
Perc with neg assets	44.8%	44.9%	44.75%
CC balance	11,614	11,664	11,590
CC burden	0.206	0.21	0.205
SL burden	0.036	0.036	0.03

4.4.2 Alternative repayment proposals

The generosity of the proposal has been criticized, in particular with respect to the low level of per period payments and the maximum time limit after which payments are no longer required. Opponents of the policy have argued that paying a larger percentage of income or allowing for a longer time to repay could considerably lower the cost of the policy and therefore be more beneficial. To shed some light on this debate we consider two alternative proposals to the current policy: 1) borrowers pay 15 percent of income (as opposed to 10 percent in the current policy) with any remaining debt after 20 years in repayment forgiven and 2) borrowers pay 10 percent of income as in the proposal, but with any remaining debt after 30 years in repayment forgiven (as opposed to 20 years in the proposal). Table 7 presents a summary of the predictions delivered by these two policies compared to the IBR policy. All of these experiments are considered in economy T (with tight credit card markets).

Findings show that increasing per period payments deliver a lower welfare relative to the current policy even though taxes are lower in the economy. In contrast, a policy that allows for a longer period to repay improves welfare relative to the current policy. While both of these experiments deliver less dischargeability and therefore less taxes that need to be collected relative to the current policy, policy 2 (with longer maximum time to repay) decreases the per period amount owed, whereas policy 1 increases it. In addition, the amount discharged delivered by policy 2 is lower, on average, relative to the amount discharged delivered by policy 1.

5 Conclusion

We developed a quantitative theory of unsecured credit and default behavior of young U.S. households based on the interplay between two forms of unsecured credit and we analyzed the im-

plications of this interaction for default incentives. Our theory is motivated by facts related to borrowing and repayment behavior of young U.S. households with college and credit card debt, and in particular by recent (alarming) trends in the default rates for student loans. Specifically, different financial market arrangements and in particular, different bankruptcy rules in these two markets alter incentives to default conducing to increased default on student loans.

We built a general equilibrium economy that mimics features of student and credit card loans. In particular, our model accounts for 1) bankruptcy arrangement differences between the two types of loans and 2) differences in pricing default risk in the two markets. Our theory explains borrowing and default behavior of young U.S. households: the incentive to default on student loans increases in college debt and the incentive to default on credit card debt increases in credit card debt.

Our model predicts that the likelihood to default on credit card debt increases in the amount of student loans. Also, individuals with a default flag in the student loan market have higher default probabilities of default in the credit card market than individuals who have not defaulted on their student loans. In the quantitative part of our paper we also show that individuals with high levels of income are less likely to default in both the student loan and in the credit card markets relative to individuals with low levels of income. This result is consistent with empirical evidence, however it is not straightforward to obtain in models of unsecured credit. The fact that individuals in our model also have other types of loans produces this result. Lastly, having more credit card debt induces higher incentives to default in the student loan market. These four results are new in the literature and reveal the importance of accounting for interactions between different financial markets in which individuals participate when one analyses default behavior for unsecured credit.

Our theory reveals that borrowers prefer defaulting on their student loans rather than on credit card debt. Our main theoretical result shows that a borrower with high enough college debt and credit card debt will choose to default in the student loan market. Our research innovates in demonstrating how differences in market arrangements can lead to amplification of default in the student loan market. In the quantitative part of the paper we show that while an increase in college debt is necessary to deliver an increase in the default rate on student loans, this effect is amplified by indebtedness in the credit card market. However, once individuals access the credit card market, loose credit conditions help reduce default on student loans, whereas tight credit induces more student loan default. We explore the policy implications of our model and study the impact of the recent proposal on income-based repayments on student loans. We find that the proposal is justified on welfare grounds, and in particular in an economy where individuals face tight credit markets. Our findings are particularly important in the current market conditions when due to a significant increase in college costs, students borrow more than ever in both the student loan and the credit card markets, and at the same time, they face stringent terms on their credit

card accounts. An alternative policy which assumes a longer time for repayment before forgiveness of student loans occurs relative to the current reform is superior (in a welfare improving sense). It delivers both lower per period payments and less dischargeability, and therefore lower taxes.

References

- AIYAGARI, S. (1994): “Uninsured idiosyncratic risk and aggregate saving,” *The Quarterly Journal of Economics*, 109(3), 659–684.
- ATHREYA, K. (2002): “Welfare implications of the Bankruptcy Reform Act of 1999,” *Journal of Monetary Economics*, 49(8), 1567–1595.
- ATHREYA, K., X. TAM, AND E. YOUNG (2009): “Unsecured credit markets are not insurance markets,” *Journal of Monetary Economics*, 56(1), 83–103.
- CHATTERJEE, S., D. CORBAE, M. NAKAJIMA, AND J. RIOS-RULL (2007): “A Quantitative theory of Unsecured Consumer Credit With Risk of Default,” *Econometrica*, 75(6), 1525–1589.
- CHATTERJEE, S., D. CORBAE, AND J. RIOS-RULL (2008): “A finite-life private-information theory of unsecured consumer debt,” *Journal of Economic Theory*, 142(1), 149–177.
- (2010): “Credit scoring and competitive pricing of default risk,” Working paper.
- COLLEGE BOARD (2009): *Trends in student aid*. College Board Publication.
- DYNARSKY, M. (1994): “Who defaults on student loans? Findings for the National Postsecondary Student Aid Study,” *Economic of Education Review*, 13(1), 55–68.
- EDELBERG, W. (2006): “Risk-based pricing of interest rates for consumer loans,” *Journal of Monetary Economics*, 53(8), 2283–2298.
- GROSS, D., AND N. SOULELES (2002): “An empirical analysis of personal bankruptcy and delinquency,” *Review of Financial Studies*, 15(1), 319–347.
- HUGGETT, M. (1993): “The risk-free rate in heterogeneous-agent incomplete-insurance economies,” *Journal of economic Dynamics and Control*, 17(5-6), 953–969.
- IONESCU, F. (2008): “Consolidation of Student Loan Repayments and Default Rates Across Income Groups,” *The B.E. Journal of Macroeconomics*, 8(1), Topics.

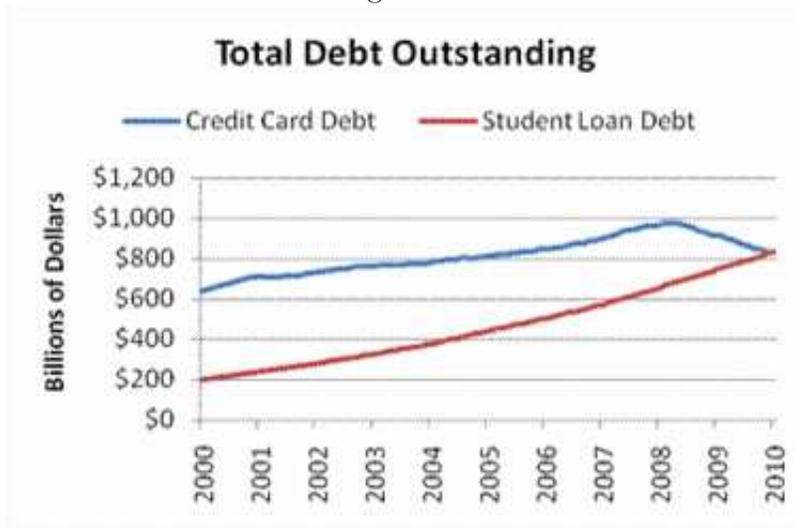
- (2010): “Risky Human Capital and Alternative Bankruptcy Regimes for Student Loans,” Working paper.
- IONESCU, F., AND N. SIMPSON (2010): “Credit Scores and College Investment,” Working paper.
- LIVSHITS, I., J. MACGEE, AND M. TERTILT (2007): “Consumer bankruptcy: A fresh start,” *The American Economic Review*, 97(1), 402–418.
- LOCHNER, L., AND N. MONGE (2010): “The Nature of Credit Constraints and Human Capital,” *American Economic Review*, Forthcoming.
- MCGRATTAN, E., AND E. PRESCOTT (2000): “Is the Stock Market Overvalued?,” *Federal Reserve Bank of Minneapolis Manuscript*.
- MUSTO, D., AND N. SOULELES (2006): “A portfolio view of consumer credit,” *Journal of Monetary Economics*, 53(1), 59–84.
- REED, M., AND B. SIMON (1972): *Methods of modern mathematical physics. I. Functional analysis*. Academic Press, New York.
- RUDIN, W. (1987): *Real and complex analysis*. McGraw-Hill Book Co., New York, third edn.
- SALLIE MAE (2009): “How Undergraduate Students Use Credit Cards: Sallie Mae’s National Study of Usage Rates and Trends,” Discussion paper.
- SULLIVAN, T., E. WARREN, AND J. WESTBROOK (2001): *The fragile middle class: Americans in debt*. Yale Univ Pr.

A Appendix

Data details

Figure 1 presents trends in student loan and credit card debt in the U.S. and Figure 2 presents trends in the two year cohort default rate for student loans according to press releases from the Department of Education.

Figure 2:



Source: Federal Reserve's G19 Consumer Credit

Figure 3: Trends in default rates
National Student Loan Default Rates

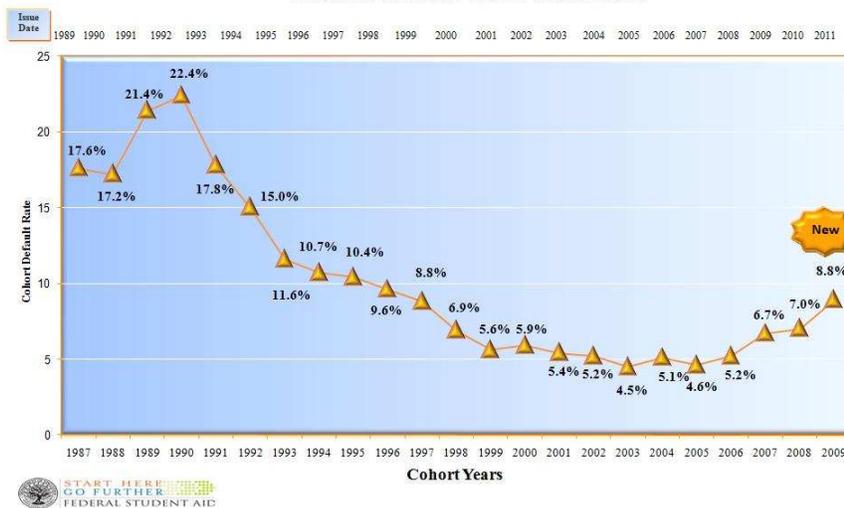


Table 8 provides details on facts related to college debt and credit card debt burdens documented in Section 2.

Table 8: Summary statistics for borrowers with student loans aged 20-30 years old

Year	SCF 2004	SCF 2007
Amt. college debt outstanding	17,670 (15,587)	23,776 (20,291)
Income	48,041 (38,519)	61,632 (69,625)
College debt-to-income ratio	0.0486	0.055
Credit card rate	12.07 (6.06)	13.34 (5.38)
Credit card limit	14,993 (22,567)	15,465 (19,183)
Credit card balance	2,906 (4,330)	3,605 (6,312)

Note: the first number represents the mean and the number in parenthesis the standard deviation. Numbers are provided in 2004 dollars.

Proofs of theorems

A.1 Proofs of Theorems 1 and 2

Let $c_{\min} = y_{\min}(1 - \gamma)$ and $c_{\max} = y_{\max} + b_{\max} - b_{\min}$. Then, if c is the consumption in any of the cases in the definition of T , we have that $U(c_{\min}) \leq U(c) \leq U(c_{\max})$ and that c_{\min} is a feasible consumption. Recall that $\mathcal{S} = B \times F \times H$ is a finite set and let $N_{\mathcal{S}}$ be the cardinality of \mathcal{S} .

Definition A1. Define \mathcal{V} to be the set of continuous functions $v : D \times Y \times Q \rightarrow \mathbb{R}^{N_{\mathcal{S}}}$ such that

1. For all $(b, f, h) \in \mathcal{S}$ and $(d, y, q) \in D \times Y \times Q$

$$\frac{U(c_{\min})}{1 - \beta\rho} \leq v(d, y, q)(b, f, h) \leq \frac{U(c_{\max})}{1 - \beta\rho}. \quad (3)$$

2. v is increasing in b and y .
3. v is decreasing in f : $v(d, y, q)(b, 0, h) \geq v(d, y, q)(b, 1, h)$ for all d, y, q, b, h .

Let $(C(D \times Y \times Q; \mathbb{R}^{N_{\mathcal{S}}}), \|\cdot\|)$ denote the space of continuous functions $v : D \times Y \times Q \rightarrow \mathbb{R}^{N_{\mathcal{S}}}$ endowed with the supremum norm

$$\|v\| = \max_{(d,y,q)} \|v(d, y, q)\|,$$

where the norm of a vector $w = (w(b, f, h)) \in \mathbb{R}^{N_S}$ is

$$\|w\| = \max_{(b,f,h) \in \mathcal{S}} |w(b, f, h)|.$$

Then \mathcal{V} is a subset of $C(D \times Y \times Q; \mathbb{R}^{N_S})$. Define also $C(D \times Y \times Q \times \mathcal{S})$ to be the set of continuous real valued functions $v : D \times Y \times Q \times \mathcal{S} \rightarrow \mathbb{R}$ with the norm

$$\|v\| = \max_{(d,y,q,b,f,h)} |v(d, y, q, b, f, h)|.$$

In the first lemma we show that the two spaces of functions that we defined above are interchangeable.

Lemma A1. *The map $V : C(D \times Y \times Q; \mathbb{R}^{N_S}) \rightarrow C(D \times Y \times Q \times \mathcal{S})$ defined by*

$$V(v)(d, y, q, b, f, h) = v(d, y, q)(b, f, h)$$

is a surjective isomorphism.

Proof. We prove first that if $v \in C(D \times Y \times Q; \mathbb{R}^{N_S})$ then $V(v)$ is continuous. Let $(d_n, y_n, q_n, b_n, f_n, h_n)_{n \in \mathbb{N}}$ be a sequence that converges to (d, y, q, b, f, h) and let $\varepsilon > 0$. Since \mathcal{S} is a finite set it follows that there is some $N_1 \geq 1$ such that $b_n = b$, $f_n = f$, and $h_n = h$ for all $n \geq N_1$. Since v is continuous then there is $N_2 \geq 1$ such that if $n \geq N_2$ then

$$\|v(d_n, y_n, q_n) - v(d, y, q)\| < \varepsilon.$$

Thus $|v(d_n, y_n, q_n)(b, f, h) - v(d, y, q)(b, f, h)| < \varepsilon$ for all $n \geq N := \max\{N_1, N_2\}$. Therefore

$$|V(v)(d_n, y_n, q_n, b_n, f_n, h_n) - V(v)(d, y, q, b, f, h)| < \varepsilon \text{ for all } n \geq N$$

and $V(v)$ is continuous. It is clear from the definition of the norms that $\|V(v)\| = \|v\|$ for all $v \in C(D \times Y \times Q; \mathbb{R}^{N_S})$. Thus V is an isomorphism. Finally, if $w \in C(D \times Y \times Q \times \mathcal{S})$ then one can define $v \in C(D \times Y \times Q; \mathbb{R}^{N_S})$ by

$$v(d, y, q)(b, f, h) = w(d, y, q, b, f, h).$$

Then $T(v) = w$ and T is surjective. □

In the following we are going to tacitly view \mathcal{V} either as a subset of $C(D \times Y \times Q; \mathbb{R}^{N_S})$ or as a subset of $C(D \times Y \times Q \times \mathcal{S})$ via $V(\mathcal{V})$. For example, we are going to prove in the following lemma that $(\mathcal{V}, \|\cdot\|)$ is a complete metric space by showing that it (‘s image under V) is a closed subspace of $C(D \times Y \times Q \times \mathcal{S})$, which is a complete metric space.

Lemma A2. $(\mathcal{V}, \|\cdot\|)$ is a complete metric space.

Proof. We are going to show that \mathcal{V} is a closed subspace of $C(D \times Y \times Q \times \mathcal{S})$. Notice first that \mathcal{V} is nonempty because any constant function that satisfies (3) is in \mathcal{V} . Let now $\{v_n\}_{n \in \mathbb{N}}$ be a sequence of functions in \mathcal{V} that converge to a function v . Then, since $C(D \times Y \times Q \times \mathcal{S})$ is complete, it follows that v is continuous. Since inequalities are preserved by taking limits it follows immediately that v satisfies the conditions of Definition A1, because each v_n satisfies those conditions. Therefore $v \in \mathcal{V}$ and, thus, $(\mathcal{V}, \|\cdot\|)$ is a closed subspace of $C(D \times Y \times Q \times \mathcal{S})$ and, hence, a complete metric space. \square

Lemma A3. The operator T defined on $C(D \times Y \times Q; \mathbb{R}^{N_S})$ maps \mathcal{V} into \mathcal{V} and its restriction to \mathcal{V} is a contraction with factor $\beta\rho$.

Proof. We will show first that if $v \in \mathcal{V}$ then $Tv \in \mathcal{V}$. Since $v \in \mathcal{V}$ we have that

$$\frac{U(c_{\min})}{1 - \beta\gamma} \leq v(d, y', q)(b', f', h') \leq \frac{U(c_{\max})}{1 - \beta\gamma}$$

for all $(d, y', q) \in D \times Y \times Q$ and $(b', f', h') \in \mathcal{S}$. Integrating with respect to y' we obtain that

$$\frac{U(c_{\min})}{1 - \beta\gamma} \leq \int v_{(b', f', h')}(d, y'; q) \Phi(dy') \leq \frac{U(c_{\max})}{1 - \beta\rho},$$

because $\int \Phi(dy') = 1$. Since $U(c_{\min}) \leq U(c) \leq U(c_{\max})$ for all c appearing in the definition of T , it follows that

$$U(c) + \beta\rho \int v_{(b', f', h')}(d, y'; q) \Phi(dy') \leq U(c_{\max}) + \frac{\beta\rho U(c_{\max})}{1 - \beta\rho} = \frac{U(c_{\max})}{1 - \beta\rho},$$

and, similarly

$$\frac{U(c_{\min})}{1 - \beta\rho} \leq U(c) + \beta\rho \int v_{(b', f', h')}(d, y'; q) \Phi(dy').$$

Thus the condition (3) of Definition A1 is satisfied. To prove that Tv is increasing in b and y and decreasing in f , note that the sets $B_{b, f, h}(d, y; q)$ are increasing with respect to b and y , and decreasing with respect to f . These facts coupled with the same properties for v (which are

preserved by the integration with respect to y') imply that Tv satisfies the remaining conditions from Definition A1, with the exception of the continuity, which we prove next.

Since B, F, H and D are finite spaces, it suffices to show that Tv is continuous with respect to y and q . Since Q is compact and v is uniformly continuous with respect to q , it follows by a simple $\varepsilon - \delta$ argument that the integral is continuous with respect to q . Since $U(\cdot)$ is continuous with respect to c and c is continuous with respect to d and y , it follows that $T(v)$ is continuous.

Finally we prove that T is a contraction with factor $\beta\rho$ by showing that T satisfies Blackwell's conditions. For simplicity, we are going to view \mathcal{V} one more time as a subset of $C(D \times Y \times Q \times \mathcal{S})$. Let $v, w \in \mathcal{V}$ such that $v(d, y, q, b, f, h) \leq w(d, y, q, b, f, h)$ for all $(d, y, q, b, f, h) \in D \times Y \times Q \times \mathcal{S}$. Then

$$\beta\rho \int v_{(b',f',h')}(d, y'; q)\Phi(dy') \leq \beta\rho \int w_{(b',f',h')}(d, y'; q)\Phi(dy')$$

for all (d, y, q, b', f', h') . This implies that $Tv \leq Tw$. Next, if $v \in \mathcal{V}$ and a is a constant it follows that

$$\beta\rho \int (v_{(b',f',h')}(d, y'; q) + a)\Phi(dy') = \beta\rho \int v_{(b',f',h')}(d, y'; q)\Phi(dy') + \beta\rho a.$$

Thus $T(v + a) = Tv + \beta\rho a$. Therefore T is a contraction with factor $\beta\rho$. \square

Theorem 1. *There exists a unique $v^* \in \mathcal{V}$ such that $v^* = Tv^*$ and*

1. v^* is increasing in y and b .
2. Default decreases v^* .
3. The optimal policy correspondence implied by Tv^* is compact-valued, upper hemi-continuous.
4. Default is strictly preferable to zero consumption and optimal consumption is always positive.

Proof. The first two parts follows from Definition A1 and Lemmas A2 and A3. The last part follows from our assumptions on U . So we need only to prove the third part of the theorem. The optimal policy correspondence is

$$\Xi_{(d,y,q,b,f,h)} = \{(c, b', h', f', \lambda_d, \lambda_b) \in B_{b,f,h}(d, y; q) \text{ that attain } v_{b',f',h'}^*(d, y, q)\}.$$

For simplicity of our notation we will write $x = (d, y, q, b, f, h)$. For a fixed x we need to show that if Ξ_x is nonempty then it is compact. First notice that

$$\Xi_x \subset [c_{\min}, c_{\max}] \times B \times H \times F \times \{0, 1\} \times \{0, 1\}$$

and, thus, it is a bounded set. We need to prove that it is closed. Let $\{(c_n, b'_n, h'_n, f'_n, \lambda_d^n, \lambda_b^n)\}_{n \in \mathbb{N}}$ be a sequence in Ξ_x that converges to some

$$(c, b', h', f', \lambda_d, \lambda_b) \in [c_{\min}, c_{\max}] \times B \times H \times F \times \{0, 1\} \times \{0, 1\}.$$

Since $B, F,$ and $\{0, 1\}$ are finite sets it follows that there is some $N \geq 1$ such that $b'_n = b', h'_n = h', f'_n = f', \lambda_d^n = \lambda_d,$ and $\lambda_b^n = \lambda_b$ for all $n \geq N$. Define

$$\phi(c) = U(c) + \beta \rho \int v_{(b', f', h')} (d, y'; q) \Phi(dy').$$

Then ϕ is continuous and, since $\phi(c_n) = v_{(b, f, h)}^*(d, y; q)$ for all $n \geq 1$, we have that

$$\phi(c) = \lim_{n \rightarrow \infty} \phi(c_n) = v_{(b, f, h)}^*(d, y; q).$$

Thus $(c, b', h', f', \lambda_d, \lambda_b) \in \Xi_x$ and Ξ_x is a closed and, hence, compact set.

To prove that Ξ is upper hemi-continuous consider $x = (d, y, q, b, f, h) \in D \times Y \times Q \times S$ and let $\{x_n\} \in D \times Y \times Q \times \mathcal{S}$, $x_n = (d_n, y_n, q_n, b_n, f_n, h_n)$ be a sequence that converges to x . Since $D, B, F,$ and H are finite sets it follows that there is $N \geq 1$ such that if $n \geq N$ then $d_n = d, b_n = b, f_n = f,$ and $h_n = h$. Let $z_n = (c_n, b'_n, h'_n, f'_n, \lambda_d^n, \lambda_b^n) \in \Xi_{x_n}$ for all $n \geq N$. We need to find a convergent subsequence of $\{z_n\}$ whose limit point is in Ξ_x . Since $B, H, F,$ and $\{0, 1\}$ are finite sets we can find a subsequence $\{z_{n_k}\}$ such that $b'_{n_k} = b', h'_{n_k} = h', f'_{n_k} = f', \lambda_d^{n_k} = \lambda_d, \lambda_b^{n_k} = \lambda_b$ for some $b' \in B, h' \in H, f' \in F, \lambda_d, \lambda_b \in \{0, 1\}$. Since $\{c_{n_k}\} \subset [c_{\min}, c_{\max}]$ which is a compact interval, there must be a convergent subsequence, which we still label c_{n_k} for simplicity. Let $c = \lim_{k \rightarrow \infty} c_{n_k}$ and let $z_{n_k} = (c_{n_k}, b', h', f', \lambda_d, \lambda_b)$ for all k . Then $\{z_{n_k}\}$ is a subsequence of $\{z_n\}$ such that

$$\lim_{k \rightarrow \infty} z_{n_k} = z := (c, b', h', f', \lambda_d, \lambda_b).$$

Moreover, since

$$\phi(c_{n_k}) = v_{b', f', h'}^*(d_{n_k}, y_{n_k}; q_{n_k}) \text{ for all } k$$

and since ϕ and v^* are continuous functions it follows that

$$\phi(c) = \lim_{k \rightarrow \infty} \phi(c_{n_k}) = \lim_{k \rightarrow \infty} v_{b', f', h'}^*(d_{n_k}, y_{n_k}; q_{n_k}) = v_{b', f', h'}^*(d, y; q).$$

Thus $z \in \Xi_x$ and Ξ is an upper hemi-continuous correspondence. \square

Theorem 2. *For any $q \in Q$ and any measurable selection from the optimal policy correspondence*

there exists a unique $\mu_q \in \mathcal{M}(x)$ such that $\Gamma_q \mu_q = \mu_q$.

Proof. The Measurable Selection Theorem implies that there exists an optimal policy rule that is measurable in $X \times \mathcal{B}(X)$ and, thus, T_q^* is well defined. We show first that T_q^* satisfies Doeblin's condition. It suffices to prove that TN_q^* satisfies Doeblin's condition (see Exercise 11.4g of Stokey, Lucas, Prescott (1989)). If we let $\varphi(Z) = TN_q^*(y, d, b, f, h, Z)$ for any $(y, d, b, f, h) \in X$ it follows that if $\varepsilon < 1/2$ and $\varphi(Z) < \varepsilon$ then $1 - \varepsilon > 1/2$ and

$$TN_q^*(y, d, b, f, h, Z) < \varepsilon < \frac{1}{2} < 1 - \varepsilon$$

for all $(y, d, b, f, h) \in X$. Thus Doeblin's condition is satisfied.

Next, notice that if $\varphi(Z) > 0$ then $TN_q^*(y, d, b, f, h, Z) > 0$ and, thus,

$$T_q^*(y, d, b, f, h, Z) = \rho TS_q^*(y, d, b, f, h, Z) + (1 - \rho)TN_q^*(y, d, b, f, h, Z) > 0.$$

Then Theorem 11.10 of Stokey, Lucas, Prescott (1989) implies the conclusion of the theorem. \square

A.2 Proofs of Theorems 3-8

Let $(b, f, h) \in \mathcal{S}$ and $q \in Q$ be fixed. Before proving the theorem we will introduce some notation which will ease the writing of our proofs. For $y \in Y$, $d \in D$ we define the following maps:

$$\psi_{nodef}(y, d)(c, b', f', h', \lambda_d = 0, \lambda_b = 0) := U(c) + \beta\rho \int v_{b', f', h'}(d, y'; q)\Phi(dy')$$

for all $(c, b', f', h', 0, 0) \in B_{b, f, h}(d, y; q)$;

$$\psi_{sl}(y, d)(c, b', f', h', \lambda_d = 1, \lambda_b = 0) = U(c) + \beta\rho \int v_{b', f', 1}(d, y'; q)\Phi(dy')$$

for all $(c, b', f', h', 1, 0) \in B_{b, f, h}(d, y; q)$;

$$\psi_{cc}(y, d)(c, b', f', h', \lambda_d = 0, \lambda_b = 1) = U(c) + \beta\rho \int v_{b, 1, h'}(d, y'; q)\Phi(dy')$$

for all $(c, b', f', h', 0, 1) \in B_{b, f, h}(d, y; q)$; and

$$\psi_{both}(y, d)(c, b', f', h', \lambda_d = 1, \lambda_b = 1) = U(c) + \beta\rho \int v_{0, 1, 1}(d, y'; q)\Phi(dy')$$

for all $(c, b', f', h', 0, 1) \in B_{b,f,h}(d, y; q)$. Note that these functions are continuous in y and d . Also, these functions depend on b, f , and q . Also, we will write $\omega_{b,f,h}(q, d)$ for the expected utility of an household that starts next period with (b, f, h, q, d) .

Theorem 3. *Let $q \in Q$, $f \in F$, $b \in B(f)$. If $h = 1$ and the set $D_{b,f,1}^{SL}(q)$ is nonempty, then $D_{b,f,1}^{SL}(q)$ is closed and convex. In particular the sets $D_{b,f,1}^{SL}(d; q)$ are closed intervals for all d . If $h = 0$ and the set $D_{b,f,0}^{SL}(d; q)$ is nonempty, then $D_{b,f,0}^{SL}(d; q)$ is a closed interval for all d .*

Proof. If $h = 1$ then $D_{b,f,1}^{SL}(q)$ is the combinations of earnings y and student loan amount d for which $B_{b,f,1}(d, y; q) = \emptyset$. Then they satisfy the inequality $y(1 - \gamma) + b(1 - \lambda_b) - d - q_{b',d,h}b' \leq 0$ for all $\lambda_b \in \{0, 1\}$ and $b' \in B$. Thus $D_{b,f,1}^{SL}(q)$ is closed. Moreover, if (y_1, d_1) and (y_2, d_2) are elements in $D_{b,f,1}^{SL}(q)$ then if $(y, d) = t(y_1, d_1) + (1 - t)(y_2, d_2)$ with $t \in (0, 1)$ it follows easily that

$$y(1 - \gamma) + b(1 - \lambda_b) - d - q_{b',d,h}b' \leq 0$$

and, thus, $(y, d) \in D_{b,f,1}^{SL}(q)$. So $D_{b,f,1}^{SL}(q)$ is convex.

Assume now that $h = 0$ and let $d \in D$ be fixed. Let y_1 and y_2 with $y_1 < y_2$ be in $D_{b,f,0}^{SL}(d; q)$. Therefore

$$\begin{aligned} \psi_{sl}(y_i, d)(c_i^*, b_i^*, f_i^*, h_i^*, 1, 0) &\geq \max \left\{ \psi_{nodef}(y_i, d)(c, b', f', h', 0, 0), \right. \\ &\quad \psi_{cc}(y_i, d)(c, b', h', 0, 1), \\ &\quad \left. \psi_{both}(y, d)(c, b', h', 1, 1) \right\} \end{aligned} \quad (4)$$

for all $(c, b', f', h', 0, 0), (c, b', f', h', 0, 1), (c, b', f', h', 1, 1) \in B_{b,f,0}(d, y_i; q)$, $i = 1, 2$. Let $y \in (y_1, y_2)$ and assume, by contradiction, that $y \notin D_{b,f,0}^{SL}(d; q)$. Assume, without loss of generality, that the agent choses not to default on either market, i.e.

$$\psi_{sl}(y, d)(c, b', f', h', 1, 0) < \psi_{nodef}(y, d)(c^*, b^*, f'^*, h'^*, 0, 0), \quad (5)$$

for all $(c, b', f', h', 1, 0) \in B_{b,f,0}(d, y; q)$, where $(c^*, b^*, f'^*, h'^*, 0, 0) \in B_{b,f,0}(d, y; q)$ is the optimal choice for the maximization problem. Let $\bar{c}_1 = c^* - (y - y_1)$. If $\bar{c}_1 \leq 0$ then $\bar{c}_1 < y_1 + b$ and thus

$$c^* = \bar{c}_1 + (y - y_1) < y_1 + b + (y - y_1) = y + b. \quad (6)$$

If $\bar{c}_1 > 0$ we have that $(\bar{c}_1, b^*, f'^*, h'^*, 0, 0) \in B_{b,f,0}(d, y_1; q)$ and, thus,

$$\psi_{sl}(y_1, d)(c_1^*, b_1^*, f_1^*, h_1^*, 1, 0) \geq \psi_{nodef}(y_1, d)(\bar{c}, b^*, f'^*, h'^*, 0, 0).$$

Therefore

$$U(y_1 + b) + \beta\rho \int v_{b_1^*, f_1^*, 1}(d, y'; q)\Phi(dy') \geq U(\bar{c}_1) + \beta\rho \int v_{b^*, f^*, 0}(d, y'; q)\Phi(dy'), \quad (7)$$

Subtracting (7) from (5) we have that

$$U(y + b) - U(y_1 + b) < U(c^*) - U(\bar{c}_1).$$

Since $(y + b) - (y_1 + b) = y - y_1 = c^* - \bar{c}_1$ and U is strictly concave it follows that $c^* < y + b$.

Consider now $\bar{c}_2 = c^* + (y_2 - y)$. Then $(\bar{c}_2, b^*, f^*, h^*, 0, 0) \in B_{b, f, 0}(d, y_2; q)$ and thus

$$U(y_2 + b) + \beta\rho \int v_{b_2^*, f_2^*, 1}(d, y'; q)\Phi(dy') \geq U(\bar{c}_2) + \beta\rho \int v_{b^*, f^*, 0}(d, y'; q)\Phi(dy'). \quad (8)$$

Using inequalities (5), and (8) we obtain that

$$U(y_2 + b) - U(y + b) > U(\bar{c}_2) - U(c^*).$$

Thus $c^* > y + b$, and we obtain a contradiction with $c^* < y + b$. Therefore $y \in D_{b, f, 0}^{SL}(d; q)$ and, thus, $D_{b, f, 0}^{SL}(d; q)$ is an interval. It is also a closed set because the maps ψ_{sl} , ψ_{both} , ψ_{cc} , and ψ_{nodef} are continuous with respect to y . Thus, $D_{b, f, 0}^{SL}(d; q)$ is a closed interval. \square

Theorem 4. *Let $q \in Q$, $(b, f, 0) \in \mathcal{S}$. If $D_{b, f, 0}^{CC}(d; q)$ is nonempty then it is a closed interval for all d .*

Proof. If $b \geq 0$ then $D_{b, f, 0}^{CC}(d; q)$ is empty. If $b < 0$ the proof of the theorem is very similar with the proof of Theorem 3 and we will omit it. \square

Theorem 5. *Let $q \in Q$, $(b, f, 0) \in \mathcal{S}$. If the set $D_{b, f, 0}^{Both}(d; q)$ is nonempty then it is a closed interval for all d .*

Proof. If $b \geq 0$ then the set $D_{b, f, 0}^{Both}(d; q)$ is empty. For $b < 0$ the proof is similar with the proof of Theorem 3. \square

Theorem 6. *For any price $q \in Q$, $d \in D$, $f \in F$, and $h \in H$, the sets $D_{b, f, h}^{CC}(d; q)$ expand when b decreases.*

Proof. Let $b_1 > b_2$. Then

$$\begin{aligned} \{(c, b', f', h', 0, 1) \in B_{b_1, f, h}(d, y; q)\} &= \{(c, b', f', h', 0, 1) \in B_{b_2, f, h}(d, y; q)\}, \\ \{(c, b', f', h', 1, 1) \in B_{b_1, f, h}(d, y; q)\} &= \{(c, b', f', h', 1, 1) \in B_{b_2, f, h}(d, y; q)\}, \\ \{(c, b', f', h', 0, 0) \in B_{b_1, f, h}(d, y; q)\} &\supseteq \{(c, b', f', h', 0, 0) \in B_{b_2, f, h}(d, y; q)\}, \\ \{(c, b', f', h', 1, 0) \in B_{b_1, f, h}(d, y; q)\} &\supseteq \{(c, b', f', h', 1, 0) \in B_{b_2, f, h}(d, y; q)\}. \end{aligned}$$

Thus, if for b_1 ,

$$\begin{aligned} \psi_{cc}(y, d)(c^*, b'^*, f'^*, h'^*, 0, 1) &\geq \max \{ \psi_{nodef}(y, d)(c, b', f', h', 0, 0), \\ &\psi_{sl}(y, d)(c, b', h', 1, 0), \\ &\psi_{both}(y, d)(c, b', h', 1, 1) \}, \end{aligned}$$

it follows that the same inequality will hold for b_2 as well. Therefore, $D_{b_1, f, h}^{CC}(d; q) \subseteq D_{b_2, f, h}^{CC}(d; q)$. \square

Theorem 7. *For any price $q \in Q$, $b \in B$, $f \in F$, and $h \in H$, the sets $D_{b, f, h}^{CC}(d; q)$ shrink and $D_{b, f, h}^{Both}(d; q)$ expand when d increases.*

Proof. Let $d_1 < d_2$. Then

$$\begin{aligned} \{(c, b', f', h', 0, 1) \in B_{b, f, h}(d_1, y; q)\} &\supseteq \{(c, b', f', h', 0, 1) \in B_{b, f, h}(d_2, y; q)\}, \\ \{(c, b', f', h', 1, 1) \in B_{b, f, h}(d_1, y; q)\} &= \{(c, b', f', h', 1, 1) \in B_{b, f, h}(d_2, y; q)\}, \\ \{(c, b', f', h', 0, 0) \in B_{b, f, h}(d_1, y; q)\} &\supseteq \{(c, b', f', h', 0, 0) \in B_{b, f, h}(d_2, y; q)\}, \\ \{(c, b', f', h', 1, 0) \in B_{b, f, h}(d_1, y; q)\} &= \{(c, b', f', h', 1, 0) \in B_{b, f, h}(d_2, y; q)\}. \end{aligned}$$

Thus, if

$$\begin{aligned} \psi_{both}(y, d_1)(c^*, b'^*, f'^*, h'^*, 1, 1) &\geq \max \{ \psi_{nodef}(y, d_1)(c, b', f', h', 0, 0), \\ &\psi_{sl}(y, d_1)(c, b', h', 1, 0), \\ &\psi_{cc}(y, d_1)(c, b', h', 0, 1) \}, \end{aligned}$$

it follows that the same inequality holds for d_2 . Therefore, $D_{b, f, h}^{Both}(d_1; q) \subseteq D_{b, f, h}^{Both}(d_2; q)$. On the

other hand, if

$$\begin{aligned} \psi_{cc}(y, d_1)(c^*, b^*, f'^*, h'^*, 0, 1) \geq \max \{ & \psi_{nodef}(y, d_1)(c, b', f', h', 0, 0), \\ & \psi_{sl}(y, d_1)(c, b', h', 1, 0), \\ & \psi_{both}(y, d_1)(c, b', h', 1, 1)\}, \end{aligned}$$

the inequalities can reverse for d_2 . Therefore $D_{b,f,h}^{CC}(d_1; q) \supseteq D_{b,f,h}^{CC}(d_2; q)$. \square

Theorem 8. *For any price $q \in Q$, $b \in B$, $d \in D$, and $f \in F$, the set $D_{b,f,0}^{CC}(d; q) \subset D_{b,f,1}^{CC}(d; q)$.*

Proof. Let $y \in Y$. For $h = 1$ we have that

$$\{(c, b', f', h', 1, 1) \in B_{b,f,1}(d, y; q)\} = \emptyset$$

and

$$\{(c, b', f', h', 1, 0) \in B_{b,f,1}(d, y; q)\} = \emptyset.$$

Therefore, if for $f = 0$ we have that

$$\begin{aligned} \psi_{cc}(y, d_1)(c^*, b^*, f'^*, h'^*, 0, 1) \geq \max \{ & \psi_{nodef}(y, d_1)(c, b', f', h', 0, 0), \\ & \psi_{sl}(y, d_1)(c, b', h', 1, 0), \\ & \psi_{both}(y, d_1)(c, b', h', 1, 1)\}, \end{aligned}$$

then the same inequalities hold for $f = 1$. \square

A.3 Proofs of Theorems 9 and 10

Theorem 9. Existence *A steady-state competitive equilibrium exists.*

We see that once q^ is known, then all the other components of the equilibrium are given by the formulas in Definition 2. We can rewrite part 5 of the Definition as*

$$\begin{aligned} q_{d,h,b'}^* &= \frac{\rho}{1+r}(1 - p_{d,h,b'}^b) \\ &= \frac{\rho}{1+r} \left(1 - \int \lambda_b^*(y', d, 0, b', h', q^*) \phi(dy') H^*(h, dh') \right), \end{aligned}$$

where λ_b^* and f'^* are measurable selections guaranteed by Theorem 1, and H^* is the transition matrix provided by Theorem 1. Thus q^* is a fixed point of the map $T : [0, q_{\max}]^{N_D \times N_H \times N_B} \mapsto$

$[0, q_{\max}]^{N_D \times N_H \times N_B}$

$$T(q)(d, h, b') = \frac{\rho}{1+r} \left(1 - \int \lambda_b^*(y', d, 0, b', h', q) \phi(dy') H^*(h, dh') \right). \quad (9)$$

Since $Q := [0, q_{\max}]^{N_D \times N_H \times N_B}$ is a compact convex subset of $\mathbb{R}^{N_D \times N_H \times N_B}$ we can apply the Schauder theorem (Theorem V.19 of Reed and Simon (1972)) if we prove that the map

$$q \mapsto \int \lambda_b^*(y', d, 0, b', h', q) \phi(dy') H^*(h, dh')$$

is continuous.

Before starting the proof we remark that the above map is well defined because even though a priori the transition matrix H^* depends on (y, d, b, f, q) , in fact, knowing the pair (h, b') completely determines $H^*(h, dh')$ when $b' < 0$. If $b' < 0$ then $f = 0$, $\lambda_d^* = 0$. Thus $H^*(0, 0) = 1$, $H^*(0, 1) = 0$, $H^*(1, 0) = p_h$ and $H^*(1, 1) = 1 - p_h$. Also, if $b' \geq 0$ then $p_{d,h,b'}^b = 0$ by definition.

We begin by showing that the sets of discontinuities of $\lambda_b^*(\cdot, q)$ and $b^*(\cdot, q)$, $q \in Q$, and $\lambda_b^*(x, \cdot)$ and $b^*(x, \cdot)$, $x \in X$, have measure 0. This will follow from the following lemmas. Let us begin by noticing that the sets of discontinuities of these functions are contained in the sets of indifference.

We fix $b \in B$, $f \in F$, $h \in H$, $d \in D$, and $q \in Q$ and we will suppress the dependence of functions on these variables. That is, we study the behavior with respect to y . Since B, F, H , and D are finite sets this will suffice to prove the continuity of $\lambda_b^*(\cdot, q)$. The first step is to study in more detail the maximization problem on the no default path. Recall that

$$\psi_{\text{nodef}}(y, d)(c, b', f', h', 0, 0) = U(c) + \beta \rho \int v_{b',0,0}(d, y'; q) \Phi(dy')$$

for all $(c, b', 0, 0, 0, 0) \in B_{b,f,h}(d, y; q)$. For $y \in Y$ we write $b'(y)$ for the values of b' that maximize ψ_{nodef} . Recall that b, f, h, d , and q are fixed and that $b'(y)$ can be a correspondence.

Lemma A4. *Let $b \in B$, $f \in F$, $h \in H$, $d \in D$, and $q \in Q$ be fixed. Then for any $y_0 \in Y$ there is $\varepsilon > 0$ such that the following holds:*

1. *If $b'(y_0)$ is a single valued then b' is constant and single valued on $(y_0 - \varepsilon, y_0 + \varepsilon)$.*
2. *If $b'(y_0)$ is multi-valued then either $b'(y)$ is single valued on $(y_0 - \varepsilon, y_0 + \varepsilon) \setminus \{y_0\}$ and there is $\bar{b} \in b'(y_0)$ such that $b'(y) = \bar{b}$ for all $y \in (y_0 - \varepsilon, y_0 + \varepsilon) \setminus \{y_0\}$, or $b'(y) = b'(y_0)$ for all $y \in (y_0 - \varepsilon, y_0 + \varepsilon)$.*

Proof. If $b'(y_0)$ is single valued, then

$$\begin{aligned} U(y_0 + b - d - q_{d,h,b'(y_0)}b'(y_0)) + \beta\rho \int v_{b'(y_0),0,0}(d, y'; q)\Phi(dy') &> \\ U(y_0 + b - d - q_{d,h,b'}b') + \beta\rho \int v_{b',0,0}(d, y'; q)\Phi(dy'), \end{aligned} \quad (10)$$

for all $b' \in B \setminus \{b'(y_0)\}$ (the right hand side is $-\infty$ if $(c, b', 0, 0, 0, 0) \notin B_{b,f,h}(y_0, d; q)$, where, here, $c = y_0 + b - d - q_{d,h,b'}b'$). Then, since $B(f)$ is finite and U is continuous with respect to y , we can find $\varepsilon > 0$ such that if $|y - y_0| < \varepsilon$ then

$$\begin{aligned} U(y + b - d - q_{d,h,b'(y_0)}b'(y_0)) + \beta\rho \int v_{b'(y_0),0,0}(d, y'; q)\Phi(dy') &> \\ U(y + b - d - q_{d,h,b'}b') + \beta\rho \int v_{b',0,0}(d, y'; q)\Phi(dy'), \end{aligned} \quad (11)$$

for all $b \in B(f) \setminus \{b'(y_0)\}$. Thus $b'(y) = b'(y_0)$ for all $|y - y_0| < \varepsilon$.

Suppose now that $b'(y_0)$ is multi-valued. WLOG, assume that $b'(y_0)$ consists of two elements b'_1 and b'_2 (we can assume this since B is finite). Then

$$\begin{aligned} U(y_0 + b - d - q_{d,h,b'_1}b'_1) + \beta\rho \int v_{b'_1,0,0}(d, y'; q)\Phi(dy') &= \\ U(y_0 + b - d - q_{d,h,b'_2}b'_2) + \beta\rho \int v_{b'_2,0,0}(d, y'; q)\Phi(dy') \end{aligned}$$

and they both satisfy inequality (10) for all $b' \in B \setminus \{b'_1, b'_2\}$. There is $\varepsilon > 0$ such that if $|y - y_0| < \varepsilon$, then (11) is satisfied for both b'_1 and b'_2 . We need to compare, thus, $U(y + b - d - q_{d,h,b'_1}b'_1) + \beta\rho \int v_{b'_1,0,0}(d, y'; q)\Phi(dy')$ and $U(y + b - d - q_{d,h,b'_2}b'_2) + \beta\rho \int v_{b'_2,0,0}(d, y'; q)\Phi(dy')$. If $q_{d,h,b'_1}b'_1 = q_{d,h,b'_2}b'_2$, then it follows that $\int v_{b'_1,0,0}(d, y'; q)\Phi(dy') = \int v_{b'_2,0,0}(d, y'; q)\Phi(dy')$. Therefore

$$\begin{aligned} U(y + b - d - q_{d,h,b'_1}b'_1) + \beta\rho \int v_{b'_1,0,0}(d, y'; q)\Phi(dy') &= \\ U(y + b - d - q_{d,h,b'_2}b'_2) + \beta\rho \int v_{b'_2,0,0}(d, y'; q)\Phi(dy') \end{aligned}$$

for all y . Thus $b'(y) = b'(y_0)$ for all $y \in (y_0 - \varepsilon, y_0 + \varepsilon)$. Suppose now that $q_{d,h,b'_1}b'_1 < q_{d,h,b'_2}b'_2$. Then

$$s_0 := y_0 + b - d - q_{d,h,b'_1}b'_1 > y_0 + b - d - q_{d,h,b'_2}b'_2 =: t_0.$$

Assume that ε is so that $t_0 + \varepsilon < s_0 - \varepsilon$. Then, if $|y - y_0| < \varepsilon$ we have that $t_0 < y + b - d - q_{d,h,b'_1}b'_1 =: s_1$,

$t_1 := y + b - d - q_{d,h,b_2}b_2' < s_0$, and $t_1 < s_1$. Then we have

$$\begin{aligned}
U(t_1) + \beta\rho \int v_{b_2,0,0}(d, y'; q)\Phi(dy') &= U(t_1) - U(t_0) + U(t_0) + \beta\rho \int v_{b_2,0,0}(d, y'; q)\Phi(dy') \\
&= U(t_1) - U(t_0) + U(s_0) + \beta\rho \int v_{b_1,0,0}(d, y'; q)\Phi(dy') \\
&= U(t_1) - U(t_0) + U(s_0) - U(s_1) \\
&\quad + U(s_1) + \beta\rho \int v_{b_1,f,0}(d, y'; q)\Phi(dy').
\end{aligned}$$

Since U is strictly concave, $t_0 < s_0$, $t_0 < s_1$, $t_1 < s_1$, $t_1 < s_0$, and $t_1 - t_0 = s_1 - s_0 = y - y_0$, it follows that $U(t_1) - U(t_0) > U(s_1) - U(s_0)$. Thus

$$U(t_1) + \beta\rho \int v_{b_2,f,0}(d, y'; q)\Phi(dy') > U(s_1) + \beta\rho \int v_{b_1,f,0}(d, y'; q)\Phi(dy')$$

and b_2' is *the only* solution to the maximization problem. Therefore b' is single valued and equals b_2' on $(y_0 - \varepsilon, y_0 + \varepsilon) \setminus \{y_0\}$. The case $q_{d,h,b'}b_1' > q_{d,h,b_2}b_2'$ is similar. \square

Lemma A5. *Let $b \in B$, $f \in F$, $h \in H$, $d \in D$, and $q \in Q$ be fixed. Suppose that y_1 is a point of indifference between not defaulting and defaulting on student loans. Then, if ε is small enough, either there is no other point y of indifference with $|y - y_1| < \varepsilon$ or all $y \in (y_1 - \varepsilon, y_1 + \varepsilon)$ are points of indifference.*

Proof. Let $\varepsilon > 0$ be such that for all $y \in Y$ with $|y - y_1| < \varepsilon$ we have that $b'(y) = b'(y_1) =: b'$. We can find such an ε by Lemma (A4): if $b'(y_1)$ is single-valued, then this is the first part of the lemma; if $b'(y_1)$ is multi-valued, the second part of the lemma implies that we can pick $\bar{b} \in b'(y_1)$ such that $\bar{b} \in b'(y)$ or $b'(y) = \bar{b}$ for all $y \in (y_1 - \varepsilon, y_1 + \varepsilon)$. We will consider $b'(y) = \bar{b}$ in both cases (note that this choice does not alter the measurability of b^*). Assume first that $d \neq q_{d,h,b'}b'$, which implies that $c_1 \neq y_1 + b$, and assume, by contradiction, that y_2 is another point of indifference and the distance between y_1 and y_2 is smaller than ε . Then

$$U(c_1) + \beta\rho \int v_{b,0,0}(d, y'; q)\Phi(dy') = U(y_1 + b) + \beta\rho \int v_{0,0,1}(d, y'; q)\Phi(dy')$$

and

$$U(c_2) + \beta\rho \int v_{b,0,0}(d, y'; q)\Phi(dy') = U(y_2 + b) + \beta\rho \int v_{0,0,1}(d, y'; q)\Phi(dy').$$

Therefore $U(c_1) - U(c_2) = U(y_1 + b) - U(y_2 + b)$. However, we have that

$$c_1 - c_2 = y_1 - y_2 = (y_1 + b) - (y_2 + b).$$

This is a contradiction with U being strictly concave. If $d = q_{d,h,b'}b'$ then $c_1 = y_1 + b$, and, hence, $c = y + b$ for all y , then all points y with $|y - y_1| < \varepsilon$ are indifference points. \square

The above lemma holds also for for all types of indifference. Thus, since Y is compact, if we fix d and q , there are only a finite number of earning levels that are discontinuity points for λ_d^ , λ_b^* , and b^* .*

Lemma A6. *The set of pairs $\{y, d\}$ that are points of discontinuity for λ_d^* , λ_b^* , and b^* has measure 0.*

Proof. Lemma A5 implies that we can change the maps in a Borel way so that for each $d \in D$ the set of $y \in Y$ for which these maps are discontinuous is finite. The conclusion follows now since D is finite. \square

Proof. of Theorem 9 Let $\{q_n\}_{n \in \mathbb{N}} \subset Q$ be a sequence that converges to q . We will show that $\lim_{n \rightarrow \infty} \lambda_b^*(y, d, f, b, h, q_n) = \lambda_b^*(y, d, f, b, h, q)$ almost everywhere. Since the sequence $\{q_n\}$ is countable, by Lemma A5 we can find a set $E \subset X$ of measure 0 that contains all the points of indifference for the prices q_n , $n \in \mathbb{N}$, and q . Let $(y, d, f, b, h) \in X \setminus E$ be fixed. Since $v_{b,f,h}(d, y; \cdot)$ is continuous and Q is a compact space it follows that $v_{b,f,h}(d, y; \cdot)$ is uniformly continuous. Therefore, since B is finite, there is $\delta > 0$ such that if $\|q' - q''\| < \delta$ and

$$\psi_{nodef}(q')(c^*, b^*, f', h', 0, 0) > \max \left\{ \begin{aligned} & \max_{(c,b',f',h',1,0)} \psi_{sl}(q')(c, b', f', h', 1, 0), \\ & \max_{(c,b',f',h',0,1)} \psi_{cc}(q')(c, b', f', h', 0, 1), \\ & \max_{(c,b',f',h',1,1)} \psi_{both}(q')(c, b', f', h', 1, 1) \end{aligned} \right\}$$

then the same inequality holds for q'' . In the inequality above we suppressed the dependence on (y, d, f, b, h) to simplify the notation. Thus, if $\lambda_b^*(y, d, f, b, h, q') = 0$ and $\lambda_d^*(y, d, f, b, h, q') = 0$ then $\lambda_b^*(y, d, f, b, h, q'') = 0$ and $\lambda_d^*(y, d, f, b, h, q'') = 0$. Similar statements hold for all possible combinations of values of λ_b^* and λ_d^* . Therefore, by shrinking δ if necessary, we have that if $\|q' - q''\| < \delta$ then $\lambda_b^*(y, d, f, b, h, q') = \lambda_b^*(y, d, f, b, h, q'')$. This implies that $\lim_{n \rightarrow \infty} \lambda_b^*(y, d, f, b, h, q_n) = \lambda_b^*(y, d, f, b, h, q)$ for all $(y, d, f, b, h, q) \in X \setminus E$. Finally, since $|\lambda_b^*(y, d, f, b, h, q)| \leq 1$ and X is a

compact space, the Lebesgue's Dominated Convergence Theorem (see, for example, (Rudin, 1987, Theorem 1.34)) implies that

$$\lim_{n \rightarrow \infty} \int \lambda_b^*(y', d, f', b', h', q_n) \Phi(dy) H(h, dh') = \int \lambda_b^*(y', d, f', b', h', q_n) \Phi(dy) H(h, dh').$$

Thus the map T defined in (9) is continuous and, hence, has a fixed point. \square

Theorem 10. *In any steady-state equilibrium the following is true:*

1. For any $b' \geq 0$, $q_{d,h,b'}^* = \rho/(1+r)$ for all $d \in D$ and $h \in H$.
2. If the grids of D and B are sufficiently fine, and $h = 0$ there are $\underline{d} > 0$ and $\underline{b}' < 0$ such that $q_{d,h,b'}^* = \rho/(1+r)$ for all $d < \underline{d}$ and $b' > \underline{b}'$.
3. If the set of income levels for which the household is indifferent between defaulting on credit card debt and any other available option is of measure zero, then $d_1 < d_2$ implies $q_{d_1,h,b'}^* > q_{d_2,h,b'}^*$ for any $h \in H$ and $b' \in B$.
4. If the set of income levels for which the household is indifferent between defaulting on credit card debt and any other available option is of measure zero, then $q_{d,h=1,b'}^* > q_{d,h=0,b'}^*$ for any $d \in D$ and $b' \in B$.

Proof. The first part follows from part 5) of the definition of an equilibrium.

For the second part, assume that there are $b_1 < 0$ and $\underline{d} > 0$ such that $y + b_1 - d_1 > 0$ for all $y \in Y$ and consider any household with $b_1 < b < 0$ and $0 < d < \underline{d}$. In particular the household must have a clean default flag on the credit card market and on the student loan market. If an household with debt $b < 0$ defaults only on the credit card market then its utility is

$$\begin{aligned} u(y - d) - \tau_b + \beta\rho \int u(y' - d - q_{b^*(d,y';q)(b,0,0),d,0}^* b^*(d, y'; q)(b, 0, 0)) \Phi(dy') \\ + (\beta\rho)^2 \int (1 - p_f) \omega_{b^*(d,y';q)(b,0,0),1,0}(q^*, d) + p_f \omega_{b^*(d,y';q)(b,0,0),0,0}(q^*, d) \Phi(dy'). \end{aligned}$$

On the other hand, one feasible action of the household is to not default on any market, pay off the debt and save in the following period $b^*(d, y'; q)(b, 0, 0)$. The utility from this course of action is

$$\begin{aligned} u(y + b - d) + \beta\rho \int u(y' - d - q_{b^*(d,y';q)(b,0,0),d,0}^* b^*(d, y'; q)(b, 0, 0)) \Phi(dy') \\ + (\beta\rho)^2 \int \omega_{b^*(d,y';q)(b,0,0),0,0}(q^*, d) \Phi(dy'). \end{aligned}$$

Then property 3) of Definition A1 implies that the utility gain by not defaulting is at least

$$u(y + b - d) - u(y - d) + \tau_b.$$

Assuming that the grid of B is sufficiently fine so that we can find $\underline{b} > b_1$ such that the above expression is positive for all $b > \underline{b}$ and $d < \underline{d}$ the conclusion follows. The proof for the case when the household defaults on both markets is similar.

Assuming that the set of income levels for which the household is indifferent between defaulting on credit card debt and any other available option, Theorem 7 implies that if $d_1 < d_2$ then $p_{d_1, h, b'}^{b^*} \leq p_{d_2, h, b'}^{b^*}$ for any $h \in H$ and $b' \in B$. The third part of the theorem follows. One can similarly prove the last part of the theorem. \square

A.4 Proof of Theorem 11

Theorem 11. *If the grids of D and B are fine enough, then we can find $d_1 \in D$ and $b_1 \in B$ such that the agent defaults. Moreover, we can find $d_2 \geq d_1$ and $b_2 \leq b_1$ such that the agent defaults on student loans.*

Proof. Suppose that D is fine enough so that we can find $d_1 > 0$ such that given $A > 1$ to be specified below we have that $|u'(y - d_1)| \geq A$ for all $y \in Y$ such that $y > d_1$. Since $q_{\max} < 1$ then we can find $b_1 < 0$ such that $b - q_{\max} b' < 0$ for all $b' \in B$. The utility from defaulting on the credit card for b_1 is

$$u(y - d_1) - \tau_b + \beta \rho \omega_{0,1,0}(q^*, d_1)$$

and the utility from not defaulting on either path is

$$u(y + b_1 - d_1 - q_{b'^*(d,y;q)(b,f,h),d_1,h} b'^*(d,y;q)(b,f,h)) + \beta \rho \omega_{b'^*(d,y;q)(b,f,h),d_1,h}(q^*, d_1).$$

Using the mean value theorem we can find c' such that $y + b_1 - d_1 - c_{b'^*(d,y;q)(b,f,h),d_1,h} b'^*(d,y;q)(b,f,h) < c' < y - d_1$ and

$$u(y - d_1) - u(y + b_1 - d_1 - q_{b'^*(d,y;q)(b,f,h),d_1,h} b'^*(d,y;q)(b,f,h)) = u'(c')(b_1 - q_{b'^*(d,y;q)(b,f,h),d_1,h} b'^*(d,y;q)(b,f,h)).$$

In particular, $|u'(c')| > A$. We chose A such that

$$A(q_{b'} b' - b_1) > \tau_b + \beta \rho (\omega_{b'^*(d,y;q)(b,f,h),d_1,h}(q^*, d_1) - \omega_{0,1,0}(q^*, d_1)),$$

for all $b' \in B$. It follows that the utility from defaulting on credit card is higher than the utility of

not defaulting at all.

Suppose now that the grids of D and B are fine enough so that we can find d_2 and b'_2 such that $u(y + b'_2) - u(y - d_2) - \tau_d + \tau_b$ is zero or as close to zero as we want. That is, the agent's current utility from defaulting on student loans or credit card are basically the same. Then, if an agent chooses to default on the credit card market today, in the next period her utility will be

$$u(y' - d_2 - q_{d_2,0,b_{CC}^*} b_{CC}^*) + \beta \rho \left((1 - p_f) \omega_{b_{CC}^*,0,1}(d_2, q^*) + p_f \omega_{b_{CC}^*,0,0}(d_2, q^*) \right),$$

where $b_{CC}^* \geq 0$. If the agent chooses to default on student loans, she can chose to borrow $b''_2 < 0$ such that $y'(1 - \gamma) - d_2 - q_{b''_2} b''_2 > y' - d_2 - q_{d_2,0,b_{CC}^*} b_{CC}^*$ and $|u'(y'(1 - \gamma) - d_2 - q_{b''_2} b''_2)| > B$, where B is so that

$$\begin{aligned} u'(c')(-\gamma y' - q_{b''_2} b''_2 + q_{d_2,0,b_{CC}^*} b_{CC}^*) &\geq (1 - p_h) \omega_{b''_2,0,1}(d_2, q^*) + p_h \omega_{b''_2,0,0}(d_2, q^*) \\ &\quad - \left((1 - p_f) \omega_{b_{CC}^*,0,1}(d_2, q^*) + p_f \omega_{b_{CC}^*,0,0}(d_2, q^*) \right). \end{aligned}$$

Thus, if $b_2 = \min\{b'_2, b''_2\}$ it follows that the agent chooses to default on student loans. \square