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Abstract

We develop an equilibrium model of on-the-job search with ex-ante heterogeneous workers and firms, aggregate uncertainty and vacancy creation. The model produces rich dynamics in which the distributions of unemployed workers, vacancies and worker-firm matches evolve stochastically over time. We prove that the surplus function, which fully characterizes the match value and the mobility decision of workers, does not depend on these distributions. We estimate the model on US labor market data from 1951-2007 and predict the fit for 2008-12. We use the model to measure the cyclicality of mismatch between workers and jobs.

Keywords: On-the-job search; Heterogeneity; Aggregate fluctuations; Mismatch

JEL codes: E24; E32; J63; J64

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1 Introduction

The 2008-12 recession in the US is characterized by several striking features that make it appear quite different from previous recessions. While output fell and unemployment rose labor productivity remained stable. While unemployment rose, the number of long term unemployed increased well beyond what has been seen since 1951. Finally, the level of unemployment remained high, even in the face of an increase in vacancies, suggesting that the pool of unemployed workers may not be well suited to fill these vacancies. These observations suggest that there may be strong interactions between the heterogeneity in worker skills and in job requirements that depend on the level of aggregate activity. How does the distribution of skills among the unemployed vary with the business cycle? How does the quality of matches for workers transiting from unemployment vary with the aggregate state? Similarly, how is the reallocation of currently employed workers to more appropriate matches related to the business cycle? To answer these questions and understand the interactions requires a model of the labor market which incorporates both worker and firm level heterogeneity and aggregate uncertainty. Existing equilibrium search models of the labor market with heterogeneous workers and firms, such as Burdett and Mortensen (1998) and Postel-Vinay and Robin (2002), rely heavily on the stationarity for tractability, which means the equilibrium distributions can effectively be treated as parameters rather than state variables.

In this paper, we develop a stochastic model of random search on the job, with ex-ante heterogeneous workers and firms and aggregate productivity shocks in which firms make state contingent offers and counter offers to workers. The model extends the work of Postel-Vinay and Robin (2002), incorporating aggregate productivity shocks and non-stationary distributions of unemployed workers and worker-firm matches. We obtain tractability by working with the function that defines the joint surplus of a worker-firm match, rather than the individual value functions of the worker and firm separately. The model delivers rich dynamics in which the distributions of unemployed workers, posted vacancies, and worker-firm matches evolve stochastically over time. We prove that the surplus function, which defines the value of a match and fully characterizes the mobility decision of workers, does not depend on the distributions of unemployed workers, posted vacancies or worker-firm matches. The implication is that the fixed point defining the surplus can be solved for independently from the current distributions of unemployed workers and worker-firm matches. The evolution of these distributions in the stochastic economy can then be solved for exactly, given the initial conditions.

In a series of influential papers Menzio and Shi (2010a,b, 2011) show that in a model with on-the-job search and aggregate shocks in which search is directed, the equilibrium is such that agents' value and policy functions are independent of the endogenous distribution of workers across employment states.[1] We have a closely related result for a class of random search models. In our model, although the agents' value functions do depend on the distribution of workers across employment states, the joint surplus of a match does not. The surplus function is sufficient to fully characterize the mobility decisions of workers between employment states and across jobs. In addition, although the firms' vacancy creation decision do depend on these distributions, the fact that the surplus function can be solved independently makes the vacancy creation decision effectively a static problem.

Moscarini and Postel-Vinay (2013) provide the first analysis of aggregate stochastic dynamics in a wage posting model with heterogeneous firms and random search by homogeneous workers. They show that in the class of models they analyze, the equilibrium is rank preserving; more productive firms offer higher wages in all states of the worlds. As a result the mobility decision of (identical) workers is independent of the aggregate state. Coles and Mortensen (2012) build on these results in a model where firms cannot fully commit to a wage policy but instead use reputation. They analyze transitional dynamics, but not aggregate stochastic dynamics. Robin (2011) develops a version of Postel-Vinay and Robin (2002) with heterogeneous workers and aggregate shocks but identical firms. In our model both workers and firms are ex-ante heterogeneous which allows us to analyze the cyclical patterns of mismatch between workers and firms. We also have an explicit vacancy creation decision which means the contact rate between workers and firms changes endogenously with the aggregate state. Additionally, we allow for production technologies in which workers may not agree on the ranking of firms. As far as we are aware this is the first tractable model of equilibrium search with two sided ex-ante heterogeneity and aggregate shocks.[2]

[1]Examples of papers making use of directed search in this way include Kaas and Kircher (2011) and Schaal (2012).

[2]Barlevy (2002) presents a model with worker and firm heterogeneity and aggregate shocks. However workers (firms) are identical in terms of potential outcomes and the heterogeneity is effectively match specific. Additionally, the full stochastic model is not tractable and is solved by replacing the

In the quantitative section of the paper we confront our model with the facts about the relative volatility of output, unemployment, vacancies and transitions rates (see Shimer, 2005; Hall, 2005; Hagedorn and Manovskii, 2008, among others). We fit the model to moments on the level and volatility of output, unemployment, vacancies, transition rates, as well as moments on unemployment duration and the cross sectional standard deviation of value added per job. The model is estimated using the 1951-2007 US series and we then predict these series for 2008-2012. The model fits these moments well; the interaction between the two sided heterogeneity and aggregate shocks amplify productivity shocks in terms of unemployment and vacancies, and additionally can explain the coexistence of rising unemployment, falling output and constant (or increasing) labor productivity during the 2008-12 period. Finally, we use simulations from the estimated model to analyze the cyclical patterns of sorting/mismatch between workers and firms (see related work by Barlevy, 2002; Sahin et al., 2012, among others).

The rest of the paper is organized as follows: In Section 2 we present the model and our main theoretical result. In Section 3 we describe the data used for estimation. In Section 4 we present our parametric specification and model fit. We discuss the model interpretation, and present simulation estimates for the cyclical behavior of mismatch between worker- and firm-types in Section 5. Section 6 concludes.

2 The Model

2.1 Agents and Technology

The economy is populated by a continuum of infinitely-lived workers indexed by type $x \in [\underline{x}, \overline{x}]$. The measure of worker types in the population is exogenous and denoted by $\ell(x)$, with $\int \ell(x)\,\mathrm{d}x = 1$. There is a continuum of firms indexed by type $y \in [\underline{y}, \overline{y}]$. The size of firms and the distribution of jobs in the economy is endogenous, and determined by firms' choice of recruiting activity. Similarly, the aggregate state of the economy is indexed by $z_t \in [\underline{z}, \overline{z}]$. At the beginning of each period the aggregate state changes from z to z' according to the Markov transition probability $\pi(z, z')$, $\pi(z, z') \geq 0$, $\int \pi(z, z')\,\mathrm{d}z' = 1$.

Workers seek to maximize the expected sum of lifetime income, discounted at

dynamic bargaining with a fixed piece rate wage.

rate r. Unemployed workers have access to a home production technology $b(x, z)$, which depends on a worker's own type and the current aggregate productivity of the economy. When matched with a firm the worker receives a wage that is, in general, type-, state-, and history-dependent.

Firms seek to maximize the discounted stream of expected profits. They have access to a production technology, indexed by their type y, that combines the skills of each employed worker with aggregate productivity to create value added $p(x, y, z) \in [\underline{p}, \overline{p}] \subset \mathbb{R}$ for each match (x, y, z). The contribution of z is common to all production in the current period, the contribution of y is common to all matches in the same firm, while match-specific productivity is governed by the interaction of a worker's type x and the firm type y. We will assume throughout that market output is increasing in the aggregate shock: $p_z > 0$. We will allow for the possibility that positive value added may require a threshold level of inputs. For example, a given production function indexed by y may require workers of sufficient skill before value added is positive. Since the production technology is defined at the level of the match, there is no complementarity between workers within a firm. We allow for complementarities between worker and firm types: $p_{xy} \geq 0$; any correlations in output between workers at the same firms are attributed to the common firm component. When producing, firms pay each worker a wage that will depend on the firm's type, the worker's type, and, in general, the history of aggregate shocks and the history of the worker's outside options. Each period firms engage in recruiting new workers by choosing recruiting effort (posting vacancies) $v_t(y)$, and paying convex (flow) recruiting cost $c[v_t(y)]$. In addition to recruiting new workers, firms may make new wage offers to their existing workers in an attempt to retain those with outside offers, or in response to the aggregate shock.

2.2 Contractual Environment

We consider employment contracts that stipulate a fixed wage that the employer commits to pay per unit of time. A contract can be renegotiated only if both parties agree to that. Employers can fire workers and workers can quit at will. There is no severance payment or experience rating, and unemployment benefit is not contingent on previous work history.

Let $W_t(w, x, y)$ denote the present value of a wage w for a worker of type x

employed by a firm of type y, where the t subscript indicates that this value function depends on all the time varying states, including the aggregate productivity z_t, the distribution of unemployed workers $u_t(x)$, and the distribution of matches $h_t(x, y)$. Similarly, let $\Pi_t(w, x, y)$ denote the expected profit to a firm. Let $B_t(x)$ denote the value of unemployment. This is the value of the outside option for the worker if the negotiation does not succeed. On the employer side, we assume that the value of a vacancy is zero as firms can create jobs at no cost. Advertising a job is the real cost, which has to be paid per period.

Transferable utility. Define the surplus of a match as the sum of the surplus to the worker plus the surplus to the firm of being matched rather than apart:

$$S_t(w, x, y) = W_t(w, x, y) - B_t(x) + \Pi_t(w, x, y).$$

Under the assumption of transferable utility, the wage should not affect the size of the surplus, i.e. $S_t(w, x, y) \equiv S_t(x, y)$. We shall start by assuming that the value of a match (x, y) at time t is some function $S_t(x, y) + B_t(x)$, then we verify that the values $W_t(w, x, y)$ and $\Pi_t(w, x, y)$ calculated under this assumption are indeed consistent with the property that $W_t(w, x, y) + \Pi_t(w, x, y)$ does not depend on w and equals the assumed match value.

Hiring and poaching. Firms make type and state contingent offers and counter offers to workers. We adopt the sequential auction framework of Postel-Vinay and Robin (2002) and assume that employers offer unemployed workers their reservation wage, and that on-the-job search triggers Bertrand competition between firms for workers. Bertrand competition grants the worker a value that is equal to the second highest bid.

Unemployed workers are thus hired at a wage $w = \phi_{0,t}(x, y)$ which solves

$$W_t(w, x, y) - B_t(x) = 0, \tag{1}$$

and when an employed worker of type x, currently employed at a firm of type y meets a firm of type y' such that $S_t(x, y') > S_t(x, y)$, the worker ends up at the higher surplus match and is paid a wage $w' = \phi_{1,t}(x, y', y)$ which solves

$$W_t(w', x, y') - B_t(x) = S_t(x, y). \tag{2}$$

The key implication of Bertrand in the current environment is that the surplus only depends on time through the current aggregate state z_t, and not on the distributions of vacancies, unemployed workers or current matches, as we shall show below.

Lastly, if the poacher y' is such that $S_t(x, y') < W_t(w, x, y) - B_t(x)$, where w is the current wage, then no quit threat can force the incumbent employer to renegotiate, and if $W_t(w, x, y) - B_t(x) \leq S_t(x, y') \leq S_t(x, y)$ then the wage contract is renegotiated to $w' = \phi_{1,t}(x, y, y')$. Note that the maximum wage that a firm of type y can pay to a worker of ability x is $\phi_{1,t}(x, y, y)$.

Wage renegotiation after a productivity shock. Let w be the wage inherited from period $t-1$. In period t, contract continuation requires that the wage be renegotiated to a new value w', if necessary, to meet the incentive-compatibility constraints:

$$0 \leq W_t(w', x, y) - B_t(x) \leq S_t(x, y),$$

equivalently,

$$0 \leq \Pi_t(w', x, y) \leq S_t(x, y).$$

We assume that $w' = \phi_{2,t}(w, x, y)$, with $\phi_{2,t}(w, x, y) = \phi_{0,t}(x, y)$ (the firm takes all the surplus) if $W_t(w, x, y) \leq B_t(x)$; $\phi_{2,t}(w, x, y) = \phi_{1,t}(x, y, y)$ (the worker takes all the surplus) if $W_t(w, x, y) - B_t(x) \geq S_t(x, y)$; and $\phi_{2,t}(w, x, y) = w$ (status quo) otherwise, i.e. $0 \leq W_t(w', x, y) - B_t(x) \leq S_t(x, y)$.

Since production is match-specific, the profit-maximizing decision of a firm can be taken match by match and is summarized as follows: terminate any matches which produce negative surplus; compete on wages to retain workers in positive surplus matches who have outside offers; and renegotiate wages in matches where the aggregate shock pushes the current wage outside the constraints imposed by worker and firm rationality.

2.3 Labor Market Flows

2.3.1 Endogenous and Exogenous Job Separations

At the beginning of period t, a measure $u_t(x)$ of unemployed workers of type x and a measure $h_t(x,y)$ of workers of type x employed at firms of type y are inherited from period $t-1$. Then, the aggregate state changes from $z_{t-1} = z$ to $z_t = z'$.

All jobs such that $S_t(x,y) < 0$ are simultaneously destroyed, and a source of idiosyncratic shocks is also responsible for a fraction δ of the viable ones, with $S_t(x,y) \geq 0$, to be destroyed. Hence the stock of unemployed workers of type x immediately after the realization of z_t (at time $t+$) is

$$u_{t+}(x) = u_t(x) + \int \left[\mathbf{1}\{S_t(x,y) < 0\} + \delta \mathbf{1}\{S_t(x,y) \geq 0\}\right] h_t(x,y)\, \mathrm{d}y. \tag{3}$$

The stock of matches of type (x,y) is

$$h_{t+}(x,y) = (1-\delta)\mathbf{1}\{S_t(x,y) \geq 0\} h_t(x,y). \tag{4}$$

2.3.2 Meeting Technology

In the new aggregate state of period t a measure $u_{t+}(x)$ of workers of type x are unemployed and a measure $h_{t+}(x,y)$ are employed at firms of type y. Together they produce effective search effort[3]

$$L_t \equiv s_0 \int u_{t+}(x)\, \mathrm{d}x + s_1 \iint h_{t+}(x,y)\, \mathrm{d}x\, \mathrm{d}y,$$

where s_0 and s_1 are the relative search efforts of unemployed and employed workers.

Firms observe the new aggregate state and decide to post a distribution $v_t(y)$ of vacancies with aggregator

$$V_t \equiv \int v_t(y)\, \mathrm{d}y.$$

The total measure of meeting at time t is given by

$$M_t \equiv \min\{\alpha L_t^{\omega} V_t^{1-\omega}, L_t, V_t\}.$$

[3] For clarity of exposition we present in the body of the paper the case with a Cobb-Douglas aggregate matching function, linear aggregators for worker search and vacancies and a power function for vacancy creation costs. See Appendix B for the corresponding expressions with general matching, aggregation, and cost functions.

Define $\lambda_{0,t} = s_0 M_t / L_t$ as the probability an unemployed searcher contacts a vacancy and $\lambda_{1,t} = s_1 M_t / L_t$ as the probability an employed searcher contacts a vacancy in period t. Let $q_t = M_t / V_t$ be the probability per unit of recruiting effort $v_t(y)$ that a firm contacts any searching worker.

2.3.3 Laws of Motion

The law of motion for unemployment is therefore

$$u_{t+1}(x) = u_{t+}(x) \left[1 - \int \lambda_{0,t} \frac{v_t(y)}{V_t} \mathbf{1}\{S_t(x,y) \geq 0\} \, \mathrm{d}y \right], \tag{5}$$

and for employment

$$\begin{aligned} h_{t+1}(x,y) = h_{t+}(x,y) &\left[1 - \int \lambda_{1,t} \frac{v_t(y')}{V_t} \mathbf{1}\{S_t(x,y') > S_t(x,y)\} \, \mathrm{d}y' \right] \\ &+ \int h_{t+}(x,y') \lambda_{1,t} \frac{v_t(y)}{V_t} \mathbf{1}\{S_t(x,y) > S_t(x,y')\} \, \mathrm{d}y' \\ &\qquad + u_{t+}(x) \lambda_{0,t} \frac{v_t(y)}{V_t} \mathbf{1}\{S_t(x,y) \geq 0\}, \end{aligned} \tag{6}$$

subtracting those lost to more productive poachers, and adding the (x,y)-jobs created by poaching from less productive firms and hiring from unemployment.

2.4 The Value of Unemployment

Consider a worker of type x who is unemployed for the whole period t. During that period she earns $b_t(x)$ and she anticipates that, at the beginning of period $t+1$, after revelation of the new aggregate state, she will meet a vacancy of type y with probability $\lambda_{0,t+1} \frac{v_{t+1}(y)}{V_{t+1}}$. $B_t(x)$, the value to this unemployed worker, is then

$$\begin{aligned} B_t(x) = b(x,z) + \frac{1}{1+r} \mathbb{E}_t &\Bigg[(1 - \lambda_{0,t+1}) \, B_{t+1}(x) \\ &+ \lambda_{0,t+1} \int \max\{W_{t+1}(\phi_{0,t+1}(x,y), x, y), B_{t+1}(x)\} \, \frac{v_{t+1}(y)}{V_{t+1}} \, \mathrm{d}y \Bigg], \end{aligned} \tag{7}$$

where $\mathbb{E}_t$ is the expectations operator taken with respect to the information set at time t, which includes z_t, $v_t(y)$, $u_t(x)$, and $h_t(x,y)$.

Making use of the assumption that firms offer unemployed workers their reservation wages (equation (1)), whether she takes the job or not, the continuation value is the value of unemployment at $t+1$, $B_{t+1}(x)$. Hence

$$B_t(x) = b(x,z) + \frac{1}{1+r}\mathbb{E}_t B_{t+1}(x).$$

2.5 The Surplus

We are now in a position to state our main result.

Proposition 1. *The surplus from an (x,y) match at time t depends on time only through the current aggregate productivity shock z and does not depend on the distributions of vacancies, unemployed workers, or worker-firm matches. Specifically, $S_t(x,y) \equiv S(x,y,z)$ such that*

$$S(x,y,z) = s(x,y,z) + \frac{1-\delta}{1+r}\int S(x,y,z')^{+}\pi(z,z')\,\mathrm{d}z', \tag{8}$$

where $s(x,y,z) = p(x,y,z) - b(x,z)$ and we denote $x^{+} = \max\{x,0\}$.

The proof is in Appendix A.

The distribution of vacancies, unemployed workers and worker-firm matches affect the values of the worker and the firm, since they are directly relevant when calculating both the probability a worker will receive an outside offer next period and the distribution of types of vacancies she will contact next period. However, outside offers do not change the amount of surplus in the match, only how it is shared between the worker and the firm. When a firm counters an outside offer to retain the worker this is done by transferring more of the match surplus to the worker, but has no impact on the total surplus in the match. Similarly, when a worker is poached by another firm, Bertrand competition ensures that the value to the worker of moving to the new match is exactly the total surplus at the previous match. The previous firm is then left with zero since vacancies do not have a continuation value (they only last one period).

2.6 Vacancy Creation

Each period firms attempt to recruit new workers, paying convex costs $c[v_t(y)]$, with $c(0) \geq 0$, $c' > 0$, $c'(0) = 0$, and $c'' > 0$. It is optimal to post vacancies $v_t(y)$ up to the point where the marginal vacancy makes zero expected profit:

$$c'\left[v_t(y)\right] = q_t J_t(y), \tag{9}$$

where $J_t(y)$ is the expected value of a new match

$$J_t(y) = \int \frac{\lambda_{0,t} u_{t+}(x)}{M_t} S(x,y,z)^+ \,\mathrm{d}x + \iint \frac{\lambda_{1,t} h_{t+}(x,y')}{M_t} \left[S(x,y,z) - S(x,y',z)\right]^+ \mathrm{d}x \,\mathrm{d}y'. \tag{10}$$

With a Cobb-Douglas matching function and recruiting cost function of the form

$$c[v_t(y)] = \frac{c_0 v_t(y)^{1+c_1}}{1+c_1},$$

optimal vacancy creation by firm-type is given by the expression

$$v_t(y) = \left(\frac{\alpha}{\theta_t^{\omega}} \frac{J_t(y)}{c_0}\right)^{\frac{1}{c_1}}, \tag{11}$$

where aggregating over $v_t(y)$ and substituting for q_t gives equilibrium market tightness

$$\theta_t = \left(\frac{\alpha}{c_0}\right)^{\frac{1}{c_1+\omega}} \left(\frac{1}{L_t} \int \left(\frac{J_t(y)}{c_0}\right)^{\frac{1}{c_1}} \mathrm{d}y\right)^{\frac{c_1}{c_1+\omega}}.$$

2.7 Computation of the Stochastic Search Equilibrium

Following directly from the results presented above, we can solve for the stochastic equilibrium in this environment in two stages.

1. For given home production and value added technologies $b(x,z)$ and $p(x,y,z)$; discount rate r; exogenous job destruction rate δ; and stochastic process for the aggregate state $\pi(z,z')$ the surplus function $S(x,y,z)$ is sufficient to determine all decisions regarding worker mobility and is defined by the unique solution to the functional equation (8).

2. For a given vacancy cost function $c[v(y)]$, and meeting technology $M(L,V)$; and for any given initial distributions of unemployed workers $u_0(x)$ and workers-job matches $h_0(x,y)$, a sequence of aggregate productivity shocks $\{z_t\}_{t=0}^{T}$, implies a

unique sequence of distributions of vacancies, unemployed workers, and worker-firm matches

$$\{v_t(y), u_{t+1}(x), h_{t+1}(x, y)\}_{t=0}^{T}.$$

The sequence of distributions can be found by using the surplus function $S(x, y, z)$ and iterating on equations (3), (4), (9), (11), (5), and (6) starting from time 0 information $\{z_0, u_0(x), h_0(x, y)\}$.

3 Data

Wherever possible we use publicly available aggregate data. The unemployment data are from the US Bureau of Labor Statistics (BLS) and cover the period 1951m1 to 2012m12. We use the BLS series of seasonally adjusted monthly employment and unemployment levels for all persons aged 16 and over. In addition to the number of unemployed, we also use the number of unemployed with unemployment durations of more than 5, 15 and 27 weeks. We divide the unemployment levels each month by the sum of unemployment and employment to obtain rates. This gives us monthly series for U_m, U_m^5, U_m^{15}, and U_m^{27} corresponding to the fraction of individuals unemployed and the fraction unemployed more than 5, 15 and 27 weeks, where the m subscript refers to a monthly frequency.

From these series we construct monthly transition rates between unemployment and employment and between unemployment and employment as follows:

$$\begin{aligned} U2E_m &= 1 - U_{m+1}^5 / U_m \\ E2U_m &= \left(U_{m+1} - U_{m+1}^5\right) / E_m. \end{aligned}$$

The BLS does not provide a series for job-to-job transitions. We construct this series using the Current Population Survey (CPS) for the period 1994m1 to 2012m12. This series is constructed following Moscarini and Thomsson (2007).

A time series of monthly vacancies can be constructed by combining the BLS monthly Help Wanted Index (HWI) 1951m1 to 2006m12 and the Job Openings and Labor Turnover Survey (JOLTS) 2001m1-2012m12. To obtain a consistent series we project the HWI series onto the JOLTS series for the overlapping months in the years. We then obtain a combined series using predicted HWI based on the JOLTS to put them into the same scale. The level is immaterial as we are only interested in the volatility of this series.

Output data is provided on a quarterly frequency. We use the BLS quarterly series 1951q1-2012q4 of seasonally adjusted real value added in the non-farm business sector to construct aggregate output. We also construct a measure of productivity dispersion across jobs using the Bureau of Economic Analysis (BEA) series for value added per job by industry 1951q1-2012q4. Our measure of productivity dispersion is the standard deviation of the log of value added per job across 19 industries.[4]

Since the value added series are only provided at the quarterly frequency, we aggregate all series up to this frequency by taking the quarterly average for the monthly series. We remove a quadratic trend from the log-transformed quarterly series, estimated on the period 1951q1 to 2007q4.[5] We estimate the model using moments calculated on the data up to the last quarter of 2007. The post 2007 data is used to provide an out-of-sample fit of the model to data generated during the great recession. From the post 2007 series we remove the quadratic trend estimated on the earlier data.

4 Parametric Specification and Model Fit

4.1 Parametrization

We choose the following parametrization of the model. The distribution of workers is assumed to be Beta with parameters β_1 and β_2 to be estimated. We approximate the continuous heterogeneity by a grid of linearly spaced points $x_1, x_2, ..., x_{N_x}$ on $[0, 1]$. We specify the set of potential job types, $y_1, y_2, ..., y_{N_y}$ on $[0, 1]$.[6] Similarly, we specify a grid of linearly spaced points $a_1, a_2, ..., a_{N_z}$ on $[\varepsilon, 1 - \varepsilon] \subset (0, 1)$, used to define

[4]Over this time period the consistent private-non-farm industry classifications are mining; utilities; construction; durable goods manufacturing; nondurable goods manufacturing; wholesale trade; retail trade; transportation and warehousing; information; finance and insurance; real estate and leasing; professional, scientific and technical services; management; administrative services; education services; health services; arts, entertainment and recreation; accommodation and food.

[5]We have also conducted the analysis with moments based on HP filtered data with a standard smoothing parameter of 1600, or a non-standard 10^5. The model fit is essentially the same (or better) with HP filtered data. Using the HP filter has the advantage of easier comparability with existing studies, but has the undesirable implication that it produces a series in which unemployment (output) is below (above) trend by 2011.

[6]Restricting the range of x and y to the unit interval is simply a normalization of the production function. The distributional assumption on x has no affect on the value of the surplus function $S(x, y, z)$.

the aggregate productivity shocks z_i. The aggregate productivity shock is given by $z_i = F^{-1}(a_i)$, and the transition probability $\pi(z_i, z_j) \propto C(a_i, a_j)$, where C is a copula density, and we normalize $\sum_j \pi(a_i, a_j) = 1$. Specifically, we set $N_x = 51$, $N_y = 51$, $N_z = 151$, F is lognormal with parameters zero and σ, and C is a Gaussian copula density with parameter governing dependence ρ. We set the length of a period in the model to be one week. The discount rate is set to five percent annually.

For the meeting function and posting costs we assume a Cobb-Douglas meeting function

$$M_t = M(L_t, V_t) = \min\left\{\alpha L_t^{\omega} V_t^{1-\omega}, L_t, V_t\right\},$$

with $\alpha > 0$ and $\omega \in [0, 1]$. We assume the effective amount of worker search effort is given by a weighted average of unemployed and employed worker search

$$L_t = s_0 \int u_{t+}(x)\,\mathrm{d}x + s_1 \iint h_{t+}(x, y)\,\mathrm{d}x\,\mathrm{d}y,$$

with $s_0 > 0$ and $s_1 > 0$. The flow cost of posting vacancies is modeled as a power function and is assumed to be independent of the type of the firm

$$c[v_t(y)] = \frac{c_0 v_t(y)^{1+c_1}}{1 + c_1},$$

with $c_0 > 0$ and $c_1 > 0$. The expression for vacancy creation $v_t(y)$ implied by these forms is given in Equation (11).

We approximate value added at the match level by a second order Taylor series in worker and firm type, assuming proportionality to the aggregate shock z:

$$p(x, y, z) = z\left(p_1 + p_2 x + p_3 y + p_4 x^2 + p_5 y^2 + p_6 xy\right).$$

We do not place any restrictions on the polynomial coefficients, which are to be estimated. Recall that we are modeling value added (revenue minus non-labour costs), not total output. We want to allow the possibility that higher-y firms may operate with more costly non-labor inputs. In this case only workers with skill above a particular threshold would produce enough to cover the non-labor costs and hence deliver positive profit to the firm. For example, suppose $p_1 = p_2 = p_3 = p_4 = 0$, $p_5 = -1$ and $p_6 = 1$, then only matches in which $x > y$ will product positive value added. Alternatively, suppose $p_1 = 1$, $p_2 = p_3 = 0$, $p_4 = p_5 = -1$, and $p_6 = 2$, then value added is maximized when $x = y$ and decreases as x and y differ (x is not well matched to y). We model home production as comprising a fixed component plus a component that depends on the worker type and the aggregate state,

$$b(x, z) = b_0 + z\left(b_1 x + b_2 x^2\right).$$

4.2 Moments

We estimate the model by the method of simulated moments. The data moments we target in estimation, along with their model simulated values, are listed in Table 2. We target the mean and standard deviation of the unemployment rate; the number unemployed more than 5, 15, and 27 weeks; unemployment-to-employment transitions; employment-to-unemployment transitions; job-to-job transitions; the cross-sectional standard deviation of value added per match (measured in the data as valued added by sector from the BEA); the standard deviation of vacancies; the standard deviation of the vacancy-to-unemployment ratio; the standard deviation and autocorrelation of value added; the correlation between vacancies and unemployment; the correlation between unemployment-to-employment and job-to-job transitions; and the correlations between value added and unemployment, vacancies, unemployment-to-employment transitions, employment-to-unemployment transitions, and the cross-sectional standard deviation of value added per per match.

We solve and simulate data from the model at a weekly frequency. We then aggregate the weekly data exactly to obtain data series at the quarterly frequency. All moments are calculated on the quarterly data (actual and simulated). We aggregate the model simulated data exactly as it comes from the BLS and BEA.

Using the subscript t to denote weekly time-series simulated from the model, we first construct the weekly series of aggregate value added, unemployment and vacancies,

$$p_t = \iint p(x, y, z_t) h_t(x, y)\, \mathrm{d}x\, \mathrm{d}y, \quad U_t = \int u_t(x)\, \mathrm{d}x, \quad V_t = \int v_t(y)\, \mathrm{d}y,$$

and the weekly series of cross-sectional standard deviation of value added per job across firms,

$$\text{sd labor prod}_t = \left(\frac{1}{\overline{y} - \underline{y}} \int \left(\frac{p_t(y)}{E_t(y)} - \frac{p_t}{E_t} \right)^2 \mathrm{d}y \right)^{1/2},$$

where

$$p_t(y) = \int p(x, y, z_t) h_t(x, y)\, \mathrm{d}x, \quad E_t(y) = \int h_t(x, y)\, \mathrm{d}x, \quad E_t = \int h_t(x, y)\, \mathrm{d}x\, \mathrm{d}y.$$

We calculate the weekly series of the number of unemployed workers with durations of 5, 15, and 27 weeks or more as

$$U_t^s = \int u_{t-s}(x) \prod_{j=0}^{s-1} \left[1 - \int \lambda_{0,t-s+j} \frac{v_{t-s+j}(y)}{V_{t-s+j}} \mathbf{1}\{S(x, y, z_{t-s+j}) \geq 0\} \right] \mathrm{d}y\, \mathrm{d}x,$$

where U_t^s is the number of unemployed workers at period t who have been unemployed for s periods (weeks) or more. We then construct monthly transition rates exactly as we did from the BLS data,

$$\begin{aligned} U2E_m &= 1 - U_{m+1}^5 / U_m, \\ E2U_m &= \left(U_{m+1} - U_{m+1}^5 \right) / E_m, \end{aligned}$$

where the subscript month corresponds to the same week which would be sampled by the BLS.[7] The monthly job-to-job transition rate can be constructed in a similar manner,[8]

$$J2J_m = 1 - E_{m+1}^5 / E_m,$$

[7]The BLS survey is done each month in the week containing the 12th day of the month. We sample from our simulated data to replicate this strategy. For example, in the first year of the simulation we sample the monthly data from simulation weeks {2, 7, 11, 15, 19, 24, 28, 33, 37, 42, 46, 50}.

[8]We use the term job-to-job transition, but we are actually working with one minus the fraction of workers who stay at the same job, which will differ from the job-to-job transition rate due to the competing risk of moving to unemployment. We calculated the same statistic from the CPS and the simulated data.

where the number of employed workers is given by

$$E_m = \int h_m(x, y)\,\mathrm{d}x\,\mathrm{d}y,$$

and we calculate the number of workers who are still employed at the same job as they were s weeks earlier,

$$E_t^s = \int h_{t-s}(x, y) \prod_{j=0}^{s} \Bigg[1 - \int \lambda_{1,t-s+j} \frac{v_{t-s+j}(y')}{V_{t-s+j}} \times \mathbf{1}\left\{ S(x, y', z_{t-s+j}) > S(x, y, z_{t-s+j}) \right\} \mathrm{d}y' \Bigg] \mathrm{d}x\,\mathrm{d}y.$$

The resulting series are aggregated exactly as the data to obtain the corresponding moments.

We fit the model to moments of the US data from 1950q1 to 2007q4, and reserve the great recession years 2008q1 to 2012q4 for out of sample analysis. Given the specification above, we have 21 parameters to determine: α, ω, s_0, s_1, c_0, c_1, δ, σ, ρ, β_1, β_2, b_0, b_1, b_2, p_1, p_2, p_3, p_4, p_5, p_6, and r. We fix the discount rate, r, to 5 percent annually. We normalize s_0 to 1 and fix ω at 0.5.[9] This leaves 18 parameters which are estimated to best fit the 27 moments.

While none of the parameters has a one-to-one relationship to a moment, we can provide a heuristic description of identification. The main variation can be described as follows: The mobility parameters α, s_1 and δ are identified by the average rates at which workers transit between unemployment and employment, between jobs, and from employment to unemployment. The parameters of the latent productivity shock, σ and ρ, are identified by the standard deviation and auto-correlation of output (corrected for selection via the model). The flow cost of posting new vacancies, c_0 and c_1, governs the response of vacancies to changes in profitability and is identified by the standard deviation of vacancies and the correlation of vacancies with output. The last set of parameters β_i, b_j, and p_k govern the distribution of worker types in the population, home production, and value added. The distribution of worker types is identified by the pattern in the number of workers unemployed 5, 15 and 27 or more weeks (homogeneous workers would imply this distribution is exponential). The

[9] Without using direct information on the costs of vacancy creation there is little hope to separately identify the vacancy cost function from the meeting function.

contribution of firm type to value added is identified by the cross-sectional variation in value added per job. Finally, we can separate value added from home production since the decision to match or separate is tied to $p_t - b_t$, but only p_t determines value added.

4.3 Estimation Results

To obtain simulated time-series we begin with a distribution of workers across employment states and jobs implied by the economy in the absence of any aggregate shock. We then simulate the economy for 700 years at a weekly frequency, discard the first 100 years to reduce the impact of initial conditions, aggregate to quarterly data (described above) and calculate the simulated moments. The GMM objective function is non-smooth and displays many local minima. We use a variant of simulated annealing and many starting values to address these issues.

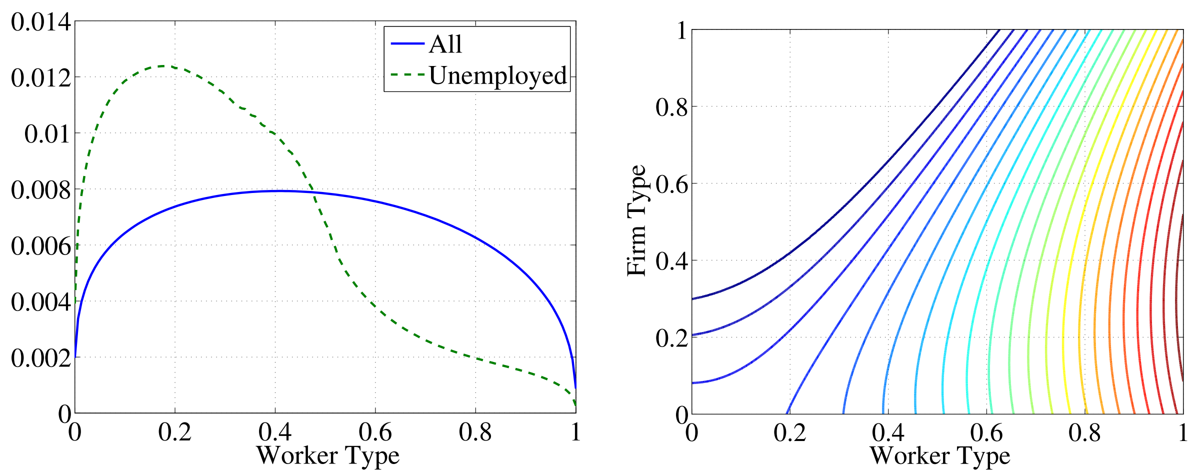
We first briefly discuss the parameter estimates in Table 1. Aggregate matching efficiency is estimated to be 1.894, with search on the job only 0.022 times as effective as unemployed search. The cost of posting a vacancy is slightly more convex than quadratic with the power term estimated to be 2.12. Job exogenously separate with a weekly probability of 0.007, or a quarterly probability of 0.09. The process for the weekly latent productivity shock has a variance of 0.049 and persistence of 0.999. This variance is higher than the quarterly variance in output, and substantially more persistent at a quarterly frequency, $0.987 = 0.999^{13}$ compared to the quarterly autocovariance for output of 0.943. The estimated distribution of worker types is presented in Figure 1(a).

The flow value of home production, $b(x, z)$, is estimated to be increasing in worker-type x. Similarly, market production, $p(x, y, z)$, is increasing in worker-type conditional on firm-type y. We can think of worker type and worker productivity as having a one-to-one mapping. At the same time market production is non-monotonic in firm-time y, conditional on worker type. In Figure 1(b) we plot the contour lines of $p_1(x, y) = p(x, y, z)/z$. The contour lines represent increasing output moving from left to right. For the most productive worker-type, output is maximized when matched with a firm with label 0.3. Moving to less productive workers the output maximizing firm-type also decreases, and the output maximizing firm for all workers below 0.4 has label 0.0. The fact that the output maximizing firm-type is estimated to be

Table 1: Parameter Estimates, Model I

Matching	α	1.894	Home production	b_0	0.553
$M = \min\{\alpha L^{\omega} V^{1-\omega}, L, V\}$	ω	0.500	$b(x,z) = b_0 + \exp(z)$	b_1	-0.095
Search intensity	s_1/s_0	0.022	$\times\left(b_1 x + b_2 x^2\right)$	b_2	4.688
Vacancy posting costs	c_0	0.055	Value added	p_1	0.612
$c[v(y)] = \frac{c_0}{1+c_1} v(y)^{1+c_1}$	c_1	1.120	$p(x,y,z) = \exp(z)$	p_2	-0.171
Exogenous separation	δ	0.007	$\times(p_1 + p_2 x + p_3 y$	p_3	-1.024
Productivity shocks	σ	0.049	$+p_4 x^2 + p_5 y^2 + p_6 xy)$	p_4	4.650
Gaussian copula (σ,ρ)	ρ	0.999		p_5	-2.995
Worker heterogeneity	β_1	1.105		p_6	3.093
Beta(β_1, β_2)	β_2	1.407	Interest rate	r	0.05

Note: r is fixed at 0.05 annually and ω is fixed at 0.5.



(a) Distribution of worker types

(b) Production function

Figure 1: Estimated distribution of worker types and production function

Notes: In Subplot (a) we also show the estimated distribution of worker types among the unemployed (averaged over all aggregate states). In Subplot (b) the contour lines are increasing from left to right. Conditional on firm-type y, $p(x,y)$ is increasing in worker-type x. However, conditional on worker-type x, $p(x,y)$ is non-monotonic in firm-type y.

increasing in worker-type, and that it is in interior implies that the equilibrium will display positive sorting. We consider this further, along with the cyclical patterns of worker-firm matches in Section 5.3.

The empirical and simulated moments are presented in Table 2 (columns labeled 'Data' and I). Overall the model fits the moments very well, the exceptions being that the model generates only 88 percent of the cross-sectional dispersion in value added per job, and the correlations of output with unemployment, vacancies and the job finding rate are somewhat stronger than in the data.

In the columns labelled II-VI we show the fit of several restricted versions of the model, designed to illustrate how identification works. In each case we re-estimate the model to find the best fit to the moments. Model II is identical to our base model with the exception that we assume all workers have identical home production and that home production is unrelated to the aggregate shock. This model fits all the moments as well as model I, with the exception of the cross-sectional productivity dispersion and the volatility of vacancy creation, where model II produces half the volatility of model I. By allowing home production to vary with worker type and the aggregate state the model can create more dispersion in value-added per job without altering the surplus which governs matching, since it is possible to hold the surplus constant and change market and home production in the same way.

Models III and IV examine the importance of worker and firm heterogeneity in matching the moments. Model III has no worker or firm heterogeneity, and by definition does not produce dispersion in value added across jobs. The model without heterogeneity is also unable to match the empirical pattern of unemployment by durations of 5, 15 and 27 plus weeks. Model IV has worker heterogeneity only, but still produces very little dispersion in value added per job (one tenth of model I) and implies a counterfactual positive correlation between this dispersion and aggregate value added.

Model V assumes that market production is additive in worker and firm type, and again produces a low degree of dispersion in value added per job. Finally, model VI assumes that value added is of the form $p(x, y, z) = xyz$. In this case all workers match will all firms and the model has difficultly matching the moments related to unemployment by duration, and the dispersions of value added per job. Models IV, V, and VI all also produce a very low correlation between unemployment and vacancies (a Beveridge curve that is too flat).

Table 2: Model fit and specification check

Fitted Moments	Data	I	II	III	IV	V	VI
$\mathbb{E}[U]$	0.0562	0.0568	0.0573	0.0541	0.0549	0.0614	0.0615
$\mathbb{E}[U^{5p}]$	0.0324	0.0339	0.0348	0.0294	0.0309	0.0320	0.0312
$\mathbb{E}[U^{15p}]$	0.0153	0.0148	0.0155	0.0090	0.0103	0.0091	0.0089
$\mathbb{E}[U^{27p}]$	0.0078	0.0064	0.0067	0.0023	0.0032	0.0024	0.0029
$\mathbb{E}[U2E]$	0.4376	0.4188	0.4090	0.4680	0.4465	0.4881	0.5109
$\mathbb{E}[E2U]$	0.0254	0.0244	0.0240	0.0262	0.0254	0.0314	0.0323
$\mathbb{E}[J2J]$	0.0273	0.0260	0.0311	0.0277	0.0276	0.0382	0.0231
$\mathbb{E}$[sd labor prod]	0.7478	0.6623	0.3537	na	0.0683	0.1856	0.0953
sd$[U]$	0.2140	0.2063	0.2126	0.1731	0.1633	0.1678	0.2098
sd$[U^{5p}]$	0.3138	0.2670	0.2791	0.2728	0.2197	0.2238	0.2898
sd$[U^{15p}]$	0.4435	0.3699	0.3979	0.4647	0.3615	0.3344	0.4435
sd$[U^{27p}]$	0.5388	0.4740	0.5332	0.6823	0.5429	0.4601	0.6356
sd$[U2E]$	0.1257	0.1509	0.1599	0.1400	0.1228	0.1130	0.1655
sd$[E2U]$	0.1291	0.1267	0.1300	0.0573	0.1033	0.1335	0.1374
sd$[J2J]$	0.0924	0.1069	0.1037	0.1899	0.1285	0.1984	0.1288
sd[sd labor prod]	0.0166	0.0082	0.0063	na	0.0042	0.0009	0.0087
sd$[V]$	0.2291	0.1860	0.1163	0.2349	0.2384	0.2260	0.1777
sd$[V/U]$	0.4162	0.3722	0.3157	0.3964	0.3223	0.3147	0.3185
sd[VA]	0.0363	0.0379	0.0389	0.0384	0.0379	0.0344	0.0354
autocorr[VA]	0.9427	0.9553	0.9557	0.8804	0.9254	0.7976	0.8754
corr$[V, U]$	-0.7642	-0.8005	-0.8272	-0.8846	-0.2614	-0.2608	-0.3463
corr$[U, \mathrm{VA}]$	-0.7742	-0.9406	-0.9528	-0.9778	-0.3586	-0.7664	-0.7380
corr$[V, \mathrm{VA}]$	0.6372	0.9159	0.8881	0.9477	0.9315	0.7690	0.8604
corr$[U2E, \mathrm{VA}]$	0.8143	0.9010	0.9360	0.9416	0.2102	0.4501	0.6420
corr$[E2U, \mathrm{VA}]$	-0.5984	-0.5169	-0.4455	-0.9226	-0.2932	-0.3915	-0.3132
corr$[U2E, J2J]$	0.6333	0.5526	0.5494	0.9974	0.2857	0.5842	0.4270
corr[sd labor prod, VA]	-0.3902	-0.4552	-0.3910	na	0.7465	-0.2184	-0.2690

Note: The data used to construct the moments is 1951q1-2007q4. Model (I) baseline model; (II) home production is independent of worker type and aggregate state $b(x, z) = b$; (III) no worker or firm heterogeneity; (IV) only worker heterogeneity; (V) has no production complementarities: $p_{xy} = 0$; (VI) has production of the form $p(x, y, z) = xyz$.

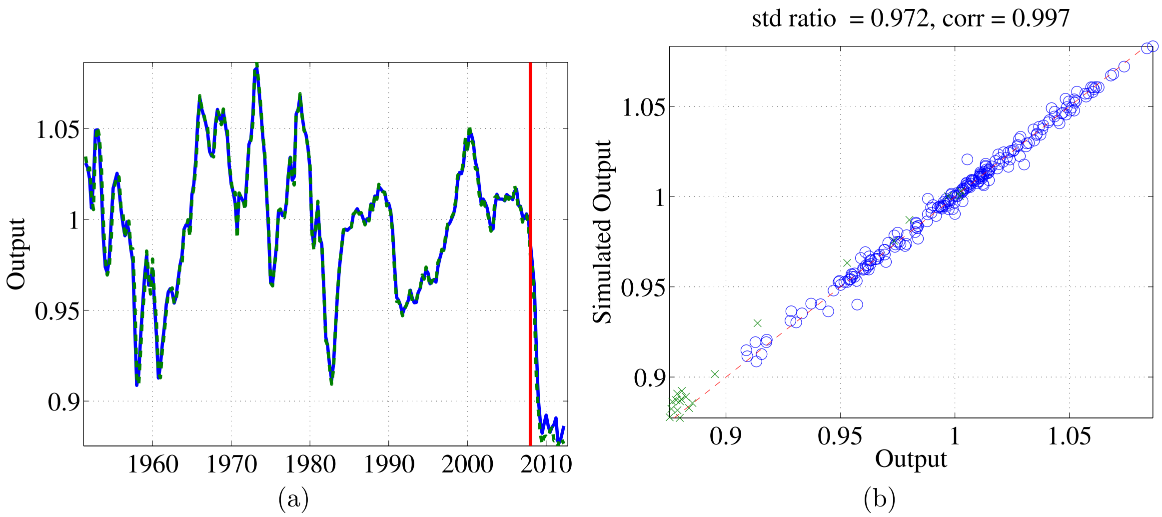


Figure 2: Estimating the latent process for z_t

Notes: In panel (a) the dashed lines is the filtered data while the solid line is the model simulated data. The vertical line indicates 2008q1. In panel (b) data from 1951q1 to 2007q4 is marked with $\circ$ and the data from 2008q1 to 2012q4 is marked by $\times$. The simulated data is produced by choosing the latent shock process z_t and the scale of output p_0 to match the time-series of output and the mean unemployment rate for 1951-2012. The ratio of standard deviations, model simulated over the data, and the correlation between the log of model simulated and data series are presented above panel (b).

In summary, in term of matching the moments of Table 2, column II indicates that heterogeneity in home production is important; columns III and IV indicate that two sided heterogeneity is important; and columns V and VI indicates that it is important to have a production technology in which there is an optimal worker for each job type.

4.3.1 Model fit 1951-2007 and prediction for 2008-2012

The model was estimated to fit the moments calculated on the 1951-2007 data. In Figure 2(a) we plot the time series for output, actual and model predicted, for 1951-2012. In Figure 2(b) we plot model predicted against actual output data, along with the 45 degree line to highlight the fit. In order to calculate the model predicted series we require an estimate of the realized process for the aggregate shock z_t. To calculate this we find the series of weekly shocks z_t that best matches the quarterly series for output, and adjust the scale of output (by adjusting p_0) to match the mean of output and unemployment for the 1951-2012 series. This second adjustment is done to ensure we match the mean when including the additional data from 2008-12 that was not used in estimation. This procedure results in an excellent fit to the output series (the ratio of standard deviations of the predicted to actual series is 0.974 and the correlation is 0.997). The small deviations from the 45 degree line in Figure 2(b) result from the combination of the discrete grids for x, y, and z (392,751 points) and requirement to match mean unemployment and output.[10]

In Figure 3 we plot the US time-series data for 1951-2012, along with the model predicted series for (a) unemployment, (b) vacancies, (c) unemployment to employment transition rate, (d) the employment to unemployment transition rate, (e) the standard deviation of value added per worker across firm types, and (f) unemployment of 27 weeks or longer.[11] In Figure 4 we present scatterplots of the model predicted data against the actual data, with the the 45 degree line to highlight model fit.

Turning to the unemployment series, the fit over the 1951-2007 period is very good, although the model is not able to match the very low periods of unemployment in the late 1960s. The dramatic drop in output during the 2008-12 recession is associated with a large spike in unemployment and the model predicts this spike well, although with somewhat higher than observed unemployment. The fact that the model predicts the correct magnitude for the rise in unemployment associated with the drop in output indicates that the model is doing well to capture the structure of this relationship, especially since these spikes are well outside the range of the 1951-2007 data used in estimation.

Looking at the fit to the series of employment-to-unemployment transitions we note several inflexibilities of the model on the job destruction front. First, while the model-predicted series rises and falls in line with the data, the model produces spikes in job destruction rather than sustained increases. In the model, a negative aggregate shock will cause a jump in the number of jobs destroyed due to negative surplus,

[10]Note that any method aiming at estimating structural parameters and filtering aggregate shocks simultaneously by fitting all series together in one single step (like an EM algorithm or particle filtering) would be clearly much more complicated to implement, and considerably slower. Our approach uses a simulation-based (or indirect inference) method to 'calibrate' structural parameters in a first step. Aggregate shocks are filtered out to maximize the fit in a second step.

[11]Recall that we removed a quadratic trend from the log of each series calculated for the 1951-2007 period. We then removed this same trend from the 2008-12 data. This procedure results in unemployment rates during the recession that are above the raw data. We experimented with several methods for filtering out the longer term trends in the data and settled on removing the quadratic as it allows us to consistently predict a de-trended series for the 2008-12 period.

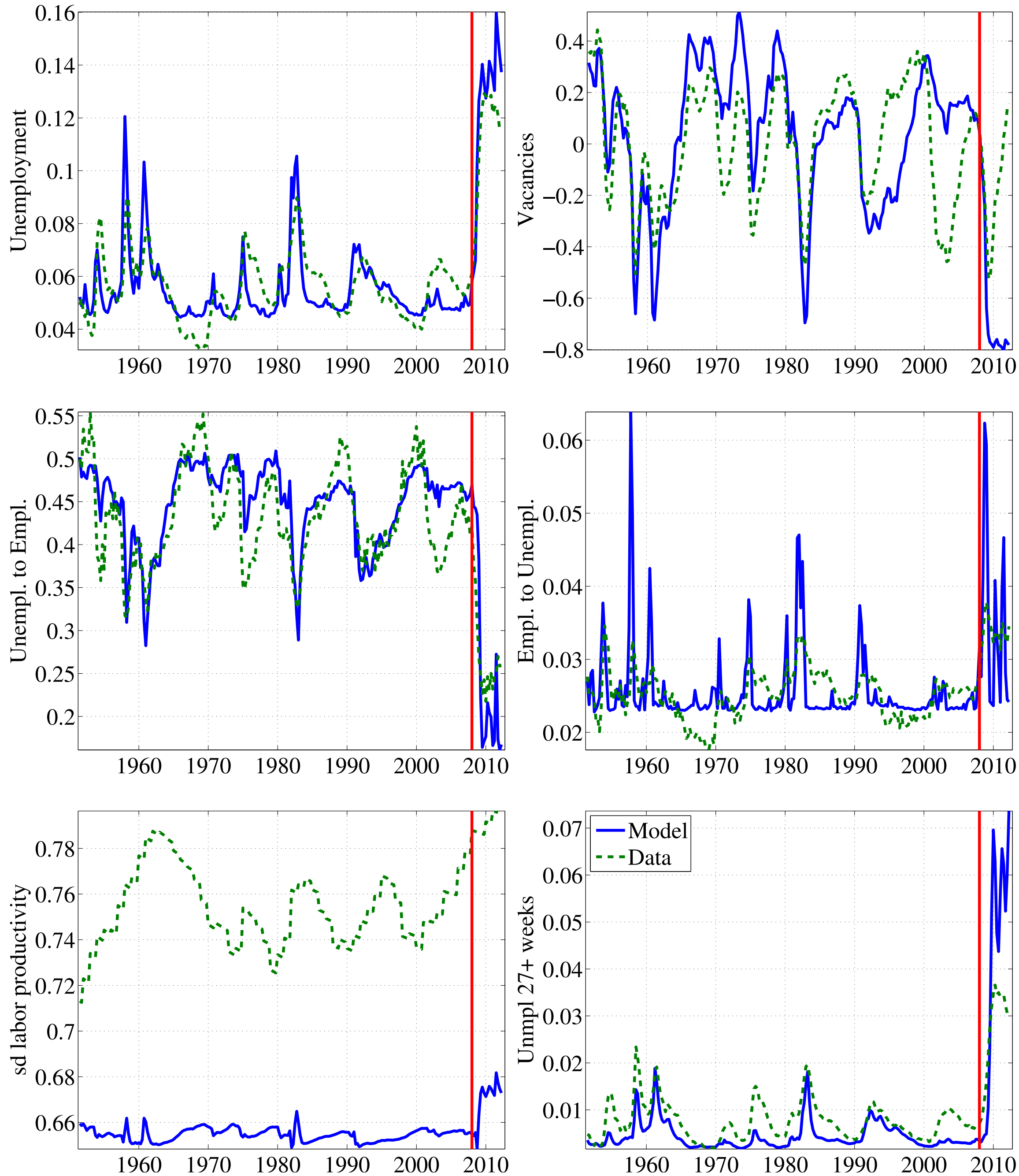


Figure 3: Data and Model Predicted Time Series

Notes: The dashed lines is the filtered data while the solid line is the model simulated data. The vertical line indicates 2008q1. The simulated data is produced by choosing the latent shock process z_t and the scale of output p_0 to match the time-series of output and the mean unemployment rate for 1951-2012.

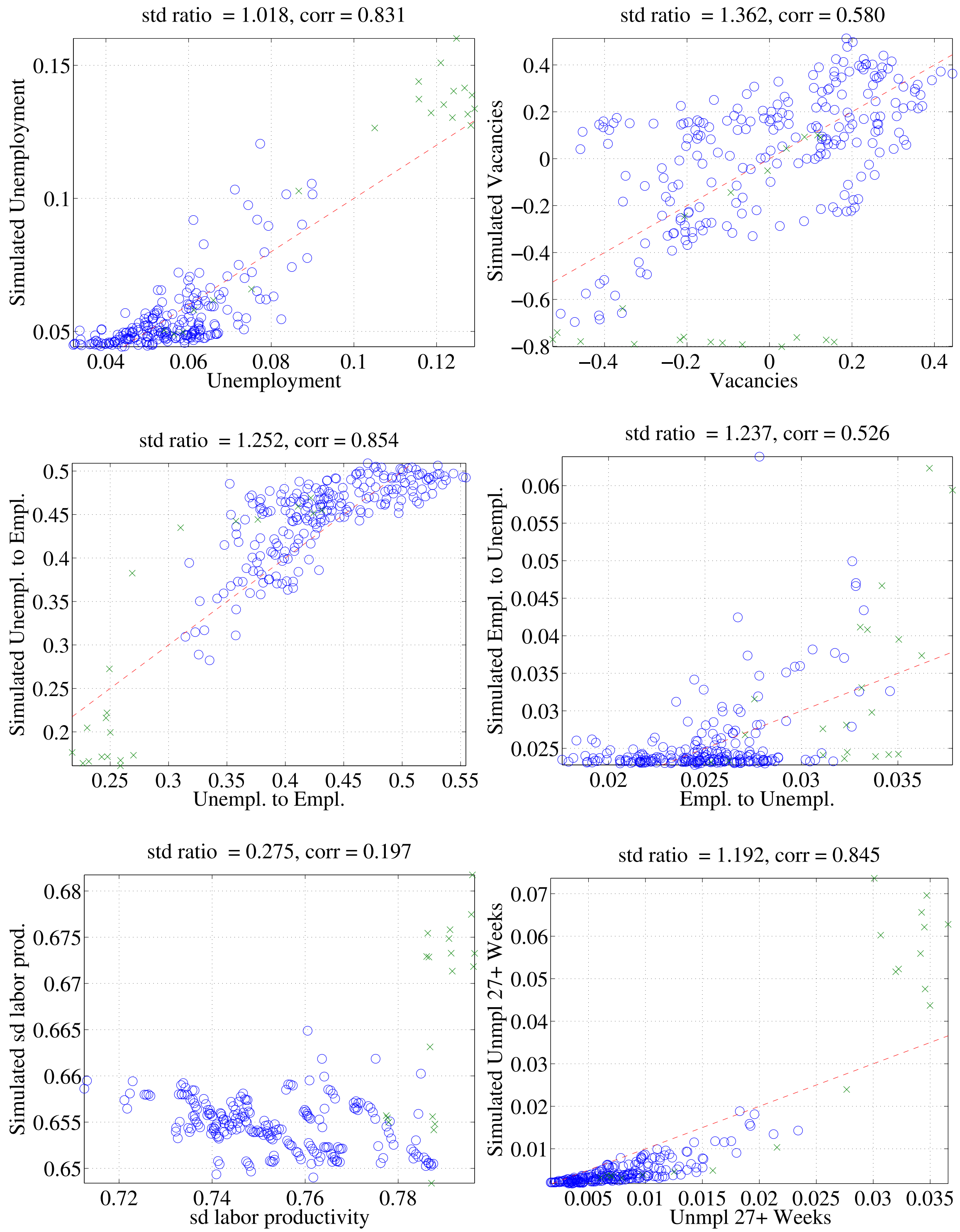


Figure 4: Data and Model Predicted Series

Note: Data from 1951q1 to 2007q4 is marked with $\circ$ and the data from 2008q1 to 2012q4 is marked by $\times$. The ratio of standard deviations, model simulated over the data, and the correlation between the log of model simulated and data series are presented above each sub figure.

however, unless the shock continues downward this is a one period effect and the destruction rate will soon return to the level of the exogenous rate δ. Second, the exogenous destruction rate δ puts a lower bound on the employment-to-unemployment flows, which make the model unable to reproduce the very low separation rates during the late 1960s, which explains the inability to match the very low unemployment during this period. Third, the model associates a sustained increase in the job destruction rate in the last recession with a much greater volatility than in the data. Small variations of aggregate output, when aggregate output remains exceptionally low for such a long time, generate large, jagged fluctuations in job destruction dynamics.

The model does better with matching the job finding rate, producing a much smoother series for transition from unemployment to employment than the other direction. The model somewhat over-predicts the drop in the job finding rate. The model is able to attain good fit to the unemployment rate during the last recession but this arises from an under prediction of the job losing rate (after the initial spike) and an over prediction of the decline in the job finding rate. The implication of this excess decline in the job finding rate shows up when we look at the predicted series for longterm unemployed (27 or more weeks). A notable feature of the 2008-12 recession is the large increase in the fraction of workers who were unemployed for 27 weeks or more. In our data series this share topped out at 3.5 percent of workers during this period, close to twice the historic high over the 1951-2007 period. The increase in the number of long-term unemployed predicted by the model is about twice as large as the actual spike. This is the result of the model over predicting the fall in the unemployment-to-employment transition rate.

The predicted fit of the model to the data for vacancies is mixed. The predicted series fits very well for the period up to the mid 2000s, then begins to break down. It is worth noting that the fit begins to deteriorate exactly at the time the vacancy series switches from the help wanted index to the JOLTS, which may be at least part of the explanation. A puzzling feature of the vacancy series is that after an initial drop in 2008, vacancies rise continuously though 2012, a feature that is clearly at odds with the model prediction of continued low level of vacancies, and implied low job finding rates.

Finally, the fit of cross-sectional dispersion of firm labor productivity is not good. The model predicts a much smaller volatility. It does however predict a sustained increased cross-sectional dispersion of the value added per job after 2008 although

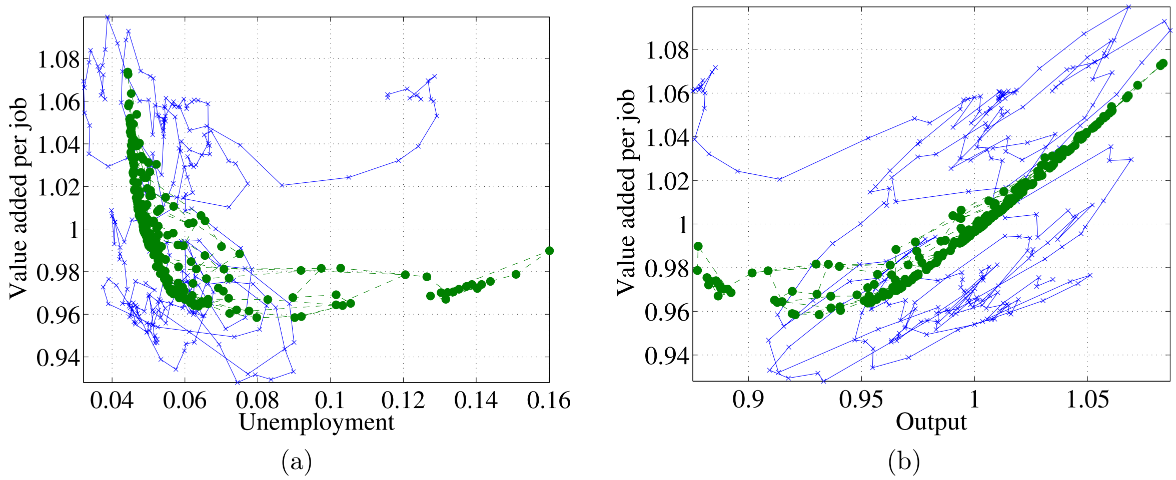


Figure 5: Labor productivity versus unemployment and output

Note: The data are marked with $\times$ and the model simulation with $\bullet$. Model simulations correspond to data in Figure 3.

in the data a steady increase started in 2000, which was not affected by the recent financial crisis.

One natural motivation for introducing worker and firm heterogeneity into a model is that the composition of workers and active jobs at different points of the cycle will have implications for measured labor productivity. Indeed, in such a model labor productivity could actually increase in a downturn as the least productive (low surplus) matches are terminated and only the most productive remain. In Figure 5 we plot the labor productivity (value added per job) against aggregate output and against the unemployment rate, for both the US time-series and our model predicted data. For most of the period there is a strong negative relationship between labor productivity and the unemployment rate. However, during the 2007-12 recession as unemployment became very high, labor productivity remained flat (or possibly rose). The model also produces this pattern. When unemployment is low or moderate (between 4 and 8 percent) there is a strong negative relationship between unemployment and labor productivity. However, when unemployment rises above 8 percent labor productivity flattens out (and tips up slightly), as only the worst matches separate. The same pattern is apparent when looking at the relationship between labor productivity and output. Again, at very low levels of aggregate output labor productivity flattens out (and turns mildly upward). One notable difference between the patterns in the data and those produced by the model is that the level of labor productivity during the 2008-12 period is notably higher than the model predicts. There is also substantially more variation in the data than the model predicts, suggesting there may be some medium term fluctuations in parameters we are treating as constant in the model.

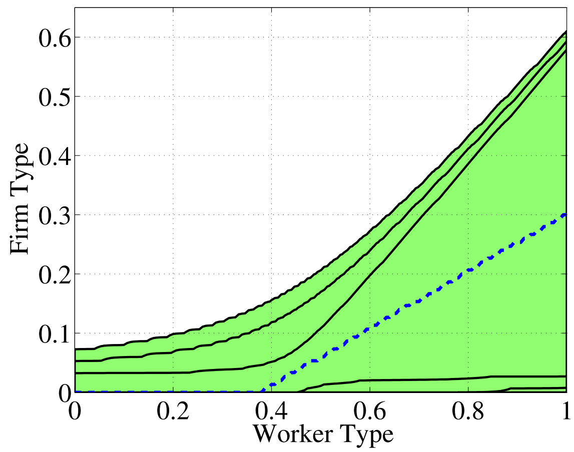
5 Discussion and Interpretation of Results

5.1 The interaction of heterogeneity and aggregate shocks.

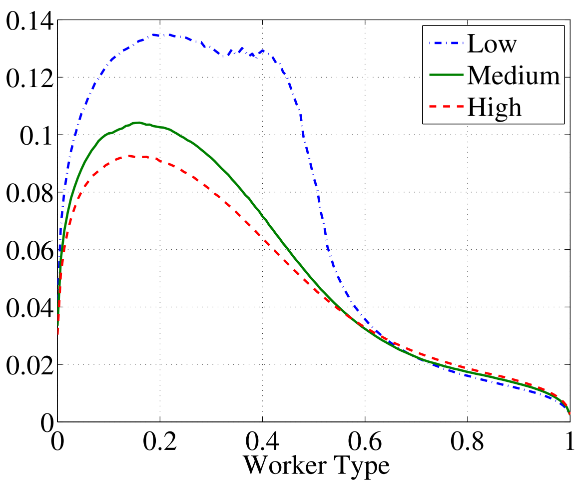
The mechanism for how the model combines two sided heterogeneity and aggregate shocks to produce the moments in Table 2 and the series in Figures 2, 3 and 5 can be readily understood by examining the shape of the estimated surplus function. In Figure 6(a) we plot the set of feasible matches for the 10th, 50th and 90th percentiles of the distribution of the aggregate shock. The shaded area bounded by the solid line represents all matches which are feasible when the aggregate shock is high (at the 90th percentile). The two solid lines inside the shaded area represent the boundaries of the feasible set of matches when the aggregate shock is at the 50th and 10th percentiles. If the aggregate state moved from the 90th to the 10th percentile, all matches outside the new bounds would immediately separate. Additionally, the set of jobs that are feasible for unemployed workers shrinks, lowering the job finding rate, especially so for low and medium type workers. When the aggregate state improves, the set of feasible matches expands substantially for the low- and especially the medium-types workers and they flow from unemployment to employment at an increased rate.

The dotted line in panel (a) represents the ideal firm type for each worker type and is calculated as $y(x, \overline{z}) = \arg\max_y S(x, y, \overline{z})$, where $\overline{z}$ is the mean aggregate state.[12] Unemployed workers initially accept any job in the feasible set (conditional on the aggregate state), and through the process of on-the-job search gradually move toward the dotted line. This process has two interesting implications for identifying which workers are most affected by aggregate fluctuations. First, the set of feasible matches for workers of low- and medium-type fluctuates substantially with the aggregate state relative to the highest type workers; the employment opportunities of low- and medium-skilled workers is much more cyclically-sensitive than the highly skilled.

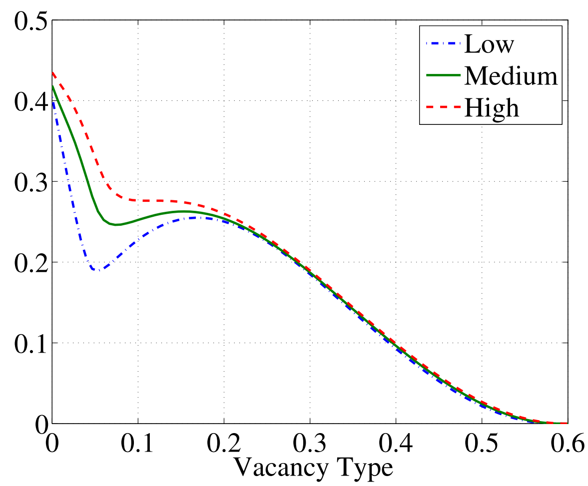
[12]It happens that $y(x, z)$ varies little with z and is close to the value of y that maximizes $p(x, y, z)$, which is indeed independent of z because of the multiplicative specification.



(a) Matching sets and optimal matches



(b) $u_t(x)$



(c) $v_t(y)$

Figure 6: Feasible matches, and the distribution of unemployment and vacancies across the cycle

Notes: The solid lines in panel (a) mark the boundaries for the set of feasible matches when the aggregate shock is at the 90th, 50th and 10th percentile (moving from outer to inner lines). The dotted line plots the optimal firm type for each worker type $y(x, \overline{z}) = \arg\max S(x, y, \overline{z})$, where $\overline{z}$ is the mean aggregate state. In panels (b) and (c) Low, Medium and High refer to periods when output is in the bottom, middle and top third of long-run distribution.

Second, as employed workers receive offers from other firms they will move toward their preferred firm, which will be far from the boundary of the feasible matching set, and hence protected from the effect of aggregate shocks; workers with short employment tenure are more cyclically-sensitive than workers with long tenures who have enjoyed a sustained period of continuous employment. Note that the model requires both worker and firm heterogeneity to produce these implications.

In panel (b) we plot the estimated distribution of worker types among the unemployed (averaged over all aggregate states). The medium- and low-skilled are clearly over represented among the unemployed. Moreover, when output is either medium or high, the shape of the distribution of unemployed worker types is similar. However, when output is very low, workers in the middle experience disproportionately high increases in their representation among the unemployed. Again, the surplus for the middle type workers is the most cyclically sensitive.

How do the types of vacancies posted and the composition of the unemployed change with the aggregate state of the economy? In panel (c) we plot the model predicted expected number of posted vacancies by firm type, $v(y)$, when output is low, medium or high. Moving from a boom to a recession, the number of vacancies posted by firms above type 0.25 barely changes. All the action is at the lower end, especially by firm in the 0.05 to 0.1 range. We can see from Figure 6(a) that these are the jobs that maximize the surplus for worker around the middle of the type distribution.

5.2 The value of non-market time, sorting, and firms' share of the surplus

In Table 3 we present, for each of the six specifications of Table 2, statistics summarizing the estimated value of home production, the degree of sorting in the economy, and the share of the match surplus that goes to the firms in new matches. We measure the value of home production as the ratio of what the worker can produce at home relative to the maximum she can produce when matched to her preferred firm, evaluated at the mean aggregate state $\overline{z}$. For our best fitting model, the mean of this measure is 0.956, with a range of 0.904 to 0.980. The mean is almost identical to both the value we obtain when we assume no heterogeneity and the value used by Hagedorn and Manovskii (2008). Indeed, most of our specifications imply that

Table 3: Relative value of home production, sorting, and firms' share of match surplus

		I	II	III	IV	V	VI
$b(x,\overline{z})/p(x,y(x),\overline{z})$	mean	0.9564	0.8350	0.9631	0.9009	0.9925	0.8665
	min	0.9040	0.1780	0.9631	0.5425	0.7204	0.3600
	max	0.9803	0.9585	0.9631	0.9726	0.9986	0.9761
$\text{corr}(x,y)$		0.736	0.709	na	na	-0.112	0.000
Firm share of surplus at matching		0.274	0.372	0.558	0.094	0.126	0.126

Notes: $b(x,\overline{z})/p(x,y(x),\overline{z})$ is home production relative the maximum market production at the mean aggregate shock $\overline{z}$. The correlation between worker and firm type, $\text{corr}(x,y)$, is measured using the distribution implied when the aggregate shock remains at its mean for a long time. The firm share of surplus at matching is calculated as the average share of the match surplus going to the firm among newly created matches (formed with either unemployed or employed workers).

the value of home production is quite close to the value of market production on average. However, when we fix the value of home production to be independent of worker type and the aggregate shock (model II) our mean estimate drops to 0.835, with a minimum of 0.178 and maximum of 0.959, covering the range of values used by Shimer (2005), Hall (2005) and Hagedorn and Manovskii (2008).

As an initial indication of the degree of sorting implied by the various models we present the correlation between worker and firm type among productive matches. The distribution of matches used is the long run distribution implied by holding the aggregate shock at the mean $\overline{z}$. The two specifications that provide the best fit to the moments (I and II) both imply a strong positive correlation, 0.74 and 0.71. Sorting is not defined in the two models without two-sided heterogeneity. We obtain negative sorting from the model without complementarity in production, and zero sorting in the model with xyz production where all matches are feasible and workers always move from lower to higher type firms.

Our assumption on wage contracts implies that workers are always offered their reservation wage. The implication is that when a firm hires a worker out of unemployment it receives the entire match surplus. However, when it hires a worker who is already employed by another firm it only receives the share of the surplus in excess of what is necessary to poach the worker. Since there are many more employed than unemployed workers, firms expect to receive a share well below one from newly formed matches. In our best fitting specifications this share is 0.27. The aggregate behavior of the model is that on average firms' bargaining position grants them just under 30 percent of the initial match surplus, although this is the result of averaging over very heterogeneous bargains with individual workers. All of our specifications imply that on average the surplus share of new matches which goes to firms is below 0.56.

5.3 The cyclicality of mismatch

Mismatch is a key feature of our model economy. Employed workers initially accept any job that produces a positive surplus and then continue to search for a match that produces a higher surplus. Even with on-the-job search relatively few workers will be employed at their surplus maximizing job at any given point in time. We define mismatch in our economy as the average difference in surplus between a worker's current match and her most preferred (surplus maximizing) match. Let mm_t^A be the average absolute difference at time t and mm_t^R be the average relative difference:

$$mm_t^A = \frac{1}{H_t^j} \int \left[S_t(x, y(x)) - S_t(x, y)\right] h_t^j(x, y)\, \mathrm{d}x\, \mathrm{d}y,$$
$$mm_t^R = \frac{1}{H_t^j} \int \left[\frac{S_t(x, y(x)) - S_t(x, y)}{S_t(x, y(x))}\right] h_t^j(x, y)\, \mathrm{d}x\, \mathrm{d}y,$$

where $h_t^j(x, y)$ for $j \in \{0, 1\}$ is either the distribution of workers across jobs who were just hired out of unemployment,

$$h_t^0(x, y) = u_{t+}(x)\lambda_{0,t}\frac{v_t(y)}{V_t}\mathbf{1}\{S_t(x, y) \geq 0\},$$

or the distribution of workers across jobs who were already employed in the previous period,

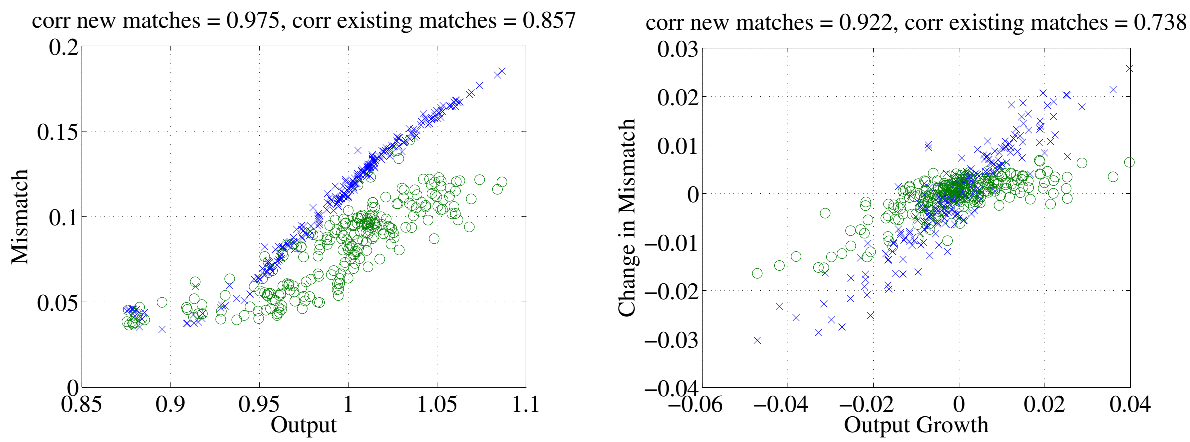
$$h_t^1(x, y) = h_{t+}(x, y)\left[1 - \int \lambda_{1,t}\frac{v_t(y')}{V_t}\mathbf{1}\{S_t(x, y') > S_t(x, y)\}\, \mathrm{d}y'\right]$$
$$+ \int h_{t+}(x, y')\lambda_{1,t}\frac{v_t(y)}{V_t}\mathbf{1}\{S_t(x, y) > S_t(x, y')\}\, \mathrm{d}y'.$$

The absolute measure of mismatch weights the degree of mismatch by current maximum match surplus. This gives greater weight to workers who have higher potential surplus, and also gives greater weight to mismatch when the aggregate state is high. This measure captures the idea that the return to optimally allocating workers across jobs differs both by worker type and by the current aggregate productivity level. The relative measure of mismatch calculates the surplus loss for each worker relative to her maximum surplus in the current period. This measure removes differences in the level of the surplus, across workers and across aggregate states, focussing only on the allocation of workers across jobs (not on which workers have a higher gain to being optimally allocated or in which aggregate states the gain is greatest).

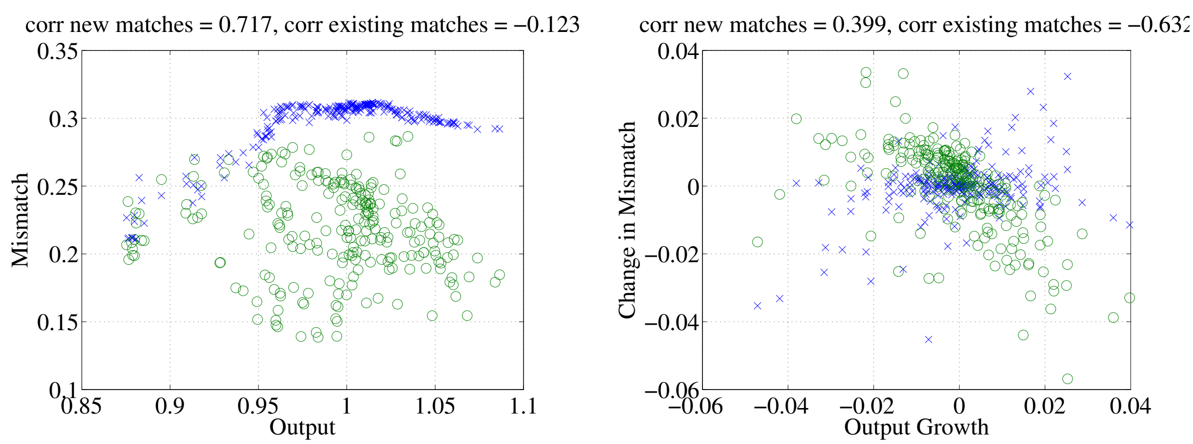
In Figure 7 we plot these measures of mismatch corresponding to the model simulation in Figure 2. Several patterns are worth commenting on. The amount of mismatch in newly formed matches is (almost) always greater than in pre-existing matches using either absolute or relative mismatch. Absolute mismatch increases with output both for workers hired from unemployment, and for workers who were already employed in the previous period. The correlation between absolute mismatch and output is stronger for the previously unemployed, 0.975, than the previously employed, 0.857. The same pattern exists if we look at the correlation between output growth and the change in absolute mismatch.

Looking at relative mismatch, we see a contrasting pattern for workers hired from unemployment and workers employed in the previous period. The correlation between relative mismatch and output for previously unemployed workers is still positive, 0.717, but the correlation for workers already employed in the previous period is negative, -0.123. This pattern is also there when looking at output growth and changes in relative mismatch, the correlation for the previously unemployed is 0.399 while for the previous employed it is -0.632. From the point of view of individual workers, when the economy is booming the jobs accepted by unemployed workers tend to be further from the workers' surplus maximizing job, while at the same time employed workers move more quickly to their optimal jobs. During recessions, mismatch for previously unemployed workers declines, since only very good quality worker-firm matches are viable due to the low aggregate state. At the same time, the reduction in the number of vacancies created combined with the increased number of unemployed searchers means that employed workers do not move as quickly to their surplus maximizing matches. Barlevy (2002) refers to this as the "sullying effect of recessions".[13] When viewed from the aggregate, or from the point of view of a social planner, average mismatch for both workers newly hired from unemployment

[13]In Barlevy's model, since workers and firms are homogeneous with respect to potential output the social return to reallocation is the same for all types.



(a) Absolute Mismatch



(b) Relative Mismatch

Figure 7: Cyclical Mismatch

Note: The values marked by $\times$ are for the distribution of worker-job pairs where the worker was hired out of unemployment. The values marked by $\circ$ are for worker-job pairs in which the worker was employed in the previous period.

and those who were already employed increases with aggregate output. The economy wide gain to reallocating workers toward their surplus maximizing jobs is higher in booms than in recessions.

6 Conclusions

We develop and estimate an equilibrium labor search model with aggregate uncertainty and ex-ante heterogeneity in worker and firm types. We show that the model has a recursive structure which makes it very tractable. We estimate the model and assess the fit to time series data for the US from 1951 to 2012. Overall the model does a good job capturing the main properties of this data. We then simulate the model to measure the cyclical properties of the distributions of vacancies, unemployed workers, and the degree of mismatch among worker-firm pairs. We find that the medium skilled workers are most cyclically sensitive in terms of their representation among the unemployed. We find that mismatch between jobs and workers hired from unemployment is pro-cyclical. For workers who were already employed the previous period, absolute mismatch is pro-cyclical, but relative mismatch is counter cyclical. Workers move more quickly to better matches in booms, while at the same time the economy wide gain from further reallocation is even higher in booms than in recessions.

APPENDIX

A Derivation of the Surplus Equation

The Value of Employment

Consider a worker of type-x who is employed at a firm of type-y with a wage contract paying w for the period t. During the period she earns the wage w and expects that at the beginning of period $t+1$ after realization of the new aggregate state she may become unemployed or remain employed, with the possibility of being contacted by another firm of type y', which happens with probability $\lambda_{1,t+1}\frac{q_{t+1}v_{t+1}(y)}{M_{t+1}}$. The value of this employment to the worker is

$$
\begin{aligned}
W_t(w,x,y) = w + \frac{1}{1+r}\mathbb{E}_t\Bigg[&\left[\mathbf{1}\{S_{t+1}(x,y)<0\} + \delta\mathbf{1}\{S_{t+1}(x,y)\geq 0\}\right]B_{t+1}(x) \\
&+ (1-\delta)\mathbf{1}\{S_{t+1}(x,y)\geq 0\}\Bigg[\lambda_{1,t+1}\int_{y'\in\mathcal{M}_{1,t+1}(x,y)} W_{t+1}(\phi_{1,t+1}(x,y',y),x,y')\frac{q_{t+1}v_{t+1}(y')}{M_{t+1}}\,\mathrm{d}y' \\
&+ \lambda_{1,t+1}\int_{y'\in\mathcal{M}_{2,t+1}(w,x,y)} W_{t+1}(\phi_{1,t+1}(x,y,y'),x,y)\frac{q_{t+1}v_{t+1}(y')}{M_{t+1}}\,\mathrm{d}y' \\
&+ \left(1-\lambda_{1,t+1}\int_{y'\in\mathcal{M}_{1,t+1}(x,y)\cup\mathcal{M}_{2,t+1}(w,x,y)}\frac{q_{t+1}v_{t+1}(y')}{M_{t+1}}\,\mathrm{d}y'\right) \\
&\times\min\left\{W_{t+1}(w,x,y),\max\left\{S_{t+1}(x,y)+B_{t+1}(x),B_{t+1}(x)\right\}\right\}\Bigg]\Bigg].
\end{aligned}
$$

Where we partition the set of potential firms a worker may meet into those that will trigger a job change $\mathcal{M}_{1,t}(x,y)\equiv\{y'|S_t(x,y')>S_t(x,y)\}$; those that will trigger a wage renegotiation, $\mathcal{M}_{2,t}(w,x,y)\equiv\{y'|W_t(w,x,y)-B_t(x)<S_t(x,y')<S_t(x,y)$; and those that will trigger neither a job change nor a wage renegotiation, $[\underline{y},\overline{y}]\setminus\mathcal{M}_{1,t+1}(x,y)\cup\mathcal{M}_{2,t+1}(w,x,y)$. In this last instance the contract may still have to be renegotiated if the shock z_{t+1} moves the current wage w out of the new bargaining set $[\phi_{0,t+1}(x,y),\phi_{1,t+1}(x,y,y)]$.

Making use of the wage determination equations (1) and (2) we can write the surplus to a worker of type x, matched with a firm of type y and currently earning a wage w as

$$
\begin{aligned}
&W_t(w,x,y) - B_t(x) = w - b(x,z) \\
&\quad + \frac{1}{1+r}\mathbb{E}_t\Bigg[(1-\delta)\mathbf{1}\{S_{t+1}(x,y) \geq 0\}\Bigg[\lambda_{1,t+1}\int_{y' \in \mathcal{M}_{1,t+1}(x,y)} S_{t+1}(x,y)\frac{q_{t+1}v_{t+1}(y')}{M_{t+1}}\,\mathrm{d}y' \\
&\qquad + \lambda_{1,t+1}\int_{y' \in \mathcal{M}_{2,t+1}(w,x,y)} S_{t+1}(x,y')\frac{q_{t+1}v_{t+1}(y')}{M_{t+1}}\,\mathrm{d}y' \\
&\qquad + \left(1 - \lambda_{1,t+1}\int_{y' \in \mathcal{M}_{1,t+1}(x,y) \cup \mathcal{M}_{2,t+1}(w,x,y)} \frac{q_{t+1}v_{t+1}(y')}{M_{t+1}}\,\mathrm{d}y'\right) \\
&\qquad\qquad \times \min\left\{\left[W_{t+1}(w,x,y) - B_{t+1}(x)\right]^+, S_{t+1}(x,y)\right\}\Bigg]\Bigg],
\end{aligned}
\tag{12}
$$

where we denote $x^+ = \max\{x, 0\}$.

The Value of a Filled Job

Consider a firm of type y who is currently matched with a worker of type x, paying a wage w for the period t. Next period, the firm may be left without a worker if the match is terminated, either for endogenous or exogenous reasons, or the worker is poached by another firm. Even if the worker does not leave, the firm may need to counter an outside offer to retain the worker. The value to the firm of this match is given by

$$
\begin{aligned}
&\Pi_t(w,x,y) = p(x,y,z) - w + \frac{1}{1+r}\mathbb{E}_t\Bigg[(1-\delta)\mathbf{1}\{S_{t+1}(x,y) \geq 0\} \\
&\quad \times \Bigg[\lambda_{1,t+1}\int_{y' \in \mathcal{M}_{2,t+1}(w,x,y)} \Pi_{t+1}(\phi_{1,t+1}(x,y,y'),x,y)\frac{q_{t+1}v_{t+1}(y')}{M_{t+1}}\,\mathrm{d}y' \\
&\qquad + \left(1 - \lambda_{1,t+1}\int_{y' \in \mathcal{M}_{1,t+1}(x,y) \cup \mathcal{M}_{2,t+1}(w,x,y)} \frac{q_{t+1}v_{t+1}(y')}{M_{t+1}}\,\mathrm{d}y'\right) \\
&\qquad\qquad \times \min\left\{\Pi_{t+1}(w,x,y)^+, S_{t+1}(x,y)\right\}\Bigg]\Bigg].
\end{aligned}
$$

Evaluating Π_{t+1} in the second line at the renegotiation wage $\phi_{1,t+1}(x,y,y')$ we have

$$\begin{aligned}\Pi_t(w,x,y) = p(x,y,z) - w + \frac{1}{1+r}\mathbb{E}_t\Bigg[&(1-\delta)\mathbf{1}\{S_{t+1}(x,y)\geq 0\} \\ &\times\Bigg[\lambda_{1,t+1}\int_{y'\in\mathcal{M}_{2,t+1}(w,x,y)}\big[S_{t+1}(x,y)-S_{t+1}(x,y')\big]\,\frac{q_{t+1}v_{t+1}(y')}{M_{t+1}}\,\mathrm{d}y' \\ &+\Bigg(1-\lambda_{1,t+1}\int_{y'\in\mathcal{M}_{1,t+1}(x,y)\cup\mathcal{M}_{2,t+1}(w,x,y)}\frac{q_{t+1}v_{t+1}(y')}{M_{t+1}}\,\mathrm{d}y'\Bigg) \\ &\times\min\big\{\Pi_{t+1}(w,x,y)^+, S_{t+1}(x,y)\big\}\Bigg]\Bigg]. \end{aligned} \tag{13}$$

The Surplus

Finally, we can combine (12) and (13) to form the total match surplus

$$\begin{aligned}S_t(x,y) &= W_t(w,x,y) - B_t(x) + \Pi_t(w,x,y) = p(x,y,z) - b(x,z) \\ &+\frac{1}{1+r}\mathbb{E}_t\Bigg[(1-\delta)\mathbf{1}\{S_{t+1}(x,y)\geq 0\}\Bigg[\lambda_{1,t+1}\int_{y'\in\mathcal{M}_{1,t+1}(x,y)}S_{t+1}(x,y)\frac{q_{t+1}v_{t+1}(y')}{M_{t+1}}\,\mathrm{d}y' \\ &+\lambda_{1,t+1}\int_{y'\in\mathcal{M}_{2,t+1}(x,y)}S_{t+1}(x,y)\frac{q_{t+1}v_{t+1}(y')}{M_{t+1}}\,\mathrm{d}y' \\ &+\Bigg(1-\lambda_{1,t+1}\int_{y'\in\mathcal{M}_{1,t+1}(x,y)\cup\mathcal{M}_{2,t+1}(w,x,y)}\frac{q_{t+1}v_{t+1}(y')}{M_{t+1}}\,\mathrm{d}y'\Bigg)S_{t+1}(x,y)\Bigg]\Bigg],\end{aligned}$$

which simplifies to

$$S_t(x,y) = p(x,y,z) - b(x,z) + \frac{1-\delta}{1+r}\mathbb{E}_t\max\{S_{t+1}(x,y),0\}.$$

We can directly verify the guess that $S_t(x,y) = S(x,y,z)$ such that

$$S(x,y,z) = s(x,y,z) + \frac{1-\delta}{1+r}\int S(x,y,z')^+\pi(z,z')\,\mathrm{d}z',$$

where $s(x,y,z) = p(x,y,z) - b(x,z)$.

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B Web Appendix (not for publication)

General Meeting Technology

In the new aggregate state of period t a measure $u_{t+}(x)$ of workers of type x are unemployed and a measure $h_{t+}(x,y)$ are employed at firms of type y. Together they produce effective search effort

$$L_t \equiv f[u_{t+}, h_{t+}].$$

Firms observe the new aggregate state and decide to post a distribution $v_t(y)$ of vacancies with aggregator

$$V_t \equiv g[v_t].$$

The aggregating functions f and g are linearly homogeneous functions. The total measure of meeting at time t is given by

$$M_t \equiv M(L_t, V_t),$$

where $M(L_t, V_t)$ is strictly increasing and concave in worker search L_t and vacancy creation V_t, and displays constant returns to scale in L_t and V_t.

Define $\lambda_{0,t}$ as the probability an unemployed searcher contacts a vacancy and $\lambda_{1,t}$ as the probability an employed searcher contacts a vacancy in period t. Let q_t be the probability per unit of recruiting effort $v_t(y)$ that a firm contacts any searching worker. We connect these meeting rates to the meeting function as

$$\begin{aligned}
\frac{q_t v_t(y)}{M_t} &\equiv \frac{\partial \log g[v_t]}{\partial \log v_t(y)}, \\
\frac{\lambda_{0,t} u_{t+}(x)}{M_t} &= \frac{\partial \log f[u_{t+}, h_{t+}]}{\partial \log u_{t+}(x)}, \\
\frac{\lambda_{1,t} h_{t+}(x,y)}{M_t} &= \frac{\partial \log f[u_{t+}, h_{t+}]}{\partial \log h_{t+}(x,y)}.
\end{aligned}$$

Note that, for linearly homogeneous aggregators f and g, the accounting equality holds:

$$M_t = \int q_t v_t(y)\,\mathrm{d}y = \lambda_{0,t} \int u_{t+}(x)\,\mathrm{d}x + \lambda_{1,t} \iint h_{t+}(x,y)\,\mathrm{d}x\,\mathrm{d}y.$$

New Jobs and Matching

Unemployed workers of type x meet vacancies of type y, and take the job in proportion

$$u_{t+}(x)\lambda_{0,t}\frac{q_t v_t(y)}{M_t}\mathbf{1}\{S_t(x,y) \geq 0\},$$

as each one of the $u_{t+}(x)$ unemployed workers meet a vacancy with probability $\lambda_{0,t}$, this vacancy is of type y with probability $\frac{q_t v_t(y)}{M_t}$, and the match is formed if $S_t(x,y) \geq 0$.

Similarly, employees of type x currently in a job of type y meet vacancies of type y', and move in proportion

$$h_{t+}(x,y)\lambda_{1,t}\frac{q_t v_t(y')}{M_t}\mathbf{1}\{S_t(x,y') > S_t(x,y)\}.$$

Note that these proportions do not change if one takes the point of view of the firms. Thus, vacancies of type y meet unemployed workers of type x, and match in proportion

$$v_t(y)q_t\frac{\lambda_{0,t}u_{t+}(x)}{M_t}\mathbf{1}\{S_t(x,y) \geq 0\}.$$

They meet employees in firms y', and match in proportion

$$v_t(y)q_t\frac{\lambda_{1,t}h_{t+}(x,y')}{M_t}\mathbf{1}\{S_t(x,y) > S_t(x,y')\}.$$

Laws of Motion

The law of motion for unemployment is therefore

$$u_{t+1}(x) \quad = \quad u_{t+}(x)\left[1 - \int \lambda_{0,t}\frac{q_t v_t(y)}{M_t}\mathbf{1}\{S_t(x,y) \geq 0\}\,\mathrm{d}y\right], \tag{14}$$

and for employment

$$\begin{aligned} h_{t+1}(x,y) = h_{t+}(x,y)&\left[1 - \int \lambda_{1,t}\frac{q_t v_t(y')}{M_t}\mathbf{1}\{S_t(x,y') > S_t(x,y)\}\,\mathrm{d}y'\right] \\ &+ \int h_{t+}(x,y')\lambda_{1,t}\frac{q_t v_t(y)}{M_t}\mathbf{1}\{S_t(x,y) > S_t(x,y')\}\,\mathrm{d}y' \\ &\qquad + u_{t+}(x)\lambda_{0,t}\frac{q_t v_t(y)}{M_t}\mathbf{1}\{S_t(x,y) \geq 0\}, \end{aligned} \tag{15}$$

subtracting those lost to more productive poachers, and adding the (x,y)-jobs created by poaching from less productive firms and hiring from unemployment.