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The Nature of Credit Constraints and Human Capital

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Abstract

We develop a human capital model with borrowing constraints explicitly derived from government student loan (GSL) programs and private lending under limited commitment. The model helps explain the persistent strong positive correlation between ability and schooling in the U.S., as well as the rising importance of family income for college attendance. It also explains the increasing share of undergraduates borrowing the GSL maximum and the rise in student borrowing from private lenders. Our framework offers new insights regarding the interaction of government and private lending as well as the responsiveness of private credit to economic and policy changes.

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Understanding the forces that shape human capital accumulation is important for many areas of economics. Economists have long thought that credit market imperfections play a crucial role in education decisions, since youth cannot generally pledge their future skills or labor as collateral.[1] This paper develops a new framework for the analysis of human capital accumulation in the presence of imperfect credit markets and uses it to examine the relationship between educational attainment, ability and family resources.

We begin by documenting two key facts from U.S. data: (1) Conditional on family income, college attendance is strongly increasing in ability. This relationship holds within all narrowly defined family income groups and has persisted for decades. (2) Conditional on ability, college attendance is strongly increasing in family income (and wealth) for recent cohorts; however, this correlation was much weaker a generation ago.

We next examine whether the standard exogenous borrowing constraint model can account for these two facts.[2] This model can account for the rising importance of family income given the rising costs of education. However, we show that it cannot generate a positive relation between college attendance and ability for constrained individuals for empirically plausible values of the intertemporal elasticity of substitution of consumption.

This motivates us to develop a new framework for human capital investment in the presence of imperfect credit markets that incorporates central features of existing government student loan (GSL) programs and private lending available for higher education. In particular, our framework captures two key features of reality: (i) GSL programs explicitly tie credit to investment in education (subject to an upper limit), and (ii) limited repayment enforcement is a major concern for private student credit. In modeling private lending, we build on recent work on credit constraints that arise endogenously when lenders have limited mechanisms for enforcing repayment.[3] We show that under standard and realistic enforcement mechanisms, the costs of default are higher for individuals with greater earnings capacity. As a result, private lenders are willing to extend more credit to individuals that invest more in their skills and/or exhibit higher ability.

Our framework is better able to explain the two key facts presented earlier. Because access to credit in our model is linked to individual ability and to investment in human capital, the model is more likely to produce a positive (and steeper) relationship between ability and investment for

[1]Becker's seminal Woytinsky Lecture (1967) provides an important early theoretical treatment of human capital investment when borrowing opportunities are limited. Hansen and Weisbrod (1969) provide an early empirical analysis of educational attainment gaps by family income. Manski and Wise (1983) emphasize borrowing constraints specifically as an explanation for their estimated family income – schooling gaps. In Section 1, we summarize more recent empirical studies on the importance of borrowing constraints and the links between family resources, cognitive achievement, and post-secondary schooling.

[2]For examples of studies assuming an exogenous borrowing constraint, see Aiyagari, Greenwood, and Seshadri (2002), Belley and Lochner (2007), Caucutt and Kumar (2003), Cordoba and Ripoll (2009), Hanushek, Leung, and Yilmaz (2003), and Keane and Wolpin (2001).

[3]The literature on endogenous credit constraints has mostly focused on risk-sharing and asset prices in endowment economies (e.g. Alvarez and Jermann 2000, Fernandez-Villaverde and Krueger 2004, Krueger and Perri 2002, Kehoe and Levine 1993, and Kocherlakota 1996) or firm dynamics (e.g. Albuquerque and Hopenhayn 2004, Monge-Naranjo 2009). Our punishments for default are similar to those in Livshits, MacGee, and Tertilt (2007) and Chatterjee, et al. (2007) in their analyses of bankruptcy.

constrained individuals, consistent with fact (1). The model is also consistent with fact (2) in predicting that differences in educational attainment by family resources should grow in response to rising schooling costs and returns (given stable GSL limits). Our model can also account for two other major changes in student borrowing: a sharp increase in the fraction of undergraduates borrowing the maximum amount from GSL programs (Berkner 2000 and Titus 2002) and a dramatic rise in student borrowing from private lenders (College Board 2005).

We extend our framework to a lifecycle economy and calibrate it to the U.S. in the early 1980s. At that time, GSL programs provided sufficient credit such that few students needed to turn to private creditors. College attendance was strongly increasing in ability and largely independent of family resources. To understand the observed changes over time in educational attainment by family income and in student borrowing behavior, we simulate responses to increases in the costs of and returns to college as observed in the U.S. during the 1980s and 1990s (holding GSL limits constant as was the case in real terms). The rising college costs and returns over time have encouraged more recent cohorts of students to invest and borrow more, with many exhausting their government loans and borrowing substantially from private lenders. Although private lenders have responded to increases in schooling by offering more credit, many students with low family resources are now constrained and unable to invest as much as they would like. While our simulations imply a weaker ability – investment relationship than found in recent data, our model performs noticeably better than the exogenous constraint model.

While the human capital literature has consistently appealed to credit constraints, little attention has been paid to the nature of those constraints.[4] An important advantage of explicitly modeling public and private lending is that it enables us to shed light on a number of different policy issues. Specifically, we use our model to study the impacts of changes in GSL programs, private loan enforcement, and education subsidies. Most interestingly, we show that expansions of public credit only partially crowd-out private lending. As a result, increases in GSL limits raise total student credit and human capital investment among youth constrained by those limits. Additionally, we show that changes in GSL limits tend to have a relatively greater impact on investment among the least able, while changes in private loan enforcement tend to impact investment more among the most able. Clearly, not all forms of credit expansion are the same, highlighting the importance of explicitly modeling different types of lenders. Finally, we show that endogenous borrowing constraints make human capital investment more sensitive to government education subsidies. Any policy that encourages investment is met with an increase in access to credit, which further encourages the investment of constrained students. This 'credit expansion effect', absent with fixed constraints, can be quite large. In our quantitative analysis, investment responds as much as 50% more than in the exogenous constraint model.

[4]Exceptions include our earlier analysis of private student lending under limited commitment (Lochner and Monge-Naranjo 2002), Andolfatto and Gervais (2006), who focus on optimal intergenerational transfers (in the form of social security and education subsidies) under limited commitment, and Ionescu (2008, 2009), who studies default in federal student loan programs.

A key message of our analysis is that private lending markets play an important role in how human capital accumulation responds to changes in policies or other changes in the economic environment. Ignoring private credit responses can lead to highly misleading conclusions. Our analysis implies that private lenders had an important incentive to expand credit for undergraduate students in the 1980s and 1990s: the rising returns to college increased the amount of debt students could credibly commit to repay while rising college costs and returns both increased student demand for credit.[5]

The rest of this paper is organized as follows. Section 1 reports U.S. evidence on the relationship between ability, family income, and college attendance. Section 2 uses a two-period model to characterize the cross-sectional implications for borrowing and investment under alternative assumptions about credit markets. Section 3 extends our framework to a multi-period lifecycle and presents our calibration and baseline quantitative analysis. Section 4 simulates the effects of increased returns to and costs of college and a number of policy experiments. Section 5 concludes.

1 Ability, Family Resources, and College Attendance

In this section, we discuss the empirical relationship between family income, cognitive ability, and college attendance in the U.S. during the early 1980s and in the early 2000s. We document two important facts using data from the 1979 and 1997 Cohorts of the National Longitudinal Surveys of Youth, NLSY79 and NLSY97 respectively. First, in both the early 1980s and the early 2000s, there is a strong positive relationship between college attendance and cognitive ability or achievement (as measured by scores on the Armed Forces Qualifying Test, AFQT) for youth from all levels of family income and wealth.[6] Second, for recent student cohorts, there is a much stronger relationship between family income (or wealth) and college attendance. Indeed, in the early 1980s, there was only a weak link between family income and college-going.

Using data for the 1980s (NLSY79), a number of empirical studies have found that family income played little role in college attendance decisions. Cameron and Heckman (1998, 1999) find that after controlling for family background, AFQT scores, and unobserved heterogeneity, family income had little effect on college enrollment rates. Carneiro and Heckman (2002) also estimate small differences in college enrollment rates and other college-going outcomes by family income after accounting for differences in family background and AFQT. Cameron and Taber (2004) and Keane and Wolpin (2001) explore different features of the NLSY79 data and also argue that credit constraints had little effect on educational outcomes in the early 1980s.

Using data for the late 1990s and early 2000s (NLSY97), Belley and Lochner (2007) show

[5] It is also likely that unrelated innovations in financial markets during the 1990s played a role in shaping higher education decisions to the extent that these innovations helped students and their families to better smooth consumption over time (e.g., see Lovenheim 2010).

[6] AFQT scores are widely used as measures of cognitive achievement by social scientists and are strongly correlated with post-school earnings conditional on educational attainment. See, e.g., Cawley, *et al.* (2000). Appendix B provides additional details on the AFQT.

that family income has become much more strongly correlated with college attendance for recent cohorts.[7] Youth from high income families in the NLSY97 are 16 percentage points more likely to attend college than are youth from low income families, conditional on AFQT scores, family composition, parental age and education, race/ethnicity, and urban/rural residence. This is roughly twice the effect observed in the NLSY79. The NLSY79 does not contain data on wealth; however, the combined effects of family income and wealth in the NLSY97 are substantially greater than the effects of income alone. Comparing youth from the highest family income and wealth quartiles to those from the lowest quartiles yields an estimated difference in college attendance rates of nearly 30 percentage points after controlling for ability and family background.

Despite changes in the relationship between family resources and college attendance, the relationship between ability and schooling has remained strong over time. Figure 1 shows college attendance rates by AFQT quartiles and either family income or family wealth quartiles in the NLSY79 and NLSY97.[8] For all family resource levels in both NLSY samples, we observe substantial increases in college attendance with AFQT. The differences in attendance rates between the highest and lowest ability quartiles range from 47% to 68% depending on the family income or wealth quartile. The figure reveals an equally strong positive ability – college attendance relationship for youth from low and high income/wealth families. In the NLSY97 data, the college attendance gap between the highest and lowest ability quartiles from both the lowest family income and wealth quartiles is 47%, compared to a 37% gap for those from both the highest family income and wealth quartiles.[9]

Of course, AFQT scores may be correlated with other family background variables that influence college attendance decisions conditional on family resources. In Lochner and Monge-Naranjo (2008), we use the NLSY79 and NLSY97 to estimate the effects of AFQT on college attendance by family income or wealth quartile after controlling for gender, race/ethnicity, mother's education, intact family during adolescence, number of siblings/children under age 18, mother's age at child's birth, urban/metropolitan area of residence during adolescence, and year of birth. These estimates confirm the general patterns observed in Figure 1: Cognitive ability is strongly positively correlated with college attendance for all family income and wealth quartiles in both NLSY samples.

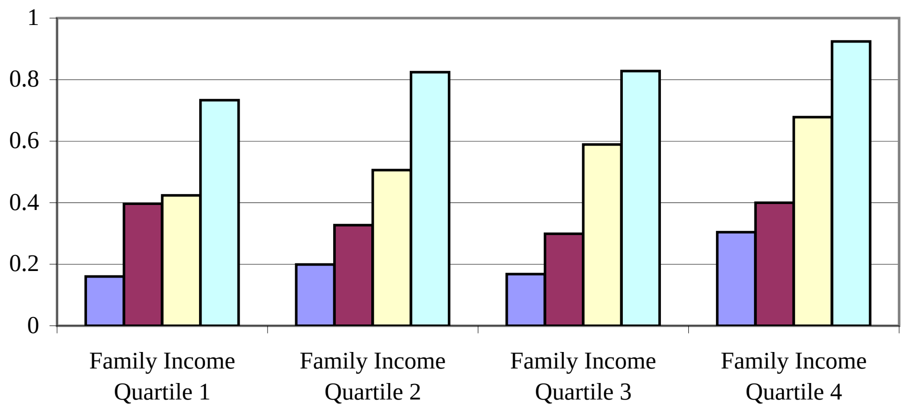
[7]Ellwood and Kane (2000) argue that college attendance differences by family income were already becoming more important by the early 1990s. Using data on youth of college-ages in the 1970s, 1980s, and 1990s (from the Health and Retirement Survey), Brown, Seshadri, and Scholz (2007) estimate that borrowing constraints limit college-going; however, they do not examine whether constraints have become more limiting in recent years. While Stinebrickner and Stinebrickner (2008) find little effect of borrowing constraints (defined by the self-reported desire to borrow more for school) on overall college dropout rates for a recent cohort of students at Berea College, they find substantial differences in dropout rates between those who are constrained and those who are not. They do not study the effects of borrowing constraints on attendance.

[8]See Appendix B for a detailed description of the data and variables used here.

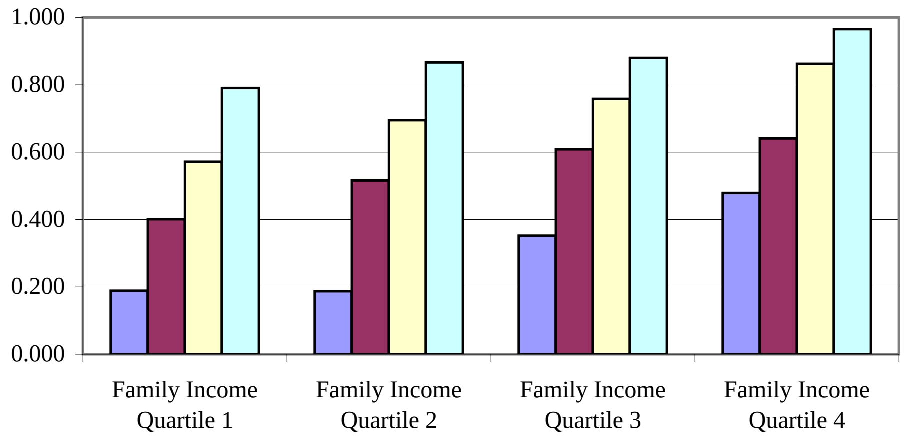
[9]We observe similar patterns in the NLSY97 for age 20 enrollment in four-year colleges/universities conditional on attendance at any post-secondary institution. Among youth from the lowest wealth quartile, the enrollment rate in four-year schools (conditional on post-secondary enrollment) is 34% higher for the most able relative to the least able. Among the highest wealth quartile, the difference is 32%. For the lowest family income quartile, the same high - low ability gap is 41%, while it is 52% for the highest income quartile.

Figure 1: College Attendance by AFQT and Family Income or Wealth (NLSY79 and NLSY97)

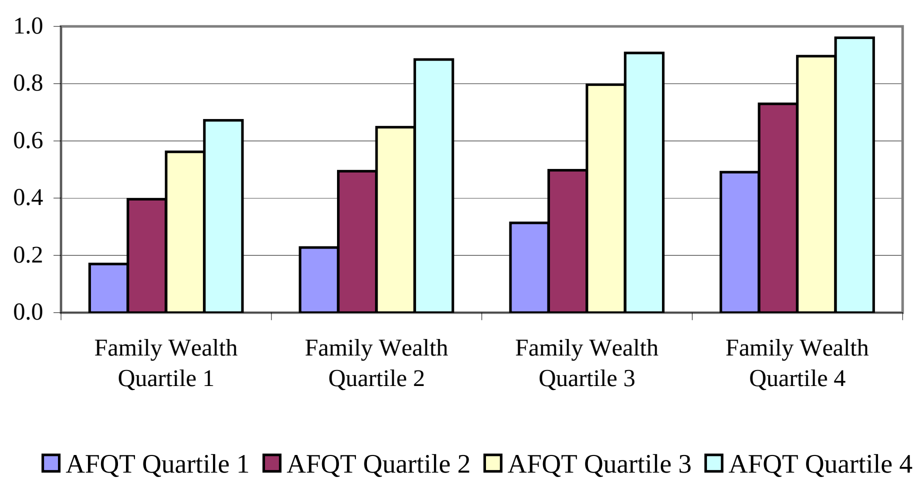
(a) Attendance by AFQT and Family Income (NLSY79)



(b) Attendance by AFQT and Family Income (NLSY97)



(c) Attendance by AFQT and Family Wealth (NLSY97)



2 Modeling Student Credit

In this section, we consider a simple two-period model to characterize analytically the implications of different forms of credit constraints for the behavior of human capital investment. We first show that the standard model of exogenous borrowing constraints cannot generate a positive relation between college attendance and ability for constrained individuals for empirically relevant preference parameters. We then show that a model that incorporates key features of public and private lending is better able to account for the data.

2.1 Preferences and Human Capital Production Technology

Consider two-period-lived individuals who invest in schooling in the first period and work in the second. Their preferences are

$$U = u\left(c_0\right) + \beta u\left(c_1\right), \tag{1}$$

where c_t is consumption in periods $t \in \{0, 1\}$, $\beta > 0$ is a discount factor, and $u\left(\cdot\right)$ is the period utility function. For expositional purposes, we assume $u\left(\cdot\right) = \frac{c^{1-\sigma}}{1-\sigma}$, so the consumption intertemporal elasticity of substitution (IES) is constant and equal to $1/\sigma$.[10]

Each individual is endowed with financial assets $w \geq 0$ and ability $a > 0$. Financial assets capture all familial transfers while ability reflects innate factors, early parental investments and other characteristics that shape the returns to investing in schooling. We take (w, a) as fixed and exogenous to focus on schooling decisions that individuals make largely on their own; however, our central results generalize naturally to an intergenerational environment in which parents endogenously make transfers to their children.[11]

Labor earnings at $t = 1$ are equal to $af\left(h\right)$, where h is schooling investment and $f\left(\cdot\right)$ is a positive, strictly increasing, strictly concave, twice continuously differentiable function that satisfies $\lim_{h \searrow 0} f'\left(h\right) = +\infty$ and $\lim_{h \nearrow \infty} f'\left(h\right) = 0$. Note that both a and h increase earnings and are complementary with each other.[12]

Human capital investment, h, is in units of the consumption good.[13] Individuals can borrow d of these units (or save, in which case $d < 0$) at a gross interest rate $R > 1$. Given w, a, h and d,

[10] Our theoretical results hold much more generally. Indeed, the proofs in Appendix C are for general preferences, allowing the IES to vary with the level of consumption.

[11] In an online appendix, we derive equivalent analytical results in three common models of parental transfers: (i) an 'altruistic' model (i.e. parents directly value the utility of their children); (ii) 'warm glow' preferences (i.e. parents directly value the resources transferred to their children); and (iii) a 'paternalistic' model (i.e. parents directly value the human capital investment of their children). In the last model, we need to impose a few additional mild conditions.

[12] We implicitly assume a constant elasticity of substitution between ability and investment equal to one. This specification is consistent with most empirical studies, which generally incorporate ability in the intercept of log wage/earnings regressions and with standard theoretical models of human capital (e.g. the widely used Ben-Porath (1967) model). In the online appendix, we extend a few key results below to the more general case of a CES production function in both ability and human capital.

[13] Our model is isomorphic to one in which foregone earnings for any given investment amount, h, are independent of ability. In the online appendix, we extend our model to allow the cost of investment to depend generally on ability. We show that our main conclusions here hold under fairly general and empirically relevant assumptions.

consumption in each of the periods is

$$c_0 = w + d - h, \tag{2}$$

$$c_1 = af(h) - Rd. \tag{3}$$

2.2 Unrestricted Allocations

Young individuals maximize utility (1) subject to (2) and (3). In the absence of financial frictions, optimal human capital investment $h^U(a)$ and borrowing $d^U(a,w)$ are characterized by

$$af'\left[h^U(a)\right] = R \tag{4}$$

and

$$u'\left(w + d^U(a,w) - h^U(a)\right) = \beta R u'\left(af\left[h^U(a)\right] - Rd^U(a,w)\right). \tag{5}$$

Unconstrained investment $h^U(a)$ equates the marginal return on human capital with the return on financial assets, is strictly increasing in ability a, and independent of initial wealth w. On the other hand, unconstrained borrowing $d^U(a,w)$ is strictly decreasing in wealth and increasing in ability. Ability increases desired borrowing for two different reasons: (i) more able individuals wish to finance a larger investment and (ii) for any given level of investment, more able individuals earn higher net lifetime income and wish to consume more in the first period. Because of (ii), unrestricted borrowing increases more steeply in ability than does unrestricted human capital investment. The following lemma formalizes this result and is used below to determine who is credit constrained.

Lemma 1 *$h^U(a)$ is strictly increasing in a, and $d^U(a,w)$ is strictly increasing in a and strictly decreasing in w. Moreover, $\frac{\partial d^U(a,w)}{\partial a} > \frac{dh^U(a)}{\partial a}$ and $\frac{\partial d^U(a,w)}{\partial w} > -1$.*

See Appendix C for all proofs and other analytical details related to this section.

2.3 Exogenous Borrowing Constraints

Credit constraints are typically introduced in models of human capital by imposing a fixed and exogenous upper bound on the amount of debt.[14] Following this approach, assume that borrowing is restricted by the exogenous constraint:

$$d \leq \bar{d}^X, \tag{EXC}$$

where $0 \leq \bar{d}^X < \infty$ is fixed and uniform across agents. We use the superscript X for all variables in this model.

[14]See, for example, Aiyagari, Greenwood, and Seshadri (2002), Belley and Lochner (2007), Caucutt and Kumar (2003), Cordoba and Ripoll (2009), Hanushek, Leung, and Yilmaz (2003), and Keane and Wolpin (2001). Instead, Becker (1975) assumes that individuals face an increasing interest rate schedule as a function of their investment. Becker's formulation yields similar predictions to those discussed here.

For each ability a, a threshold level of assets $w^X_{\min}(a)$ defines who is constrained ($w < w^X_{\min}(a)$) and who is unconstrained ($w \geq w^X_{\min}(a)$). Constrained persons have high ability relative to their wealth since $w^X_{\min}(a)$ is increasing in ability (see Appendix C). Individuals constrained by (EXC) have exhausted their possibilities to bring future resources to the early (investment) period. Their human capital investment $h^X(a,w)$ must strike a balance between increasing lifetime earnings and smoothing consumption and is uniquely determined by

$$u'\left(w + \bar{d}^X - h^X(a,w)\right) = \beta u'\left(af\left[h^X(a,w)\right] - R\bar{d}^X af'\left[h^X(a,w)\right]\right),$$

equality between the marginal cost of investing (reducing current consumption) and its marginal benefit (net return in terms of future consumption).

The next proposition highlights four empirically relevant implications of this model. Most importantly, the implied relationship between constrained investment and ability in part (iv) depends on the value of the IES.

Proposition 1 *Assume $w < w^X_{\min}(a)$, so (EXC) binds. Then: (i) $h^X(a,w) < h^U(a)$; (ii) $h^X(a,w)$ is strictly increasing in w; (iii) $af'\left[h^X(a,w)\right] > R$ and $af'\left[h^X(a,w)\right]$ is strictly decreasing in w; and (iv) if the $IES \leq 1$, then $h^X(a,w)$ is strictly decreasing in ability, a.*

Results (i)-(iii) are well-known (Becker 1975) and central to the empirical literature on credit constraints. For instance, Cameron and Heckman (1998, 1999), Ellwood and Kane (2000), Carneiro and Heckman (2002), and Belley and Lochner (2007) empirically examine if youth from lower income families acquire less schooling conditional on family background and ability (results (i) and (ii)). Lang (1993), Card (1995), and Cameron and Taber (2004) explore the prediction that the marginal return on human capital investment exceeds the return on financial assets (result (iii)).

The most interesting result is part (iv). The relationship between ability and investment for constrained individuals is determined by the balance of two opposing forces. On the one hand, there is an intertemporal substitution effect: more able individuals earn a higher return on human capital investment, so they would like to invest more. On the other hand, there is a wealth effect: more able individuals have higher lifetime earnings, which increases their desired consumption at all ages. Since constrained borrowers can only increase consumption during the initial period by investing less, the wealth effect discourages investment. With strong preferences for intertemporal consumption smoothing (i.e. IES$\leq$1), the wealth effect dominates and a negative ability – investment relationship arises.

The prediction of a *negative* relationship between ability and investment (among constrained youth) for an IES ≤ 1 is a serious shortcoming of the model.[15] Most estimates of the IES are less than one (see Browning, Hansen, Heckman 1999) and as discussed earlier, schooling is strongly increasing in ability even for youth from low-income families.

[15] An $IES \leq 1$ is only a sufficient condition. We show in the online appendix that the result is even stronger if investment is in terms of foregone earnings that increase with ability.

As shown below, the same economic logic implies that an increase in the return on human capital should lead to aggregate reductions in investment among those who are constrained.

2.4 Government Student Loan Programs

In this subsection, we consider GSL programs as the only source of credit. We then introduce private lending in the following subsection.

Federal GSL programs are an important source of finance for higher education in the U.S., accounting for 71% of the federal student aid disbursed in 2003-04. GSL programs generally have three important features. First, lending is directly tied to investment. Students (or parents) can only borrow up to the total cost of college (including tuition, room, board, books, computers, and other expenses directly related to schooling) less any other financial aid they receive in the form of grants or scholarships. Thus, GSL programs do not finance non-schooling consumption expenses. Second, GSL programs set upper loan limits on the total amount of credit available for each student. Third, loans covered by GSL programs typically have extended enforcement rules compared to unsecured private loans. See Appendix A for further details.

To capture these key features of GSL programs, we assume that individuals face two constraints on government loans. First, lending is tied to investment and cannot be used to finance non-schooling related consumption goods or activities:

$$d \leq h. \tag{TIC}$$

This condition is equivalent to $c_0 \leq w$. Second, borrowing is constrained by a fixed upper limit $0 < \bar{d}^G < \infty$, so

$$d \leq \bar{d}^G. \tag{6}$$

Combining these two constraints yields actual credit limits imposed by GSL programs:

$$d \leq \min\left\{h, \bar{d}^G\right\}. \tag{GSLC}$$

As in the exogenous constraint model, we continue to assume that repayment is fully enforced. This captures the government's superior enforcement mechanisms relative to private lenders, which we introduce below.

To isolate the role of (TIC), first assume that it is the only constraint.[16] In this case, individuals are unconstrained as long as desired borrowing does not exceed desired investment. Because unconstrained investment is increasing in ability, the (TIC) constraint is less stringent than the (EXC) constraint for higher ability individuals but more stringent for those with low ability. When borrowing is only restricted by (TIC), youth can borrow to finance any level of investment, but they cannot borrow to raise their consumption. Therefore, constrained youth (i.e. high ability/low wealth individuals with $d^U(a,w) > h^U(a)$) consume their initial wealth and choose h to maximize

[16] This would be the case if upper borrowing limits were non-existent or set very high (e.g. PLUS program for students' parents). See Appendix A for U.S. borrowing limits.

$\{u(w) + \beta u[af(h) - Rh]\}$, which is equivalent to maximizing discounted net lifetime earnings. Therefore, optimal investment equals $h^U(a)$.

By itself, (TIC) does not lead to a conflict between smoothing consumption and maximizing net lifetime resources. Although everyone invests the optimal unconstrained amount, there are still potentially large distortions to the intertemporal allocation of consumption. It follows that evidence suggesting that family resources (or credit constraints) do not affect schooling (e.g. Cameron and Heckman 1998, 2001, Carneiro and Heckman 2002, Belley and Lochner 2007) does not necessarily imply that credit constraints are not relevant along other important dimensions.

Now, consider the full GSL constraint (GSLC), denoting allocations in this model by the superscript G. To facilitate the exposition, we assume (throughout this section) that $\bar{d}^G = \bar{d}^X$. Unconstrained individuals ($w \geq w^G_{\min}(a)$) possess high assets relative to their ability.[17] The remaining population of constrained individuals falls into three categories: First, a low ability group is comprised of individuals constrained only by (TIC) and not by the maximum $\bar{d}^G$. They invest the unrestricted level $h^U(a)$ but would like to borrow to increase consumption while in school. Second, a more able group consists of individuals who borrow up to the maximum $\bar{d}^G$ and invest beyond that using some of their own available resources. For them, investment coincides with $h^X(a,w)$. A third group might emerge if $h^X(a,w)$ is decreasing in a (e.g. $IES \leq 1$.) This third group would be composed of very high ability youth who are constrained by both (6) and (TIC). We formalize this discussion as follows:

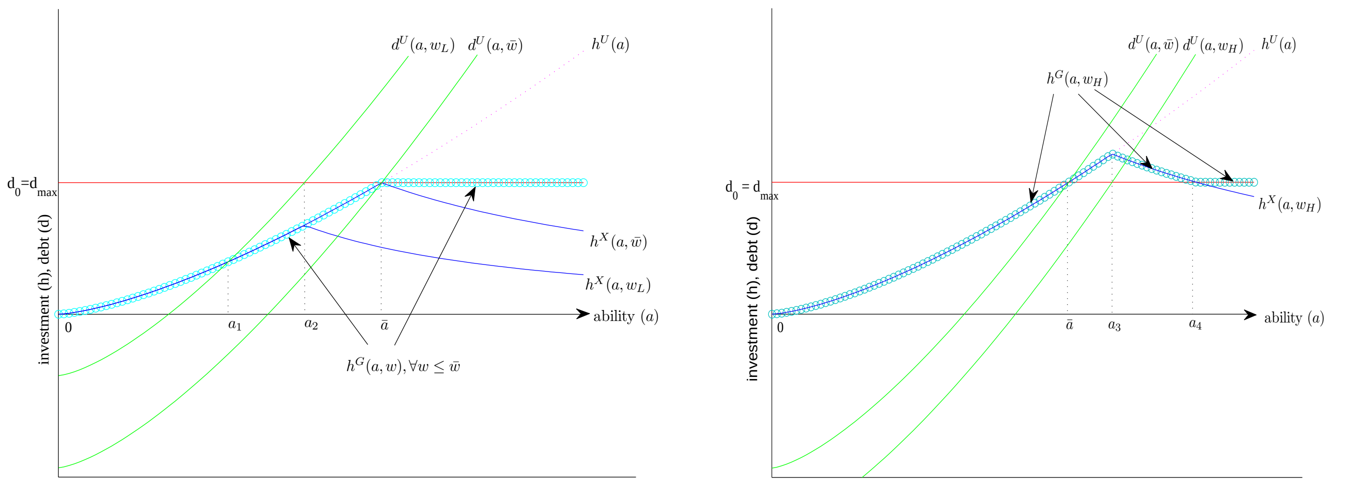
Proposition 2 *Assume that $u(\cdot)$ has IES ≤ 1. Let $\bar{d}^G = \bar{d}^X > 0$; let $\bar{a} > 0$ be defined by $h^U(\bar{a}) = \bar{d}^G$; and let $\hat{w} : [\bar{a}, \infty) \to R_+$ be defined by $h^X[a, \hat{w}(a)] = \bar{d}^G$, the (possibly infinite) wealth level that leads an exogenously constrained individual with ability a to invest $\bar{d}^G$. Then:*

$$h^G(a,w) = \begin{cases} h^U(a) & a \leq \bar{a} \text{ or } w \geq w^X_{\min}(a) \\ h^X(a,w) & a > \bar{a} \text{ and } w < \hat{w}(a) \\ \bar{d}^G & \text{otherwise.} \end{cases}$$

Figures 2(a) and (b) illustrate the behavior of $h^G(a,w)$, $h^X(a,w)$, and $h^U(a)$ for the empirically relevant case of IES ≤ 1. These figures also display unconstrained borrowing as a function of ability for different levels of wealth. Figure 2(a) displays investment and borrowing behavior for two low levels of wealth, $\bar{w}$ and $w_L < \bar{w}$, while Figure 2(b) illustrates investment behavior for a higher level of wealth $w_H > \bar{w}$.[18]

[17] In Appendix C, we show that the threshold $w^G_{\min}(a)$ is increasing in ability. We also show that when $\bar{d}^G = \bar{d}^X$, $w^G_{\min}(a) \geq w^X_{\min}(a)$ and more persons are constrained by the GSL, because it imposes an additional constraint.

[18] Note that $\bar{w} \equiv w^G_{\min}(\bar{a})$ reflects the level of wealth below which agents of ability $\bar{a}$ are constrained, where $\bar{a}$ is the ability level at which unconstrained investment equals the upper limit on borrowing (i.e. $h^U(\bar{a}) = d_{\max}$).



(a) low wealth individuals (b) high wealth individuals

Figure 2: d^U, h^U, h^X, and h^G for low and high wealth individuals

Because of the 'tied-to-investment' constraint, the implied investment – ability and investment – wealth relationships in the GSL model are more closely aligned with the empirical evidence than the simple exogenous constraint model. First, investment is equal to the unconstrained level $h^U(a)$ and increasing in ability for a larger range of lower ability and low/middle wealth individuals (e.g. individuals with wealth w_L and ability $a \in (a_2, \bar{a}]$ in Figure 2(a)). Second, among high ability individuals (i.e. $a > \bar{a}$), investment never falls below $\bar{d}^G$; this shrinks the range of abilities for which investment is negatively related to ability (e.g. individuals with ability $a > a_4$ in Figure 2(b)). Third, among high ability types, investment is weakly increasing in initial assets (e.g. individuals with ability $a \in (a_3, a_4)$ in Figure 2(b)).

2.5 GSL Programs and Private Lenders

The importance of private credit markets for students in the U.S. has increased dramatically since the early 1990s. As real tuition costs have risen (with no corresponding increase in real GSL limits), the fraction of undergraduate borrowers 'maxing out' federal Stafford student loans nearly tripled over the 1990s to 52% (Berkner 2000 and Titus 2002). Students have increasingly turned to private lending markets to finance their schooling: private student loan amounts skyrocketed from \$1.3 billion in 1995-96 to almost \$14 billion in 2004-05 (nearly 20% of all student loan dollars distributed). Credit card debt among students also rose considerably over this period (College Board 2005).

Private student loans differ from GSL programs in two important respects. First, unlike GSL programs, private lenders link credit to projected post-school earnings in addition to educational expenditures. Second, private loan repayment entails weaker enforcement under U.S. bankruptcy code than does GSL repayment.[19]

In modeling the coexistence of GSL programs and private lending, we continue to assume full enforcement of repayment in GSL programs. However, we assume that competitive private lenders face limited repayment incentives from students due to the inalienability of human capital and lack of other forms of collateral.

A rational borrower repays private loans if and only if repaying is less costly than defaulting. These limited incentives can be foreseen by rational lenders who, in response, limit their supply of credit to amounts that will be repaid.[20] Since penalties for default are likely to impose a larger monetary cost for borrowers with higher earnings and assets — only so much can be taken from someone with little to take — credit offered to an individual is directly related to his perceived future earnings. Because expected earnings are determined by ability and investment, private credit limits and investments are co-determined in equilibrium.

In the life-cycle model of Section 3, credit limits arise from temporary exclusion from credit markets and wage garnishments. Here, we derive a similar form of constraint by simply assuming that defaulting borrowers lose a fraction $0 < \tilde{\kappa} < 1$ of labor earnings.[21] In this case, optimal repayment behavior is quite simple: borrowers repay (principal plus interest on private debt d_p) if and only if the payment Rd_p is less than the punishment cost $\tilde{\kappa} a f(h)$. As a result, credit from private lenders is limited to a fraction of post-school earnings:

$$d_p \leq \kappa a f(h), \tag{7}$$

where $\kappa \equiv R^{-1}\tilde{\kappa}$. Private credit is directly increasing in both ability and investment. Moreover, ability may also indirectly affect credit through its influence on investment.

Students can borrow d_g from the GSL (subject to (GSLC)) and d_p from private lenders (subject to (7)). Because GSL repayments are fully enforced and do not affect incentives to repay private loans, total borrowing is constrained by

$$d_g + d_p \leq \min\left\{h, \bar{d}^G\right\} + \kappa a f(h). \tag{8}$$

We use the superscript $G+L$ to highlight that both sources of credit are present. Note that our GSL-only model above is a special case with no private loan enforcement (i.e. $\kappa = 0$). One could similarly define a private lender-only economy setting $\bar{d}^G = 0$. For future reference, we use the superscript L to refer to this special case.

The coexistence of both sources of credit reduces the incidence of constrained individuals relative to economies with only one of these credit sources. The threshold $w^{G+L}_{\min}(a)$ of assets below which individuals are constrained is decreasing in $\bar{d}^G$ and κ, because increases in either of these

[19] See Appendix A for details on the structure and enforcement of private loans.

[20] Gropp, Scholz, and White (1997) empirically support this form of response by private lenders.

[21] This is consistent with wage garnishments and penalty avoidance actions like re-locating, working in the informal economy, borrowing from loan sharks, or renting instead of buying a home, which are all costly to those who default.

parameters represent an expansion of total credit. Expanding either public or private credit would reduce the population of constrained individuals and change the investment behavior of those who remain constrained.

Lemma 2 *Let $h^{G+L}\left(a, w; \bar{d}^G, \kappa\right)$ denote the optimal investment for an individual with ability a and wealth w in an economy with $\bar{d}^G > 0$ and $\kappa > 0$. Then: (i) $w_{\min}^{G+L}(a) < \min\left\{w_{\min}^G(a), w_{\min}^L(a)\right\}$; (ii) For constrained individuals with abilities $a > \bar{a}$, the inequalities $\frac{\partial h^{G+L}\left(a,w;\bar{d}^G,\kappa\right)}{\partial \bar{d}^G} > 0$ and $\frac{\partial h^{G+L}\left(a,w;\bar{d}^G,\kappa\right)}{\partial \kappa} > 0$ hold.*

The two sources of credit have differential impacts on investment depending on ability. Among highly able youth constrained by the upper GSL limit and private constraints, increasing the GSL limit may increase investment more than one-for-one, since private credit expands with investment. The associated rise in private credit also yields an increase in consumption while in school. An increase in private credit (i.e. a higher κ) would also raise in-school consumption and investment. Notice that result (ii) in this lemma applies only to higher ability persons with $a > \bar{a}$ (i.e. persons with $h^U(a) > \bar{d}^G$). Less able individuals are constrained by (TIC) and not by $\bar{d}^G$, so an expansion of the GSL limit has no effect on their behavior. Moreover, as we discuss below, an increase in κ might actually reduce their investments.

Unlike models with exogenous or government constraints alone, it is possible that for the same level of familial resources w, a more able person is unconstrained while another with lower ability is constrained. That is, for large enough κ, the threshold $w_{\min}^{G+L}(a)$ may be decreasing in a, since punishment for default may be substantially more costly for the more able/higher earnings person. For the same reason, it is possible that individuals at the top of the ability distribution are always unconstrained (i.e. $w_{\min}^{G+L}(a) < 0$ for high a). These features are driven entirely by the presence of private lenders in the market.

There are other interesting interactions between GSL credit and private lending, depending on which of the GSL constraints binds, (6) or (TIC). Among the more able individuals for whom the upper GSL limit $\bar{d}^G$ binds, there is under-investment and investment is increasing in wealth (as in the previous models). For individuals in this group, the ability – investment relationship depends on the IES as well as the relative importance of the GSL and private lending. We show that if private lending is a relatively important source of funds, constrained investment is increasing in ability for empirically relevant values of the IES less than one.

Among lower ability individuals, for whom (6) is slack but (TIC) binds, investment behavior can be quite different. In the absence of private lenders, these individuals borrow and invest $h^U(a)$ as discussed earlier. With private lenders, constrained individuals actually *over-invest* in human capital (i.e. $h > h^U(a)$ and $af'(h) < R$) if (TIC) is the binding GSL constraint, since on the margin, total credit is increasing more than one-for-one with investment. This is because (i) additional marginal investments can be financed fully by the GSL, and (ii) additional investments raise earnings, which expands access to private credit and allows for greater consumption while in school.

Over-investing is socially inefficient and produces a negative relationship between investment and wealth for these individuals. Furthermore, their investment may decline with more access to private credit (i.e. an increase in κ). In any event, we show that a positive relationship between ability and investment arises in this situation.

The following proposition summarizes the relationship between investment, ability, and wealth when GSL programs and private lending co-exist. To this end, define $\varrho(a) \equiv \frac{R\bar{d}^G}{af(\bar{d}^G)}$ ($\equiv 0$ if $\bar{d}^G = 0$), the fraction of post-school earnings someone of ability a can borrow from the GSL if they invest $h = \bar{d}^G$.[22]

Proposition 3 *Assume $\bar{d}^G > 0$ and $\kappa > 0$, and consider individuals with $w < w_{\min}^{G+L}(a)$, so constraint (8) binds. Then, the following results hold: (1) If $a > \bar{a}$, then: (i) $h^{G+L}(a,w) < h^U(a)$, (ii) $h^{G+L}(a,w)$ is strictly increasing in w, (iii) $h^{G+L}(a,w)$ is strictly increasing in a if either (A) the IES $\geq \frac{1-\kappa R}{1-\varrho(a)}$ or (B) $\beta R \leq 1$ and the IES $\geq \frac{1}{1-\varrho(a)}\left(\frac{1-\kappa(R+1)}{1+\kappa(\beta^{-1}-R)}\right)$. (2) If $a < \bar{a}$, then: (i) $h^{G+L}(a,w) > h^U(a)$, (ii) $h^{G+L}(a,w)$ is strictly decreasing in w, and (iii) $h^{G+L}(a,w)$ is strictly increasing in a.*

The size of the GSL program has complicated effects on the ability – investment relationship when private lending is also available. On one hand, a larger GSL limit $\bar{d}^G$ reduces the mass of individuals for which this constraint is binding (i.e. it increases $\bar{a}$). This ensures a positive ability – investment relationship for a broader range of ability levels. On the other hand, an increase in $\bar{d}^G$ raises $\varrho(a)$, which reduces the range of IES values that ensure a positive ability – investment relationship for high ability individuals that remain constrained by the upper GSL limit.[23]

Increasing private lending (i.e. κ) weakens the conditions in part (1) for a positive ability – investment relationship, allowing for a broader range of IES values. Upon inspection of condition (A), if someone investing $h = \bar{d}^G$ can borrow more from private lenders than from the GSL program (i.e. $\bar{d}^G < \kappa a f(\bar{d}^G)$), then there is a positive ability – investment relationship for a range of IES less than one. In general, the bound in (B) is lower, so as long as individuals do not want increasing consumption profiles, a positive ability – investment relationship holds for still lower values of the IES.

2.6 Changes in the Returns to and/or Costs of Schooling

We close this section by examining the implied investment responses to increases in the returns to and costs of schooling as observed in the U.S. over the past several decades. To this end, assume that post-school earnings are now given by $paf(h)$, where $p > 0$ reflects the price of human capital. Furthermore, suppose that investing h now costs τh units of the consumption good,

[22] When $a > \bar{a}$, $\varrho(a)$ is less than the elasticity of earnings with respect to human capital investment evaluated at $h = d_{\max}$, i.e. $\varrho(a) = \frac{\bar{a}}{a}\frac{f'(d_{\max})d_{\max}}{f(d_{\max})} < \frac{f'(d_{\max})d_{\max}}{f(d_{\max})}$.

[23] If only private lending prevails (i.e. $d_{\max} = 0$), then $\varrho(a) = 0$ and only part (1) of Proposition 3 is relevant since $\bar{a} = 0$. In this case, both conditions for a positive ability – investment relationship admit a (potentially large) range of IES below one.

where $\tau > 0$ reflects factors affecting the cost of investment.[24] Our analysis thus far implicitly normalizes $p = \tau = 1$, but as shown in Appendix C (as part of the proof for Corollary 1 below), our specification is isomorphic to this extension as long as p and τ remain fixed. Our interest here is on the impact of changes in p and/or τ on the set of constrained individuals and the behavior of constrained investments in the different models.

Increasing p in this extended framework is equivalent to increasing ability a for everyone in our normalized model, so *all* of the qualitative properties for ability described thus far carry over to the price of skill, p. Changes in investment costs, τ, are slightly more complicated. Unconstrained investments h^U, which now satisfy $\tau = paf'\left(h^U\right)/R$, are decreasing in τ. While an increase in τ lowers desired investment levels, it also increases the desire for borrowing conditional on any level of investment and may raise total unconstrained investment expenditures τh^U. Thus, changes in τ have ambiguous effects on unconstrained borrowing d^U. It is interesting to consider what happens if both skill prices and schooling costs increase simultaneously. If both p and τ increase in the same proportion, unconstrained investment h^U is unaffected; however, the resulting increases in total investment expenditures and in post-school earnings unambiguously raise desired student debt levels d^U. Of course, h^U and d^U increase when both p and τ increase if p/τ increases. This reflects changes in the U.S. during the 1980s and 1990s when the costs of and net returns to education increased substantially (e.g., see Heckman, Lochner, and Todd 2008).[25]

By raising desired debt d^U, increases in p and τ (such that p/τ remains constant or increases) imply higher wealth thresholds w^X and w^G and more constrained individuals in the exogenous constraint and GSL-only models. In our baseline model with GSL and private lending, the expansion of private credit in response to increased future earnings dampens any increase in w^{G+L} and leads to fewer newly constrained youth. Indeed, if κ is large enough, the expansion of credit could even lead to a reduction in the set of constrained individuals.

We now turn to the response of constrained investments in the different models. To this end, the following corollary relies heavily on Propositions 1-3.

Corollary 1 *For τ fixed, the sign of dh/dp equals the sign of dh/da in all models. Moreover, if $f(\cdot)$ is Cobb-Douglas, then an increase in both p and τ (such that p/τ does not fall) has the same effects on total investment costs (τh) in all models as an increase in a.*

Corollary 1 shows another important advantage of our model. The observed rising skill prices, schooling costs, and net returns to human capital investment since the early 1980s should have had the same qualitative effects on educational expenditures as an increase in ability. Therefore, under empirically relevant IES values, the exogenous constraint model predicts that human capital investment should have *declined* for constrained youth. The GSL-only model predicts the same

[24] We also assume that the GSL's (TIC) constraint is modified to $d_g \leq \tau h$.

[25] In the following section, we consider simultaneous increases in both the costs of schooling and the elasticity of post-school earnings with respect to investment. As discussed below, this is quite similar to the case here with an increase in τ and p.

response among constrained higher ability individuals. By incorporating an endogenous response of private credit, our baseline model produces a substantially more appealing prediction. In the following section, we investigate the empirical relevance of these and earlier analytical results.

3 Quantitative Analysis

We now explore the quantitative implications of our model with public and private lending for schooling. To facilitate calibration and develop new insights on the interaction of GSL programs and private lending, we consider a multi-period lifecycle setting that incorporates government subsidies for education and additional punishments for private loan default. With a few convenient assumptions, the human capital investment decision in this model simplifies to a two-stage allocation problem nearly identical to that of the previous section. We calibrate this model to college costs, labor earnings, and other features of the U.S. economy and examine whether the model can quantitatively reproduce the main empirical patterns reported earlier. We also consider the effects of potential policy changes on human capital investment behavior.

3.1 A Lifecycle Model

We consider individuals whose post-secondary education life is represented by the time interval $[S, T]$. Letting $P \in (S, T)$ indicate the age of (full-time) labor market entry, we focus on the "schooling" stage $[S, P)$, in which education decisions are made. These decisions affect earnings and consumption over the "work" stage $[P, R)$ and consumption during "retirement" $[R, T]$.

After college, individuals work full time. Earnings $y(t)$ for all $t \in [P, R)$ depend positively on the individual's ability a, his human capital acquired through school h, and his accumulated experience $E(t-P)$ since labor market entry:

$$y(t) = ah^{\alpha} E(t-P), \tag{9}$$

with $0 < \alpha < 1$. Let the market interest rate be $r > 0$, and define $\Phi \equiv \int_P^R e^{-r(t-t_0)} E(t-P)\, dt$. Then, the present value lifetime labor income (as of date $t = P$) is $\Phi a h^{\alpha}$, which is increasing in both ability a and schooling human capital h. As in the previous section, a and h are complementary factors.

We assume individuals are endowed with an initial stock of human capital $h_0 \geq 0$, which they can augment through schooling investments.[26] Investing a flow $x(t) \geq 0$ during "youth" $[S, P)$ yields a total private investment of $h_I \equiv \int_S^P e^{-r(t-S)} x(t)\, dt$. To incorporate government education subsidies, we assume that the government matches every unit of privately financed investment at

[26] Our results extend to the case where h_0 and/or $E(t-P)$ are increasing in a. We have estimated the model allowing h_0 to depend on ability a. These estimates suggest that h_0 is about 25% higher for the top AFQT quartile relative to the bottom quartile; other parameter estimates are very similar to our baseline values. Most importantly, simulation results for the more general model are quite similar to those presented below.

the rate $s \geq 0$. Total human capital h accumulated at the end of school is, therefore,

$$h = h_0 + (1+s)\, h_I. \tag{10}$$

Here, as well as in our quantitative exercises, h, h_I and h_0 are denoted in present value units as of the beginning of "youth" ($t = S$).

For any $t_0 \in [S, T]$, a consumption flow $c(t)$ generates utility

$$U(t_0) = \int_{t_0}^{T} e^{-\rho(t-t_0)} u\,[c(t)]\, dt, \tag{11}$$

where $\rho > 0$ is a subjective discount rate and $u(c) = \frac{c^{1-\sigma}}{1-\sigma}$ with $\sigma > 0$. Given initial wealth $w > 0$, optimal investment and borrowing decisions maximize the value of (11) for $t_0 = S$, subject to the lifetime budget constraint[27]

$$\int_{S}^{T} e^{-r(t-S)} c(t)\, dt + h_I \leq w + e^{-r(P-S)} \Phi a h^{\alpha}. \tag{12}$$

We consider restrictions on borrowing next.

3.2 Human Capital Decisions Under Public and Private Lending

We focus on constraints that limit the amount of debt that can be accumulated during the "schooling" period. Our benchmark quantitative model allows youth to borrow from GSL programs, d_g, and from private lenders, d_p, such that total borrowing at the end of school is given by $d = d_g + d_p$.

Credit from the GSL is tied to schooling-related expenses, subject to a maximum cumulative amount:

$$d_g \leq \min\left\{e^{r(P-S)} h_I, \bar{d}^G\right\}, \tag{13}$$

for some $0 < \bar{d}^G < \infty$.[28] Here, government credit is linked to personal out-of-pocket investment expenditures h_I rather than total human capital h. We continue assuming that the repayment of d_g is fully enforced regardless of whether individuals default on private loans.

Private lenders restrict student credit due to their limited ability to punish default. We assume that lenders employ two punishments commonly assumed in the literature on consumer bankruptcy (e.g. Livshits, MacGee, and Tertilt (2007), Chatterjee, et al. (2007)). First, defaulting borrowers are reported to credit bureaus, disrupting (at least temporarily) their access to formal credit markets. The resulting inability to smooth consumption can be quite costly when the IES is low and the earnings profile is steep in experience. Second, defaulting borrowers must forfeit a fraction $\gamma \in [0, 1)$ of their labor earnings. The fraction γ encompasses direct garnishments from

[27] Assuming goods (e.g. tuition, books) and time investments (i.e. foregone earnings) are perfect substitutes in the production of schooling human capital, the value of w includes family transfers plus the discounted present value of earnings an individual could receive if he worked (rather than attended school) full-time during "youth". We make this assumption in our calibration below, where we discuss it in further detail.

[28] Note that d_g is denominated in time $t = P$ units while h_I is in time $t = S$ units, which explains why h_I is multiplied by $e^{r(P-S)}$ in equation (13).

lenders and/or the costs of actions taken by borrowers to avoid direct penalties (e.g. working in the informal sector, renting instead of owning a house, etc.). Both penalties are assumed to last for a period of length $\pi \in [0, R-P)$ that begins the moment default takes place.

We make three additional assumptions that greatly simplify the analysis. Specifically: (1) individuals can only default on private loans at the time of labor market entry; (2) individuals that choose to repay their private student loans have access to perfect financial markets upon entry into the labor market; and (3) individuals that default on private loans can access frictionless and fully enforceable credit markets after the punishment period. In short, we abstract from issues related to the optimal timing of default and the enforcement of post-school loans.[29]

Given these additional assumptions, the analysis of human capital decisions in our lifecycle model can be mapped into the two-period model of the previous section. Within each of the sub-intervals $[S, P)$ and $[P, T]$, consumption can be allocated optimally and grows at the rate $\frac{r-\rho}{\sigma}$. If credit constraints bind, consumption will exhibit a discrete jump at the end of schooling ($t = P$). Discounted lifetime utility can be written compactly (up to a multiplicative constant) as

$$u\left(w + e^{-r(P-S)}d - h_I\right) + \hat{\beta} u\left(\Phi a h^{\alpha} - d\right), \tag{14}$$

where the constant $\hat{\beta} > 0$ reflects the role of both time discounting and the relative length of the schooling vs. post-schooling period.[30] See Appendix D for details.

Borrowers repay private debt d_p only if the cost of repaying is less than the cost of being punished (or the cost of taking actions to avoid punishment). This implies a maximum private credit limit as a function of a, h, and d_g. The timing of GSL repayment also affects the cost of defaulting on private loans, since it affects the amount of resources available for consumption during the punishment period. For analytical tractability, we assume that during the punishment period, individuals must repay a constant fraction $\delta > 0$ of their earnings to service their GSL debt. Further restricting this minimum GSL repayment rate yields a simple and intuitive representation of the private credit constraint (see Appendix D).

Lemma 3 *If the minimum GSL repayment rate δ is set such that individuals repay a constant fraction of their income (net of garnishments in the case of default) over their entire working lives, then private credit d_p available during schooling is constrained by*

$$d_p \leq \kappa_1 \Phi a h^{\alpha} + \kappa_2 d_g, \tag{15}$$

where $0 \leq \kappa_1 \leq 1$ and $\kappa_2 > -1$.

We adopt the private lending constraints defined by equation (15) as our baseline.[31] The values of κ_1 and κ_2 depend on preferences (σ, ρ), the interest rate r, and enforcement parameters (γ, π).

[29]For many parameter values, (3) is not an assumption but an equilibrium outcome (e.g. see Lochner and Monge-Naranjo 2002). See Monge-Naranjo (2009) for a continuous time model in which default can take place in any period and the optimal contract must satisfy a continuum of participation constraints.

[30]Recall that wealth w, human capital h, and private investment h_I are all denoted in time $t = S$ units while borrowing d is denoted in time $t = P$ units.

[31]See Appendix D for the formulas for δ, κ_1 and κ_2 and for private lending constraints in the more general case.

The punishment of exclusion from financial markets introduces an important interaction between public and private lending through κ_2 that does not exist in the two-period model. A few key properties of the private lending constraint warrant discussion. First, even if wage garnishments are not allowed ($\gamma = 0$), private lending can be sustained ($\kappa_1 > 0$) by the threat of exclusion from credit markets; only when $\pi = 0$ does private credit dry up entirely (i.e. $\kappa_1 = \kappa_2 = 0$). Second, the amount of sustainable borrowing (as determined by κ_1 and κ_2) is generally higher with: (i) tougher punishments (higher values of γ and π); (ii) more patient individuals (lower ρ); (iii) a stronger desire to smooth consumption (lower IES, higher σ), and (iv) higher growth in earnings with experience. Third, κ_1 and κ_2 do not depend on government subsidies s or the initial human capital level h_0 – these only affect private constraints through total human capital h and GSL borrowing d_g. Fourth, we find that $\kappa_2 > -1$, so private credit does not decrease one-for-one with expansions of GSL credit. However, $\kappa_2 < 0$ does imply partial 'crowding out'.

Optimal schooling investment decisions maximize discounted lifetime utility (14) subject to credit constraints (13) and (15). It is straightforward to show that unconstrained private investment, $h_I^U(a)$, maximizes discounted lifetime income net of private investment. Given $h_0 > 0$, there exists an ability level a_0 (defined in Appendix D), below which unconstrained individuals do not wish to invest. For $a > a_0$, unconstrained investment is strictly increasing in ability and independent of wealth as in the two-period model.

Youth with initial wealth less than the ability-specific threshold $w_{\min}^{G+L}(a)$ will be constrained. To characterize their investment behavior, it is useful to re-define the following analogues from Section 2: let $\bar{a}$ reflect the ability level for which unconstrained private investment equals $\bar{d}^G$, and let $\varrho(a)$ equal the fraction of lifetime earnings that can possibly be borrowed from the GSL. Also, let θ reflect the fraction of discounted lifetime resources an unconstrained individual chooses to consume over the schooling period.[32] With these, we derive a version of Proposition 3 for our quantitative model:

Proposition 3 ***(Lifecycle Model)*** *Consider individuals with ability $a > a_0$ (i.e. $h_I^U(a) > 0$) and whose wealth $w < w_{\min}^{G+L}(a)$, so constraints (13) and (15) bind. Then, the following holds: (1) If $a > \bar{a}$, then: (i) $h^{G+L}(a,w) < h^U(a)$, (ii) $h^{G+L}(a,w)$ is strictly increasing in w, (iii) $h^{G+L}(a,w)$ is strictly increasing in a if either* (A) *$\kappa_1 \geq \theta$ or* (B) *$\sigma \leq \left[1 - \left(\frac{1+\kappa_2}{1-\kappa_1}\right)\varrho(a)\right]\left[1-\kappa_1/\theta\right]^{-1}$ hold. (2) If $a < \bar{a}$, then: (i) $h^{G+L}(a,w) > h^U(a)$, (ii) $h^{G+L}(a,w)$ is strictly decreasing in w, and (iii) $h^{G+L}(a,w)$ is strictly increasing in a.*

This proposition provides sufficient conditions in terms of parameters that can be calibrated. As with the two-period model, the nature of private lending constraints, especially the link between private credit and future earnings (κ_1), plays a critical role in determining the relationship between ability and constrained investment. However, this proposition incorporates important economic forces that are absent from the two-period model due to the lifecycle nature of the underlying

[32] See Appendix D for precise formulas for $w_{\min}^{G+L}(a)$, $\bar{a}$, $\varrho(a)$, and θ.

Table 2: Baseline Model Parameters

Calibrated Parameters			GMM estimates		
Parameter	Value	To match:	Parameter	Value	Coefficient on:
S	19		g_0	0.03	Experience
P	26	US Demographics			
R	65		α	0.70	Schooling investment
T	80		h_0	160,312	Min. human capital
π	10	U.S. Legal environment			
$\rho = r$	0.05	See text			AFQT Quartiles:
σ	2	IES = 0.5	a_1	1.51	1
$d_{\max}$	35,000	GSL Loan Limits	a_2	1.55	2
γ	0.2	Garnishments & other costs	a_3	1.60	3
s	0.80	Subsidy school grades 10+	a_4	1.72	4

problem and the resulting nature of κ_1 and κ_2. First, partial crowd-out of private lending by the GSL program (embodied in $\kappa_2 < 0$) weakens the link between investment and total credit, which makes it less likely that constrained investment is increasing in ability. Second, κ_1 is increasing in σ, because the cost of disrupting consumption smoothing is increasing in the curvature of the utility function. This implies that both sufficient conditions ensuring that investment is increasing in ability may be more likely to hold for higher values of σ (i.e. lower values of the IES).[33] Third, the effect of $\bar{d}^G$ on the relationship between ability and investment is complicated: On one hand, a higher $\bar{d}^G$ increases $\bar{a}$, which signals that more individuals can directly finance $h^U(a)$ with GSL programs alone. On the other hand, a higher $\bar{d}^G$ increases the fraction of lifetime earnings that can be borrowed from the GSL, $\varrho(a)$, which crowds-out some private lending and makes it more difficult for the sufficient condition 1(iii)(B) to hold.

As in the two period model, among less-abled constrained individuals for whom $h^U(a) \leq \bar{d}^G$, there is 'over-investment', and investment is strictly decreasing in w and strictly increasing in a.

3.3 Parameter Values

We now discuss the parameter values used to study the quantitative implications of our model. We normalize time so that a unit interval represents a calendar year. All monetary amounts are denominated in 1999 dollars using the Consumer Price Index (CPI-U). As a measure of ability, we use quartiles of the AFQT distribution in our sample. This facilitates comparison with the empirical patterns discussed earlier in Section 1. Baseline parameter values, reported in Table 2, are chosen to match basic features of the U.S. economy, while others are estimated using data on earnings and educational attainment from the random sample of males in the NLSY79.

With our focus on college education, we assume that youth (investment period) begins at age $S = 19$; maturity (labor market participation) begins at age $P = 26$; and retirement runs from age $R = 65$ until death at age $T = 80$. These values roughly capture the demographics and the

[33] For example, if $r = \rho$, then θ is independent of σ, while κ_1 is increasing in σ.

timing of college education and labor market decisions in the U.S.

We assume an annual interest rate $r = 0.05$ based on historical averages of the risk-less rate and the return to capital in the U.S. We also set $\rho = r$. Given our calibration strategy, reasonable variations of ρ and r, including differences between them, have little impact on our results. We set $\sigma = 2$, which implies an IES of 0.5 – an intermediate value in the estimates reported in Browning, Hansen, and Heckman (1999). Values of σ inside the interval $[1.5, 3]$ yield similar results.

We set reasonable values for parameters that define borrowing opportunities ($\bar{d}^G$, γ, π) and show that our results are not overly sensitive to changes in them. We assume $\bar{d}^G = 35,000$ based on loan limits for Perkins and Stafford Loan Programs. As discussed in Appendix A, there are a number of different government loan limits depending on the type of loan, dependency status, and whether the student is an undergraduate or graduate student. Our choice of $35,000 is in the range between the limit for dependent undergraduates borrowing only from the Stafford program and the limit for independent undergraduates or for graduate students.

We calibrate the length of the penalty period π based on the U.S. legal environment. According to U.S. bankruptcy code, individuals must wait for at least 7 years after filing for Chapter 7 before they qualify to file again, while default records remain in an individual's credit history for a period of 10 years. Thus, π should range between 7 and 10. In our baseline, we set $\pi = 10$, but $\pi = 7$ produces similar conclusions.

Regarding γ, the effective earnings lost in the event of private loan default, regulations provide little direct guidance. For private unsecured loans, an explicit garnishment rule does not exist. Moreover, actual costs of default – via either direct penalties or avoidance actions – extend beyond simple garnishments (e.g. poor credit ratings can limit employment, rental, and home ownership options, and force individuals to pay high interest rates for day-to-day transactions, etc.) These costs are likely to be non-trivial, but are difficult to quantify. Rather than attempting to directly measure these distortions, we instead choose $\gamma = 0.2$ as our baseline to yield reasonable credit and borrowing levels in our model. While higher values of γ produce a more positive ability – investment relationship among those that are constrained, much higher values imply too much private credit and borrowing and too few constrained individuals. Much lower values for γ result in too little private credit and borrowing, which implies investment allocations more similar to those in the exogenous constraint model.[34]

Finally, for the sake of comparison, we also report the predictions of an exogenous constraint model with a limit equal to $\bar{d}^X = 70,000$, a value we justify below.

[34] Our baseline value of γ is a bit higher than in Livshits, MacGee, and Tertilt (2007) and Chatterjee, et al. (2007) for two reasons. First, we abstract from the benefits of financial markets in smoothing out temporary earnings and preference shocks, which implies a smaller 'cost' of default in our framework for any level of γ. Second, in the context of human capital formation, a higher γ helps capture any disruptions in career possibilities that may arise as a result of default.

3.3.1 Estimation of the earnings function

We use data on wage income, education, age, and AFQT quartile from the NLSY79 (1979-2006) to estimate parameters of the labor earnings function. Our sample includes all men ages 19+ with at least 12 years of completed schooling from the random sample. We associate different levels of investment with different levels of reported schooling, calculating the total expenditures associated with each level of schooling separately by AFQT quartile. These costs include both foregone earnings and direct expenditures as discussed below. Consistent with the formulation of the model, we make no distinction for investment in time costs (foregone earnings) or purchased inputs.[35] Implicitly, they are perfect substitutes in the production of human capital, an issue we discuss further in the online appendix. We also abstract from investment differences related to differences in college quality. While an interesting margin of choice, we leave this to future work.

Estimation of the labor earnings function proceeds in three separate steps (details on steps 1 and 2 are provided in Appendix E):

Step 1: Estimating foregone earnings. Foregone earnings reflect the present value of average earnings relative to someone with 12 years of completed schooling, taking into account earnings during college.

Step 2: Determining total costs of schooling and the government subsidy matching rate. For individuals attending college, we add foregone earnings determined in Step 1 to direct costs to determine total schooling expenditures. Direct expenditures are based on current-fund expenditures per full-time equivalent student in all institutions of higher education (1999 Digest of Education Statistics, Table 342). We used expenditures for the 1980s corresponding to the years most students in our NLSY79 sample attended college. Table E1 in Appendix E reports measures of direct expenditures, foregone earnings, and total expenditures.

To calculate the subsidy rate s used in our analysis, we calculate the average marginal subsidy rate for an additional year of college. The resulting government subsidy matching rate is $s = 0.799$. In simulating the 'year 2000' economy below, we use a lower subsidy matching rate of $s = 0.710$, consistent with the observed rise in current fund revenue that came from tuition.

Step 3: Estimating the parameters. With Step 2, we have imputed total investment expenditures $h(q, C)$ for each AFQT quartile q and years of completed schooling C. Since we include total expenditures in calculating $h(q, C)$, it reflects total private investment plus public subsidies, $h_I(a)(1 + s)$. Assuming human capital grows at rate g with labor market experience $x = age - 26$ (i.e. $E(x) = exp(gx)$), we estimate α, h_0, g, and ability parameters $a_1, ..., a_4$ using NLSY79 data on wage income, schooling, and age.

Taking logs and incorporating measurement error ε_i, we obtain the following specification for

[35] It is worth noting that GSL programs typically allow borrowing against purchased inputs but not foregone earnings. GSL programs also generally allow individuals to borrow against modest living expenses, which range from $6,000-10,000 per year in the U.S. Since our estimated measures of foregone earnings are mostly in this range (and the tied-to-investment constraint is generally quite slack), incorporating this feature would have negligible effects on our quantitative results below.

individual i earnings as a function of AFQT quartile q_i, experience x_i, and years of college C_i:

$$ln(y_i) = ln[a_{q_i}] + gx_i + \alpha\ ln[h_0 + h(q_i, C_i)] + \varepsilon_i. \tag{16}$$

The model also implies unconstrained investment $h^U\left(a_{q_i}\right) = \max\left\{h_0,\ \left[\alpha\left(1+s\right)\Phi a_{q_i} e^{-r(P-S)}\right]^{\frac{1}{1-\alpha}}\right\}$ with $\Phi = \frac{e^{(g-r)(R-P)}-1}{g-r}$.

We use GMM to estimate our parameters using moments based on both earnings and unconstrained investment:

$$E\left\{[ln(y_i) - (ln[a_{q_i}] + gx_i + \alpha\ ln[h_0 + h(q_i, C_i)])]Z_i\right\} = 0$$
$$E\left\{h_0 + h(q_i, C_i) - h^U(a_{q_i})|q_i\right\} = 0,$$

where Z_i includes indicators for each year of schooling from grades 12 to 20, experience x_i, and AFQT quartile indicators.[36] The first set of moments using the wage equation simply estimates parameters to best fit average earnings conditional on schooling, age, and AFQT quartile. Using only this set of moments is nearly identical to non-linear least squares estimation of equation (16). With the second set of moments, we also match average schooling expenditures by AFQT quartile to the unconstrained optimal levels as implied by the model.[37]

The strategy of targeting unconstrained investments when estimating (a_q, α, g, h_0) is consistent with evidence in the NLSY79 (e.g. Cameron and Heckman (1998, 1999) and Carneiro and Heckman (2002)) suggesting that most individuals were not constrained in their schooling investments at that time. However, it is important to note that this does not guarantee that simulations of our baseline model will lead to these unrestricted investments. None of our assumptions about the credit environment (i.e. the GSL program and private lending under limited commitment) imply sufficient credit for everyone. Therefore, one metric for evaluating our model is whether anyone is constrained in our baseline calibration.

Finally, it is important to discuss the nature and correct interpretation of an individual's available resources w in our simulations. Because foregone earnings are an important part of investment expenditures in our calibration, w includes at least the amount an individual could earn if he began working immediately after high school. These amounts depend on ability, since foregone earnings depend on ability (see Appendix Table E1). The relevant range of available resources, therefore, begins at $\$36,000$ for the least able, $\$73,000$ for AFQT quartile 2, $\$76,000$ for AFQT quartile 3, and $\$79,000$ for the top quartile. Any available resources above these amounts must be interpreted as transfers from parents or others.

[36] We do not attempt to address concerns about unobserved heterogeneity in estimating the wage equation (i.e. we assume ε_i is orthogonal to completed schooling conditional on AFQT).

[37] We could have estimated parameters of the human capital production function using only moments based on the wage equation. However, this produces fairly noisy estimates of most parameters, especially h_0. Since the model implies an optimal unconstrained investment that is quite sensitive to all parameter values, including the second set of moments provides much more precise and robust estimates. We do not lose much in terms of mean squared error (MSE) for the log wage equation (0.593 vs. 0.601) when estimating the model using both sets of moments.

3.4 Baseline Simulations

We now report the model's main implications given our baseline parameterization. Figure 3 shows the wealth threshold $w^{G+L}(a)$ for our benchmark model. It also shows $w^{G}(a)$ and $w^{L}(a)$, the special cases when we shut down private or GSL credit, respectively. Individuals with ability-wealth pairs above and to the left of the thresholds are unconstrained, while those with pairs below and to the right are constrained. The x-marks indicate the point estimates for each ability quartile as reported in Table 2. Finally, the dotted horizontal lines reflect estimated potential earnings (PE) for these same ability levels.

For all estimated ability types a_q, the dotted $PE(a_q)$ lines lie above the corresponding wealth threshold $w^{G+L}(a_q)$, which implies that even youth who receive zero transfers (from their parents or other sources) attain unconstrained consumption and investment allocations. Regardless of individual resources, our parameterization yields investments of \$8,000, \$22,300, \$44,600, and \$100,900 for AFQT quartiles 1, 2, 3 and 4, respectively.[38] The fact that our baseline model predicts that individuals in the NLSY79 were unconstrained justifies our estimation strategy of matching unconstrained investment with average investment in the data.

Figure 3 also reveals that ability quartiles 2 and 3 would be unconstrained by the GSL alone; thus, middle ability individuals would not need to borrow from private lenders. Lower ability individuals lie in the flat region of w^G and w^{G+L}, indicating that the GSL's tied-to-investment constraint (i.e. $d_g \leq e^{r(P-S)}h_I$) may bind. The fact that $w^G(a_1) < PE(a_1) < w^{G+L}(a_1)$ implies that the least able poor would be constrained (low consumption during school) under the GSL alone, but they receive enough credit from private lenders to enable full consumption smoothing. Among the most able, the upper GSL loan limit (i.e. $d_g \leq \bar{d}^G$) binds for those receiving no family transfers. They would under-invest without access to private lenders; however, private lenders provide enough credit to ensure unconstrained maximization.

Figure 4 reports total borrowing $d_g + d_p$ for each level of ability as a function of initial wealth minus potential earnings (i.e. family or outside transfers). Only youth from the top AFQT quartile with very low resources wish to borrow more than the upper GSL limit (reflected in the dashed horizontal line at \$35,000).[39] Among the most able, roughly \$35,000 in family transfers (received over ages 19-26) would be enough to ensure unconstrained consumption and investment without any need for private loans. All other youth wish to borrow less than the GSL maximum. As noted above, youth from the lowest ability quartile would like to borrow more than they invest, which the GSL does not accommodate. As a result, the least able receiving less than \$20,000 in family transfers (cumulative over ages 19-26) would like to borrow small amounts from private lenders

[38] These reflect total expenditures for post-secondary education and are very close to average total expenditures by AFQT quartile in the NLSY79 (from least to most able): \$8,800, \$29,000, \$47,400, \$107,700. See Appendix Table E1 for a mapping between these amounts and years of college attendance.

[39] This group should be quite small given the strong correlation between ability and family resources observed in the NLSY79. For example, Table 2 in Belley and Lochner (2007) reveals that 70% of youth from the highest AFQT quartile have family income in the top half of the distribution.

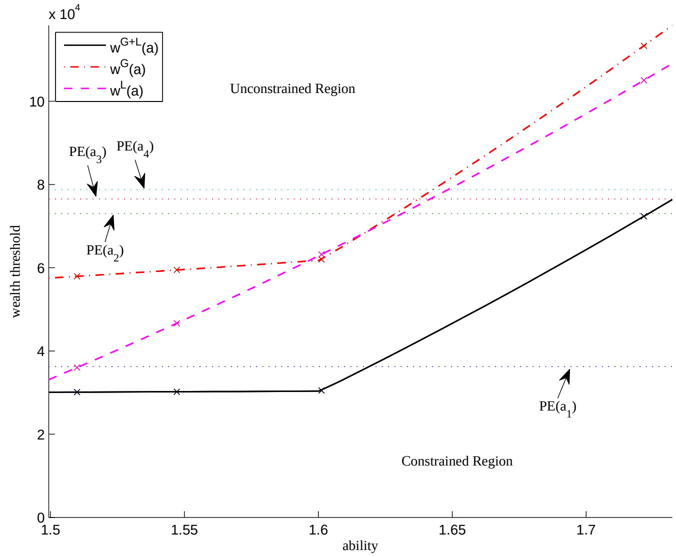


Figure 3: Thresholds for Unconstrained Allocations (Baseline)

(e.g. credit cards) in order to smooth consumption. Youth in the interquartile range invest more than they wish to borrow from the GSL and do not run up against the GSL upper loan limit; they are fully unconstrained by the GSL regardless of parental transfers.

Altogether, our baseline model fits the '1980s facts' quite well. The prediction that investment is unconstrained for all ability levels is consistent with the evidence from the NLSY79, i.e. that investments are independent of the individual's wealth and strongly increasing in ability. The model further predicts that most NLSY79 respondents should borrow less than the GSL maximum. Only youth with high ability and low family transfers would borrow up to the GSL maximum and then some from private lenders. This is consistent with the fact that early private student loan programs in the 1980s were relatively unimportant and almost exclusively served students of elite institutions and professional schools.

4 Counterfactual Exercises

We now use our model to conduct two sets of counterfactual exercises. First, we simulate an increase in both the costs of and the returns to education to see whether our model is consistent with the rising importance of family resources for college attendance and the increase in student borrowing (from both the GSL and private lenders) as observed between the 1980s and early 2000s. Second, we conduct a number of policy experiments related to the financing of college education.

4.1 A Rise in the Costs of and Returns to Schooling

We simulate the effects of an increase in the costs of and returns to schooling — two major economic changes that took place between the early 1980s and early 2000s. We model an increase in the

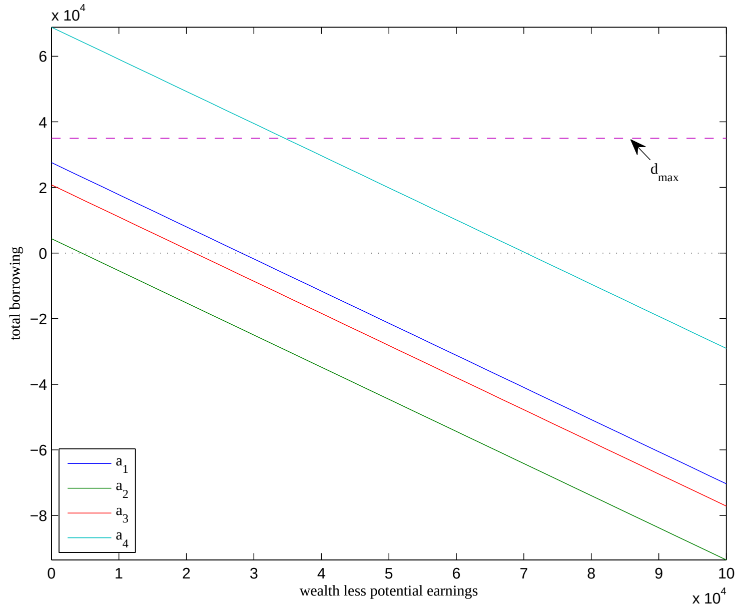


Figure 4: Total Borrowing as a Function of Ability and Wealth less Potential Earnings (Baseline)

wage returns to education by assuming that α increases by 0.01 to 0.71, which leads to a modest increase in the college – high school log wage differential.[40] We model the rise in net tuition costs by assuming that the government subsidy rate, s, falls from 0.799 to 0.71. This reduction reflects the increased importance of tuition and fees as a fraction of total current-fund revenue for public and private universities in the U.S. Our simulations capture the observed stability of maximum GSL loans in real terms by assuming that $\bar{d}^G$ remains unchanged at $\$35,000$. We refer to the baseline parameterization as the "1980s economy" and to the counterfactual parameterization as the "2000s economy." The results in this section are closely related to our theoretical results of Section 2.6 regarding the role of schooling costs and returns. However, unlike increases in the price of skill, an increase in α not only raises the returns to schooling but it also increases the sensitivity of earnings to investment.

The model suggests that the higher returns to investment led to an increase in the amount of available private credit. However, the demand for credit rose even more such that the $w^{G+L}(a)$ thresholds increased substantially relative to their 1980 levels as shown in Figure 5. A much larger set of wealth-ability pairs lies in the constrained region in the 2000s. The model suggests that many youth receiving little or moderate transfers from their parents are likely to be borrowing constrained in the more recent period. Finally, the flat region for the threshold that was present in the

[40]There is some disagreement in the literature regarding the underlying cause for the increase in estimated college – high school wage differentials. Some argue that much of the rise is due to a rising 'return to ability', while others argue that most of the rise is due to an increase in the actual 'return to school'. See Cawley, et al. (2000) and Taber (2001) for detailed discussions of the empirical difficulties and evidence. Changing α more closely reflects the latter, but we increase α less than the amount needed to fully account for the rise in the college – high school log wage differential. An increase in the 'return to ability' is equivalent to shifting the ability distribution upwards in our framework (see Section 2.6), which produces qualitatively similar effects to those discussed here.

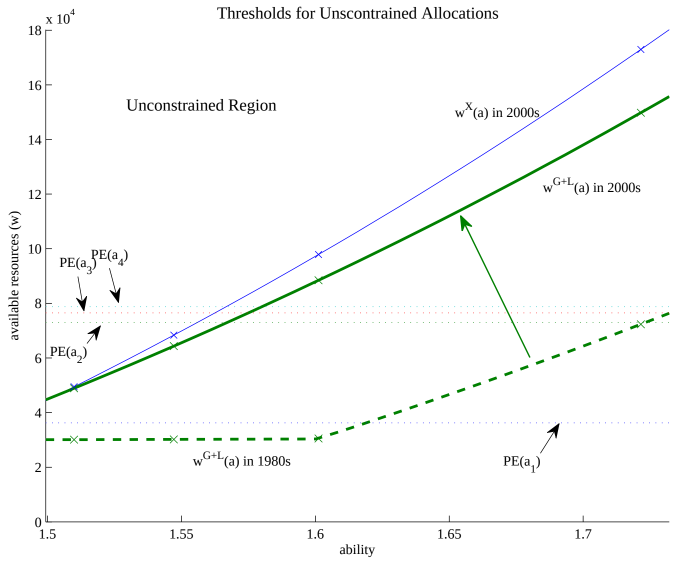


Figure 5: Thresholds for Unconstrained Allocations

1980s economy disappears completely in the 2000s economy, indicating that the only potentially binding constraint in the GSL is $d_g \leq \bar{d}^G$. This rules out the possibility of over-investment.

In the 2000s economy, wealth becomes an important determinant of human capital investment. Figure 6 shows total investment, $h_I(1+s)$, as a function of available resources, w. The solid lines represent investment for the estimated ability levels by AFQT quartile; dotted vertical lines indicate potential earnings and delineate the empirically relevant regions of w for each ability quartile. Consistent with the predictions of Proposition 3 (Lifecycle Model), constrained investment is steeply increasing in wealth until it reaches the unconstrained level. Constraints are binding for a wide range of wealth levels. Most notably, the top ability individuals can only reach unconstrained investments and consumption if their parents give them at least \$70,000 during college.

Credit available from the GSL is no longer sufficient in the 2000s economy. As shown in Figure 7, the model predicts a significant expansion in the set of individuals borrowing beyond the maximum $\bar{d}^G$ from the GSL. Private lending expands to the point that it is comparable to or greater than GSL borrowing for youth with low-to-medium parental transfers. Among the most able, borrowing from private lenders is as much as \$50,000 for a large range of wealth (and parental transfer) levels. Private lenders are willing to provide extra credit, because the increased return to investment raises earnings and the cost of default. Interestingly, borrowing is not monotone in wealth, because constrained wealthy individuals consume and invest more. The latter expands private credit.

The endogeneity of credit limits is important. To see this, compare our baseline model with an exogenous constraint model. Figure 5 includes the threshold $w^X(a)$ assuming $\bar{d}^X = 70,000$ the exogenous limit that yields the same wealth threshold for the lowest ability quartile in the

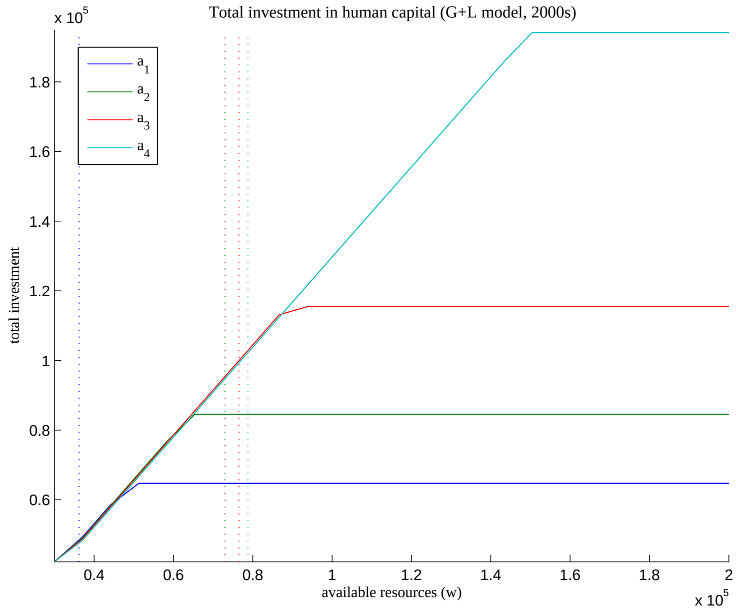


Figure 6: Total investment in human capital (2000s)

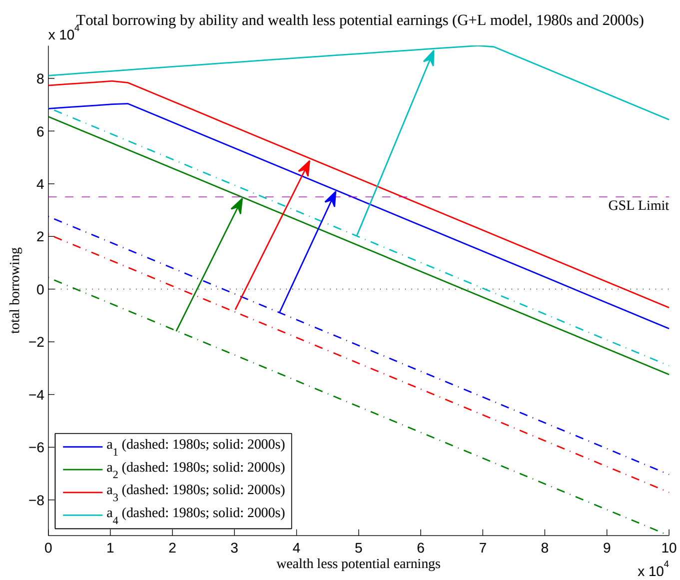


Figure 7: Total borrowing by ability and wealth less potential earnings (1980s and 2000s)

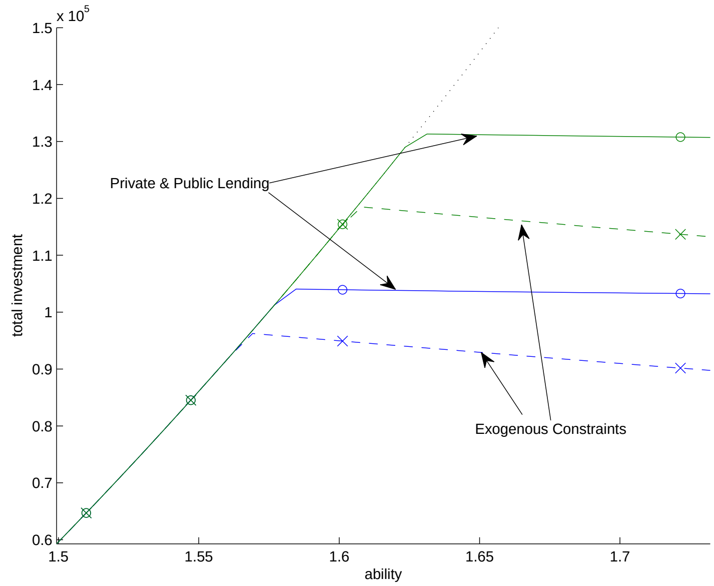


Figure 8: Total investment in human capital for $w = 80,000$ and $w = 100,000$ (2000s)

'2000s economy'.[41] The same set of low ability individuals are constrained in either model, but the steeper $w^X(a)$ curve implies that more higher ability individuals are constrained under exogenous constraints. The gap between the two thresholds is increasing in ability, since private credit endogenously increases with ability.

Figure 8 compares the relationship between ability and human capital investment (for two wealth levels) implied by exogenous constraints and our model with endogenous GSL and private credit constraints. The effects of endogenous constraints on the extensive margin are evident in the wider range of abilities for which unconstrained investment is observed. The effects on the intensive margin for those that are constrained is reflected in the different slopes between the solid and dashed lines at higher abilities. As expected from Proposition 1 and $\sigma > 1$, the exogenous constraint model predicts that constrained investment is decreasing in ability. With exogenous constraints, low-income youth from the top AFQT quartile would invest 5% *less* than youth from the third quartile. In contrast, our baseline model predicts that constrained investments are essentially flat in ability.[42]

It is noteworthy, however, that our model delivers the observed positive ability – investment relationship at the bottom of the family income distribution where family transfers are likely to be negligible. Comparing youth receiving no family or other transfers (i.e. $w = PE(a)$), the most able invest more than double the least able. This is because potential earnings (i.e. resources available to those receiving zero transfers) are increasing in ability. More generally, total investment is

[41] That is, $w^{G+L}(a_1) = w^X(a_1)$. This exogenous constraint level is also consistent with the 1980s, since it does not bind for any estimated ability levels.

[42] We can easily generate steeper ability – investment profiles for our baseline model using higher values for γ and π.

increasing in ability for any given level of transfers, $w - PE(a)$.

In sum, our model predicts that the increase in costs and returns to schooling have led to a rise in borrowing from both the GSL and private lenders. The model further predicts that while more youth have become constrained, private lenders have expanded credit opportunities in response to the higher earnings associated with a college education. These patterns are consistent with the evidence on family income – college attendance patterns in the NLSY79 and NLSY97, the increased fraction of youth constrained by upper GSL limits, and the expansion of private credit. While the model does not necessarily deliver a strong positive relationship between ability and schooling conditional on available resources for constrained youth, it performs noticeably better than the exogenous constraint model.

4.2 Policy Experiments

We next consider the response of human capital investments to three types of changes in the economy: (i) changes in the enforcement institutions underlying private lending; (ii) changes in the extent of GSL programs; and (iii) changes in government subsidies. In all exercises, our point of departure is the 2000s economy where some agents are constrained. We report the response for the lowest resources available by ability (i.e. potential earnings, $PE(a)$), and for other levels of w. For the lowest ability quartile, we report the results for lower values of w, because their potential earnings are substantially lower.

Changes in the enforcement of private lending. Columns 2-6 of Table 3 show the percentage change in human capital investment (relative to the 2000s economy benchmark in column 1) for each ability quartile and different levels of available resources w. Column 2 presents the case of $\pi = 0$, when the GSL is the only source of credit. The elimination of private lending leads to sizeable reductions in investment, as much as 50% for bright youth from poor families. Columns 3 and 4 show that variations in π closer to our benchmark value of 10 years lead to more modest responses in human capital investments. Except for the most able, a punishment period of 15 years would lead to unconstrained investments for all wealth levels; top ability students from poor backgrounds would remain constrained but would invest considerably more than under the benchmark. The punishment period would need to be extended to near retirement (i.e. $\pi \approx R-P$) for the most able with no familial income transfers to be unconstrained.

The next two columns of Table 3 show that a reduction in γ to 0.1 would reduce investment by as much as 25% for the poorest youth of different ability levels, while increasing γ to 0.3 would lead to unconstrained investment for all but the highest ability quartile. Although the latter would substantially increase investment among the most able (by nearly 30% for the very poor), γ needs to rise above 0.45 before everyone is unconstrained. Of course, simultaneously increasing π and γ would more easily ensure unrestricted investment for everyone.

Changes in the GSL program. The remaining columns of Table 3 consider changes to GSL programs. First consider eliminating the GSL program altogether ($\bar{d}^G = 0$). This would severely

Table 3: Effects of Lending Policy Changes on Human Capital Investment (in % terms)

	'Year 2000' Baseline	Private Lending Parameters:					GSL Parameters:		
		$\pi =$			$\gamma =$		$d_{\max} =$		$M = 15$
		0	7	15	.1	.3	0	$50,000$	
$h^{G+L}(a_1, w)$:									
$w = PE(a_1)$	48,239	-12.6	-12.6	19.5	-12.6	34.1	-86.7	34.1	34.1
$w = 50,000$	64,702	-34.8	-11.0	0.0	-25.3	0.0	-61.8	0.0	0.0
$w = 80,000$	64,702	-9.0	0.0	0.0	0.0	0.0	-3.2	0.0	0.0
$w = 100,000$	64,702	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
$h^{G+L}(a_2, w)$:									
$w = PE(a_2)$	84,529	-41.0	0.0	0.0	-10.5	0.0	-36.6	0.0	0.0
$w = 80,000$	84,529	-31.8	0.0	0.0	-0.3	0.0	-25.9	0.0	0.0
$w = 100,000$	84,529	-5.4	0.0	0.0	0.0	0.0	0.0	0.0	0.0
$w = 120,000$	84,529	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
$h^{G+L}(a_3, w)$:									
$w = PE(a_3)$	99,966	-48.0	-10.0	11.7	-21.0	15.5	-41.9	15.5	15.5
$w = 80,000$	104,485	-46.5	-9.7	10.5	-20.3	10.5	-40.1	10.5	10.5
$w = 100,000$	115,447	-32.4	0.0	0.0	-6.8	0.0	-23.6	0.0	0.0
$w = 120,000$	115,447	-13.2	0.0	0.0	0.0	0.0	-1.3	0.0	0.0
$h^{G+L}(a_4, w)$:									
$w = PE(a_4)$	102,213	-50.3	-10.6	12.3	-22.1	28.2	-40.3	17.5	28.3
$w = 80,000$	103,819	-49.8	-10.5	12.2	-21.9	27.9	-39.7	17.2	27.9
$w = 100,000$	129,693	-43.0	-9.0	10.4	-18.8	23.9	-32.0	13.9	22.2
$w = 120,000$	155,581	-38.4	-8.0	9.2	-16.8	21.3	-26.8	11.6	18.4

Notes: Unconstrained investments, $h^U(a)$, are \$64,702, \$84,529, \$115,447, and \$194,164.

restrict investment among the poorest and least able. However, the effects are fairly large for all poor youth regardless of ability. Comparing these results against those with only government lending (i.e. $\pi = 0$) suggests that the GSL is more important for investment among the least able, while private lending is more important for all others. This is because, unlike the GSL, private lenders base credit on ability. Next, we consider an expansion in the GSL program, increasing the upper limit to $\bar{d}^G = \$50,000$. Such a policy would disproportionately benefit the least able poor, but it would also help low income youth of high ability. As with an increase in γ to 0.3, this GSL expansion enables unconstrained investment for the bottom three-quarters of the ability distribution, while effects are comparatively weaker for the most able.

The last column of Table 3 reports the impact of changing the GSL repayment period. Recall that our baseline model allows individuals to spread their GSL re-payments over their entire working careers. Here, we consider reducing the repayment period to 15 years after the completion of school (see Appendix D for details). This change effectively increases the cost of default by reducing resources available for consumption during the period of exclusion from financial markets. Interestingly, such a policy would have nearly identical effects on private lending constraints and human capital accumulation as increasing γ to 0.3. Therefore, our baseline calibration closely mimics a model with a shorter GSL repayment period and lower γ.

Response to education subsidies. Finally, consider the effects of reducing the government subsidy rate s to its 1980s level (in our benchmark 2000s economy). As Table 4 demonstrates, a higher subsidy rate leads to substantial increases in investment with the largest responses among wealthier, unconstrained youth. Investment among constrained youth responds less, because they also want to consume more while in school. Overall, a universal subsidy to investment amplifies inequality in earnings.

Table 4 also compares the investment responses for our model (h^{G+L}) with those for an exogenous constraint model (h^X) with $\bar{d}^X = 70,000$. Since private credit expands with investment in our framework, investment responses are always greater for constrained individuals than under exogenous constraints. The main differences are for the middle ability groups, where the effects are as much as 50% higher in our model compared to the exogenous constraint model.

With respect to the impact of these policies on welfare (not shown here), we make two remarks. First, impacts on welfare tend to be smaller than on human capital investment, because borrowers only benefit from the difference between the returns and costs of additional human capital. Second, impacts on welfare (across different policies or individuals) need not correlate highly with impacts on investment, because consumption is also an important margin of response to credit constraints.

Table 4: Response of Investment to Increasing Subsidy Rate to 1980s Level (2000s Economy)

	2000s benchmark levels		% Changes from benchmark	
	h^{G+L}	h^{X}	h^{G+L}	h^{X}
$h(a_1, w)$:				
$w = PE(a_1)$	$48,239$	$49,823$	8.7	6.7
$w = 50,000$	$64,702$	$64,702$	10.0	7.2
$w = 80,000$	$64,702$	$64,702$	66.8	61.9
$w = 100,000$	$64,702$	$64,702$	66.8	66.8
$h(a_2, w)$:				
$w = PE(a_2)$	$84,529$	$84,529$	21.1	12.2
$w = 80,000$	$84,529$	$84,529$	32.4	22.0
$w = 100,000$	$84,529$	$84,529$	55.7	49.9
$w = 120,000$	$84,529$	$84,529$	55.7	55.7
$h(a_3, w)$:				
$w = PE(a_3)$	$99,966$	$91,481$	6.9	5.7
$w = 80,000$	$104,485$	$95,382$	6.8	5.6
$w = 100,000$	$115,447$	$115,447$	20.3	7.6
$w = 120,000$	$115,447$	$115,447$	43.9	28.0
$h(a_4, w)$				
$w = PE(a_4)$:	$102,213$	$89,252$	7.0	5.8
$w = 80,000$	$103,819$	$90,625$	6.9	5.8
$w = 100,000$	$129,693$	$112,766$	6.5	5.5
$w = 120,000$	$155,581$	$134,984$	6.3	5.3

5 Conclusions

GSL programs and private lending under limited commitment link the borrowing opportunities of young individuals with their cognitive ability and investments in human capital. We show that this link shapes the intertemporal trade-off between investment and consumption for those that are credit constrained and is important for understanding college attendance and borrowing patterns in recent decades. Most notably, the link is important for explaining the positive ability – schooling relationship for youth from low-income families and the rapid expansion in private student lending in recent decades. Conventional wisdom and numerous empirical studies presume that borrowing constraints always inhibit investment; however, we show that this is not the case if what constrains youth is the GSL's tied-to-investment constraint (i.e. their borrowing is restricted by their level of investment). Finally, we show that schooling is more sensitive to government policies when credit depends on investment behavior: policies that increase schooling also expand private credit opportunities, which further increases schooling among constrained youth.

A calibrated version of our model reinforces existing empirical findings that American youth were not constrained during the 1980s but suggests that many youth may be constrained today. This change is explained by rising college costs, even faster rising returns to education, and largely unresponsive GSL programs. Consistent with the evidence, our model predicts that these forces make family resources a more important determinant of higher education, cause more individuals to exhaust their government borrowing opportunities, and lead to an expansion in private lending.

Our framework enables us to study the effects of changes in government student loan programs on private lending. We show that expansions of government lending are only partially offset by reductions in private lending, so total student credit is increasing in GSL limits. In contrast, efforts to extend GSL repayment periods lead to contractions in private lending, since they reduce the costs associated with private loan default. These private credit responses, in turn, affect educational investment decisions. We also study the effects of changes in private loan enforcement or bankruptcy regulations on schooling in our framework. We show that expansions in private loan enforcement capabilities increase human capital investment, especially among the more able, while expansions in government credit tend to favor the least able.

Finally, our model can serve as a natural starting point for future empirical work and policy analysis exploring dimensions ignored here. An obvious next step is to introduce uncertainty and learning about the returns to investment, opening the door to default in equilibrium. Default may serve as insurance against adverse outcomes, and loan contracts with private lenders and the GSL must strike a balance between ensuring repayment and providing insurance against unexpected outcomes. We have also abstracted from school quality and labor supply decisions while in school. Both are likely to be important margins of response in the face of credit constraints and deserve further attention. With reliable data on schooling, borrowing, earnings, and loan repayment (an admittedly tall order), estimation of models that explicitly incorporate government and private lending should provide important new insights on the nature of endogenous borrowing constraints, who is constrained, and the effects of higher education policies and economic changes on private credit offerings and, ultimately, individual schooling and borrowing decisions.

Appendices

A Student Loans in the U.S.

This appendix describes the structure and enforcement of GSL programs and private student lending in the U.S.

A.1 GSL Programs

The largest program is the Stafford Loan program, which awarded nearly $50 billion to students in the 2003-04 academic year.[43] The Parent Loans for Undergraduate Students (PLUS) awarded $7 billion to parents of undergraduate students during the same period. On a much smaller scale, the Perkins Loan program disbursed $1.6 billion to a small fraction of students from very low-income families. See The College Board (2006) for details about financial aid disbursements and their trends over time.

Table A1: Borrowing Limits for Stafford and Perkins Student Loan Programs (1993-2007)

	Stafford Loans		
	Dependent Students	Independent Students*	Perkins Loans
Eligibility Requirements	Subsidized: Financial Need Unsubsidized: All Students		Financial Need
Undergraduate Limits:			
First Year	$2,625	$6,625	$4,000
Second Year	$3,500	$7,500	$4,000
Third-Fifth Years	$4,000	$8,000	$4,000
Cum. Total	$23,000	$46,000	$20,000
Graduate Limits:			
Annual	$18,500		$6,000
Cum. Total**	$138,500		$40,000

Notes:
* Students whose parents do not qualify for PLUS loans can borrow up to independent student limits from Stafford program.
** Cumulative graduate loan limits include loans from undergraduate loans.

Table A1 reports loan limits (based on the dependency status and class of the student) for Stafford and Perkins programs for the period 1993-2007. Dependent students could borrow up to $23,000 from the Stafford Loan Program over the course of their undergraduate careers. Independent students could borrow roughly twice that amount,

[43]The Stafford program offers both subsidized and unsubsidized loans, with the latter available to all students and the former only to students demonstrating financial need. The government waives the interest on subsidized loans while students are enrolled; it does not do so for unsubsidized loans. Prior to the introduction of unsubsidized Stafford Loans in the early 1990s, Supplemental Loans to Students (SLS) were an alternative source of unsubsidized federal loans for independent students.

although most traditional undergraduates do not fall into this category. Qualified undergraduates from low-income families could receive as much as $20,000 in Perkins loans, depending on their need and post-secondary institution. However, amounts offered through this program have typically been less than mandated limits. Since 1993-94, the PLUS loan program no longer has a fixed maximum borrowing limit; however, parents still cannot borrow more than the total cost of college net of other financial aid. Student borrowers can defer loan re-payments until six (Stafford) to nine (Perkins) months after leaving school. Repayment of PLUS loans typically begins within 60 days of loan disbursement.

In real terms, cumulative Stafford loan limits for undergraduates were nearly identical in 2002-03 to what they were twenty years earlier. (NLSY79 and NLSY97 respondents made their college attendance decisions around these two periods.) While the government nominally increased loan limits (especially for upper-year college students) in 1986-87 and 1993-94, inflation has otherwise eroded these limits away.[44]

GSL loans are more strictly enforced relative to typical unsecured private loans. Except in very special circumstances, these loans cannot be expunged through bankruptcy. If a suitable repayment plan is not agreed upon with the lender once a borrower enters default, the default status is reported to credit bureaus and collection costs (up to 25% of the balance due) may be added to the amount outstanding. (Formally, a borrower is considered to be in default once a payment is 270 days late.) Up to 15% of the borrower's wages can also be garnisheed. Moreover, federal tax refunds can be seized and applied toward any outstanding balance. Other sanctions include a possible hold on college transcripts, ineligibility for further federal student loans, and ineligibility for future deferments or forbearances.

A.2 Private Lending

The design of private lending programs is broadly consistent with the problem of lending under limited repayment incentives. Private lenders directly link credit to educational investment expenditures and indirectly to projected earnings. All private student loan programs require evidence of post-secondary school enrollment, offering students credit far in excess of what is otherwise offered in the form of more traditional uncollateralized loans. While many private student lending programs are loosely structured like federal GSL programs, they vary substantially in their terms and eligibility requirements. Some private lenders clearly advertise that they consider the school attended, course of study, and college grades in determining loan packages.

Until the 'Bankruptcy Abuse Prevention and Consumer Protection Act of 2005', individuals could discharge private student loans through bankruptcy. Thus, enforcement of private student loans was regulated by U.S. bankruptcy code. Borrowers filing for bankruptcy under Chapter 7 must pay a court and filing fees of up to a few thousand dollars and surrender any non-collateralized assets (above an exemption) in exchange for discharging all debts; however, most school-leavers considering bankruptcy have few if any assets. Furthermore, bankruptcy shows up on an individual's credit report for ten years, limiting future access to credit. Bankruptcy may spill over into other domains as well (e.g. banks, mortgage companies, landlords, and employers often request credit reports from potential customers or employees). Finally, U.S. bankruptcy requires "good faith" attempts to meet debt obligations, which may make it difficult for former students to expunge their debts if current income levels are high. After reviewing the punishments associated with Chapter 7 bankruptcy, Livshits, MacGee, and Tertilt (2007) argue that they are well-approximated by a temporary period of both wage garnishments and exclusion from credit markets.

[44]From 1982-83 to 2002-03, Stafford borrowing limits for undergraduates declined by 44% for first-year students and 25% for second-year students, while they increased by about 20% for college students enrolled in years three through five. For most of this period, loan limits for independent undergraduates remained about twice the amounts available to dependent students. Stafford loan limits for graduate students declined by about 35% in real terms from 1986–87 to 2006–07, roughly the time NLSY97 respondents would have began attending graduate school.

B NLSY79 and NLSY97 Data

The NLSY79 is a random survey of American youth ages 14-21 at the beginning of 1979, while the NLSY97 samples youth ages 12-16 at the beginning of 1997.[45] Since the oldest respondents in the NLSY97 recently turned age 24 in the 2004 wave of data, we analyze college attendance as of age 21 in both samples.

Individuals are considered to have attended college if they *attended* at least 13 years of school by the age of 21. For the 1979 cohort, we use average family income when youth are ages 16-17, excluding those not living with their parents at these ages. In the NLSY97 data, we use household income and net wealth reported in 1997 (corresponding to ages 13-17), dropping individuals not living with their parents that year. Family income includes government transfers (e.g. welfare and unemployment insurance), but it does not subtract taxes. Net wealth is the value of all assets (e.g. home and other real estate, vehicles, checking and savings, and other financial assets) less loans and credit card debt. We use AFQT as a measure of cognitive ability. It is a composite score from four subtests of the Armed Services Vocational Aptitude Battery (ASVAB) used by the U.S. military: arithmetic reasoning, word knowledge, paragraph comprehension, and numerical operations. These tests are taken by respondents in both the NLSY79 and NLSY97 during their teenage years as part of the survey process. Since AFQT percentile scores increase with age in the NLSY79, we determine an individual's quartile based on year of birth. (AFQT percentile scores in the NLSY97 have already been adjusted to account for age differences.)

C Proofs and Other Aspects of the Two-Period Model

In the text, we assume preferences with a constant IES; however, in this appendix, we allow for general preferences satisfying $u'(c) > 0$ and $u''(c) < 0$. The IES is defined as $\eta(c) \equiv \frac{-cu''(c)}{u'(c)}$.

The set of constrained individuals. For each ability level a, the various forms of credit constraints define a threshold wealth level below which the agent is constrained (and above which he is not). We now characterize those thresholds.

Exogenous Constraints: The threshold $w^X_{\min}(a)$ is defined by $d^U\left(a, w^X_{\min}(a)\right) = \bar{d}^X$, and therefore it is increasing in a. Consumption smoothing implies that $w^X_{\min}(a) \geq h^U(a) - \bar{d}^X$ (the minimum wealth needed to finance $h^U(a)$ given maximum borrowing) and that $w^X_{\min}(a)$ is steeper than $h^U(a)$ as a function of a. To see this, implicit differentiation leads to $\frac{dw^X_{\min}(a)}{da} = \frac{\partial d^U\left(a, w^X_{\min}\right)}{\partial a} / \frac{\partial d^U\left(a, w^X_{\min}\right)}{\partial w} > \frac{\partial d^U\left(a, w^X_{\min}\right)}{\partial a} > \frac{dh^U(a)}{da} > 0$.

GSL Programs: The threshold $w^G_{\min}(a) \equiv \max\{w^X_{\min}(a), \tilde{w}_{\min}(a)\}$, where $\tilde{w}_{\min}(a)$ is defined by $h^U(a) = d^U(a, \tilde{w}_{\min}(a))$. It is increasing in a because $d^U(\cdot, w)$ is steeper than $h^U(\cdot)$. To see that $w^X_{\min}(a)$ is steeper than $\tilde{w}_{\min}(a)$, use implicit differentiation to obtain $\frac{d\tilde{w}_{\min}(a)}{da} = \frac{dw^X_{\min}(a)}{da} + \frac{\partial h^U}{\partial a} / \frac{\partial d^U}{\partial w} < \frac{dw^X_{\min}(a)}{da}$.

GSL Programs Plus Private Lenders: The threshold $w^{G+L}_{\min}(a)$ is defined by $d^U\left(a, w^{G+L}_{\min}(a)\right) = \kappa a f\left(h^U(a)\right) + \min\left\{h^U(a), \bar{d}^G\right\}$. An instructive special case is when $\bar{d}^G = 0$ and only private lending is available in the economy. In this case, the threshold $w^L_{\min}(a)$ is defined by $d^U(a, w^L_{\min}(a)) = \kappa a f\left[h^U(a)\right]$, which increases at a slower rate in a than does $w^X_{\min}(a)$. Indeed, $w^L_{\min}(a)$ may even be decreasing in a if κ is large enough. Both of these facts can be seen from $\frac{dw^L_{\min}}{da} = \frac{dw^X_{\min}}{da} + \kappa\left(f(h^U) + R\frac{\partial h^U}{\partial a}\right) / \frac{\partial d^U}{\partial w} < \frac{dw^X_{\min}}{da}$ because $\frac{\partial d^U}{\partial w} < 0$. In the general case when both private and GSL credit is available, direct inspection reveals that $w^{G+L}_{\min}(a) < \min\left\{w^G_{\min}(a), w^L_{\min}(a)\right\}$. As with $w^L_{\min}(a)$, the threshold $w^{G+L}_{\min}(a)$ can be decreasing in a and may even be negative.

Proof of Lemma 1. Implicit differentiation of (4) yields $\frac{dh^U(a)}{da} = -\frac{f'[h^U(a)]}{af''[h^U(a)]} > 0$. Using expression (5), define

$$F \equiv u'\left[w + d - h^U(a)\right] - \beta R u'\left[af\left[h^U(a)\right] - Rd\right] = 0.$$

From the implicit function theorem $\frac{\partial d^U(a,w)}{\partial a} = -\frac{\partial F}{\partial a} / \frac{\partial F}{\partial d}$, then

$$\frac{\partial d^U(a,w)}{\partial a} = \frac{\partial h^U(a)}{\partial a} + \beta R \frac{u''\left[af\left[h^U(a)\right] - Rd\right] f\left[h^U(a)\right]}{u''\left[w + d - h^U(a)\right] + \beta R^2 u''\left[af\left[h^U(a)\right] - Rd\right]} > \frac{\partial h^U(a)}{\partial a} > 0,$$

[45] See Belley and Lochner (2007) for details on their sample and variables, which we employ here.

where we have used $af'\left[h^U(a)\right]=R$. Similarly,

$$\frac{\partial d^U(a,w)}{\partial w}=-\frac{u''\left[w+d-h^U(a)\right]}{u''\left[w+d-h^U(a)\right]+\beta R^2 u''\left[af\left[h^U(a)\right]-Rd\right]}=-\frac{1}{1+\beta R^2\frac{u''\left[af\left[h^U(a)\right]-Rd\right]}{u''\left[w+d-h^U(a)\right]}}.$$

Since the denominator is greater than one, the argument is complete. ■

Proof of Proposition 1. From the FOC define

$$F\equiv -u'\left(w+\bar{d}^X-h\right)+\beta af'[h]\,u'\left[af(h)-R\bar{d}^X\right]=0.$$

The second order condition implies $\partial F/\partial h<0$, which, combined with implicit differentiation, implies that $sign\left\{\frac{\partial h}{\partial w}\right\}=sign\left\{\frac{\partial F}{\partial w}\right\}$ and $sign\left\{\frac{\partial h}{\partial a}\right\}=sign\left\{\frac{\partial F}{\partial a}\right\}$. First, we have $\frac{\partial h}{\partial w}>0$ since $\frac{\partial F}{\partial w}=-u''\left(w+\bar{d}^X-h\right)>0$. Second,

$$\begin{aligned}\frac{\partial F}{\partial a}&=\beta f'[h]\,u'\left[af(h)-R\bar{d}^X\right]\left\{1+af(h)\frac{u''\left[af(h)-R\bar{d}^X\right]}{u'\left[af(h)-R\bar{d}^X\right]}\right\}\\&<\beta f'[h]\,u'\left[af(h)-R\bar{d}^X\right]\left\{1-1/\eta\left[af(h)-R\bar{d}^X\right]\right\},\end{aligned}$$

where the first results from direct derivation, and the second from $u'>0$, $u''<0$, $f'>0$, $\bar{d}^X\geq 0$, and the definition of IES$\equiv\eta(\cdot)$. If $\eta(c)\leq 1\ \forall\ c>0$, the right-hand-side (RHS) of the last line is non-positive and $\frac{\partial F}{\partial a}<0$. ■

Proof of Proposition 2. Using the FOC for the exogenous constraint model,

$$\hat{a}(w)\equiv\sup\left\{\hat{a}:u'(w)\geq\beta\hat{a}f'\left[\bar{d}^G\right]u'\left[\hat{a}f\left(\bar{d}^G\right)-R\bar{d}^G\right]\right\},$$

which in principle could be $+\infty$. If $u(c)=c^{1-\sigma}/(1-\sigma)$, then $\hat{a}(w)$ is finite and given by $\hat{a}:w\left(\beta f'\left[\bar{d}^G\right]\right)^{\frac{1}{\sigma}}=(\hat{a})^{\frac{\sigma-1}{\sigma}}f\left(\bar{d}^G\right)-R\bar{d}^G(\hat{a})^{\frac{-1}{\sigma}}$. If $\sigma>1$ (IES <1), the RHS is strictly increasing and unbounded, so $\hat{a}(w)$ is finite. The rest is direct upon examination of optimality conditions under the three different cases. ■

Proof of Lemma 2. Part (i) is from direct inspection based on the thresholds as derived above. For part (ii), use the FOC for a constrained person with $a>\bar{a}$ (i.e. $d_g=\bar{d}^G$, $d_p=\kappa af(h)$ and $h>\bar{d}^G$) to define

$$F\left(h,\bar{d}^G,\kappa\right)\equiv\left(\kappa af'(h)-1\right)u'\left[w+\bar{d}^G+\kappa af[h]-h\right]+\beta af'(h)(1-\kappa R)\,u'\left[af(h)(1-\kappa R)-R\bar{d}^G\right].$$

For constrained agents, with $a>\bar{a}$, we have that $u'(c_0)>\beta Ru'(c_1)$ and $af'(h)<R$. It is straightforward to verify that $\frac{\partial F}{\partial \bar{d}^G}>0$, and $\frac{\partial F}{\partial\kappa}>0$, and therefore, implicit differentiation implies the state results.■

Proof Proposition 3. Part (1): If $a>\bar{a}$, the FOC is given by

$$F=u'(c_0)\left[\kappa af'(h)-1\right]+\beta u'(c_1)\,af'(h)(1-\kappa R)=0,$$

where $c_0=w+\kappa af(h)+\bar{d}^G-h$ and $c_1=af(h)(1-\kappa R)-R\bar{d}^G$. Moreover, notice that $\frac{\partial c_0}{\partial h}=\kappa af'(h)-1<0$, and $\frac{\partial c_1}{\partial h}=af'(h)(1-\kappa R)>0$. To prove (i) notice that if the agent is constrained, then $u'(c_0)>\beta Ru'(c_1)$. Therefore, $F=0$ implies $\left[1-\kappa af'(h)\right]<\frac{af'(h)}{R}(1-\kappa R)\implies af'(h)>R$, i.e. there is under-investment. To prove (ii), notice that sign$\left\{\frac{dh}{dw}\right\}=$ sign$\left\{\frac{dF}{dw}\right\}$ and that $dF/dw=u''(c_0)\left[\kappa af'(h)-1\right]>0$. To prove (iii), first define for any $a\geq\bar{a}$ and $h\geq\bar{d}^G$ the fraction of labor earnings needed to pay back the maximum debt from the GSL: $\varrho(a,h)\equiv\frac{R\bar{d}^G}{af(h)}=\varrho(a)\frac{f\left(\bar{d}^G\right)}{f(h)}$, where $\varrho(a)$ is defined in the text. Next, compute the derivative, re-group, simplify, and use the definition of $\eta(\cdot)$ and $\varrho(a,h)$

$$\frac{\partial F}{\partial a}=u'(c_0)\,\kappa f'(h)+\left(1-\kappa af'(h)\right)\left[-u''(c_0)\right]\kappa f(h)+(1-\kappa R)\,\beta f'(h)\,u'(c_1)\left[1-\frac{1}{\eta(c_1)}\frac{1}{1-\varrho(a,h)}\right].\tag{17}$$

Since the agent is constrained, we have that $u'(c_0)\geq\beta Ru'(c_1)$. Using this inequality in the first term and ignoring the second term because it is always positive, obtain

$$\frac{\partial F}{\partial a}\geq\beta u'(c_1)\,f'(h)\left(R\kappa+(1-\kappa R)\left[1-\frac{1}{\eta(c_1)}\frac{1}{1-\varrho(a,h)}\right]\right).$$

The RHS is positive when $\eta(c_1) > \frac{1-\kappa R}{1-\varrho(a,h)}$, which is stated as sufficient condition (A). Next, impose sufficient condition (B) on $\frac{dF}{da}$ in equation (17). Since $\beta R \leq 1$, we have $c_0 < c_1$ and $u'(c_0) \geq u'(c_1)$. Take $u'(c_1) f'(h) > 0$ as a common factor, and in the second term use the FOC implied equality $u'(c_1) = \frac{(1-\kappa a f'(h))}{\beta(1-\kappa R) a f'(h)} u'(c_0)$. Also, divide and multiply by c_0 and simplify to obtain:

$$\begin{aligned}\frac{\partial F}{\partial a} &= u'(c_1) f'(h) \left\{ \kappa \frac{u'(c_0)}{u'(c_1)} + \frac{\kappa\left(1-\kappa a f'(h)\right) f(h)}{f'(h)} \frac{-c_0 u''(c_0)}{c_0 \frac{(1-\kappa a f'(h)) u'(c_0)}{\beta(1-\kappa R) a f'(h)}} + \beta(1-\kappa R)\left(1 - \frac{1}{\eta(c_1)\left[1-\varrho(a,h)\right]}\right)\right\}, \\ &\geq u'(c_1) f'(h) \left\{\kappa + \frac{\kappa\beta}{\eta(c_0)} \frac{1}{1-\varrho(a,h)} + \beta(1-\kappa R)\left[1 - \frac{1}{\eta(c_1)\left[1-\varrho(a,h)\right]}\right]\right\},\end{aligned}$$

where the second line uses the definition of $\varrho(a,h)$, the fact that $c_1 \geq c_0$ and that $u'(c_0) \geq u'(c_1)$. Finally, for $\eta(c)$ constant (as assumed in the text) or increasing,

$$\frac{\partial F}{\partial a} \geq u'(c_1) f'(h) \left\{\kappa + \frac{\kappa\beta}{\eta(c_0)} \frac{1}{1-\varrho(a,h)} + \beta(1-\kappa R)\left[1 - \frac{1}{\eta(c_0)\left[1-\varrho(a,h)\right]}\right]\right\}.$$

This inequality holds whenever the term inside brackets is positive, which holds if $\eta(c_0) \geq \frac{1}{1-\varrho(a,h)} \frac{1-\kappa(R+1)}{1+\kappa(\beta^{-1}-R)}$. Since $\varrho(a,h) \leq \varrho(a)$, the sufficiency of condition (B) follows.

Part (2): The FOC in this case is given by

$$F \equiv u'(c_0)\kappa a f'(h) + \beta u'(c_1)\left[a f'(h)(1-\kappa R) - R\right] = 0,$$

where $c_0 = w + \kappa a f(h)$ and $c_1 = a f(h)(1-\kappa R) - Rh$. To prove (i) notice that if the agent is constrained, then $u'(c_0) > \beta R u'(c_1)$ and $F = 0$ implies that $\kappa a f'(h) < \frac{R - a f'(h)(1-\kappa R)}{R}$. Re-arranging, we get $a f'(h) < R$, or equivalently $h > h^U(a)$ because of the strict concavity of $f(\cdot)$. To prove (ii), compute $\frac{dF}{dw} = \kappa a f'(h) u''(c_0) < 0$. The result follows from implicit differentiation $\left(\frac{\partial h^{G+L}(a,w)}{\partial w} = -\frac{\partial F/\partial w}{\partial F/\partial h}\right)$ and the second order condition $(\partial F/\partial h < 0)$. Similarly, for (iii) $\text{sign}\left\{\frac{\partial h^{G+L}(a,w)}{\partial a}\right\} = \text{sign}\left\{\frac{\partial F}{\partial a}\right\}$. First, compute the derivative

$$\frac{\partial F}{\partial a} = u'(c_0)\kappa f'(h) + \kappa a f'(h) u''(c_0) \frac{\partial c_0}{\partial a} + \beta u'(c_1)\left[f'(h)(1-\kappa R)\right] + \beta\left[a f'(h)(1-\kappa R) - R\right] u''(c_1) \frac{\partial c_1}{\partial a}.$$

Notice that only the second term in this expression can be negative. Take $\frac{1}{a}$ as a common factor and then add and subtract $R\beta u'(c_1)$ to get:

$$\frac{\partial F}{\partial a} = \begin{array}{c} \frac{1}{a}\left\{u'(c_0)\kappa a f'(h) + \beta u'(c_1)\left[a f'(h)(1-\kappa R) - R\right]\right\} \\ + \frac{1}{a}\left\{R\beta u'(c_1) + \kappa a f'(h) u''(c_0) \frac{\partial c_0}{\partial a} a + \beta\left[a f'(h)(1-\kappa R) - R\right] u''(c_1) \frac{\partial c_1}{\partial a} a\right\}. \end{array}$$

The FOC implies that the first line equals zero. Take $R\beta u'(c_1)$ as common factor and multiply and divide the second term by $u'(c_0)$:

$$\frac{\partial F}{\partial a} = \frac{R\beta u'(c_1)}{a}\left\{1 + \frac{1}{R}\left(\frac{u'(c_0)\kappa a f'(h)}{\beta u'(c_1)}\right) \frac{u''(c_0)}{u'(c_0)} \frac{\partial c_0}{\partial a} a + \frac{1}{R}\left[a f'(h)(1-\kappa R) - R\right] \frac{u''(c_1)}{u'(c_1)} \frac{\partial c_1}{\partial a} a\right\}.$$

Because of the FOC, the expression inside parentheses in the second term equals $\left[R - a f'(h)(1-\kappa R)\right]$. After dividing and multiplying the last two terms by c_0 and c_1, respectively, and using the definition of the IES, $\eta(c)$, and re-grouping:

$$\frac{\partial F}{\partial a} = \frac{R\beta u'(c_1)}{a}\left\{1 + \left[1 - \frac{a f'(h)}{R}(1-\kappa R)\right]\left[\frac{1}{\eta(c_1)}\left(\frac{\partial c_1}{\partial a}\frac{a}{c_1}\right) - \frac{1}{\eta(c_0)}\left(\frac{\partial c_0}{\partial a}\frac{a}{c_0}\right)\right]\right\}.$$

The term $1 - \frac{a f'(h)}{R}(1-\kappa R) > 0$, since there is over-investment, i.e. $a f'(h) < R$. Therefore, $\frac{\partial F}{\partial a}$ can only be negative if $\frac{1}{\eta(c_1)}\left(\frac{\partial c_1}{\partial a}\frac{a}{c_1}\right) - \frac{1}{\eta(c_0)}\left(\frac{\partial c_0}{\partial a}\frac{a}{c_0}\right)$. However, notice that since $\frac{\partial c_1}{\partial a}\frac{a}{c_1} = \frac{a f(h)(1-\kappa R)}{a f(h)(1-\kappa R) - Rh} > 1$ and $\frac{\partial c_0}{\partial a}\frac{a}{c_0} = \frac{\kappa a f(h)}{w + \kappa a f(h)} < 1$, this possibility is ruled out if $\eta(c_1) \leq \eta(c_0)$, which clearly holds if $\eta(\cdot)$ is constant. ■

Proof Corollary 1. We first show that our normalized model ($p = \tau = 1$) is isomorphic to any model with arbitrary fixed (τ, p). Define $e \equiv \tau h$, reflecting total investment expenditures measured in units of the consumption good, and re-write the (TIC) constraint as $d \leq e$. Earnings can be written as $y = paf(e/\tau)$. Defining $g(e) \equiv pf(e/\tau)$, the function $g(\cdot)$ inherits all of the relevant properties of $f(\cdot)$ (e.g. positive, increasing, concave) assumed in the text. Therefore, for any pair (p, τ), the maximization problem in terms of h and $f(\cdot)$ can be equivalently mapped into our normalized maximization problem using e and $g(\cdot)$.

Now, consider changes in p holding τ constant: the partial derivatives of h^X, h^G and h^{G+L} with respect to p are the same as the respective derivatives with respect to a (up to a multiplicative positive constant), so the proofs of Propositions 1-3 directly apply to the relationship between p and h or e. Finally, we discuss simultaneous increases in p and τ with p/τ increasing. Consider a new pair (p', τ') so that $\tau' > \tau$, and $p' = px(\tau'/\tau) > p$, where $x \geq 1$. Assuming that $f(h) = h^\alpha$ with $\alpha \in (0, 1)$, the new earnings function can be written as

$$y = p'a\left(\frac{e}{\tau'}\right)^\alpha = px\left(\frac{\tau'}{\tau}\right)a\left(\frac{e}{\tau}\frac{\tau}{\tau'}\right)^\alpha = p\hat{a}\left(\frac{e}{\tau}\right)^\alpha,$$

where $\hat{a} \equiv a \cdot \left[x(\tau'/\tau)^{1-\alpha}\right] > a$, since the term inside brackets is greater than one by assumption. Setting the problems in terms of e (expenditures on human capital investment), Propositions 1-3 establish the direction of change for constrained investment expenditures in the respective models. ■

D Proofs and Other Aspects of the Quantitative Model

First, define $\Theta_{[t_0,t_1]} \equiv \int_{t_0}^{t_1} e^{\left[\frac{r-\rho}{\sigma}-r\right](t-t_0)}dt$. This function indicates the present value of optimal unconstrained consumption growth factors $\left(\frac{r-\rho}{\sigma}\right)$ between any pair of dates t_0 and t_1. That is, $\int_{t_0}^{t_1} e^{-r(t-t_0)}c^U(t)dt = \Theta_{[t_0,t_1]}c^U(t_0)$. With CRRA preferences, the function $\Theta_{[t_0,t_1]}$ also indicates the present value of growth factors of utility flows. The variable θ in the text is defined as $\theta \equiv \frac{\Theta_{[S,P]}}{\Theta_{[S,T]}}$ and represents the ratio of discounted present value of optimal unconstrained consumption over the schooling period relative to the full lifetime. An unrestricted individual with ability a and initial wealth w enters the labor market with debt (in present value terms of $t = P$)

$$d^U(a, w) = \Phi\theta a\left[h^U(a)\right]^\alpha + e^{r(P-S)}(1-\theta)\left[h_I^U(a) - w\right]. \tag{18}$$

This function $d^U(a, w)$ shares the same essential properties as in the two-period model.

Also, under assumptions (1)-(3) in the text, even if GSL constraint (13) or private lending constraint (15) bind, the intra-period consumption allocations within intervals $[S, P)$ and $[P, T]$ are not distorted. As a result, lifetime utility can be written as

$$\Theta_{[S,P]}\frac{\left[\left(w + e^{-r(P-S)}d - h_I\right)/\Theta_{[S,P]}\right]^{1-\sigma}}{1-\sigma} + e^{-\rho(P-S)}\Theta_{[P,T]}\frac{\left[\left(ah^\alpha\Phi - d\right)/\Theta_{[P,T]}\right]^{1-\sigma}}{1-\sigma}, \tag{19}$$

which is equivalent to representation (14) (up to a positive multiplicative constant) with $\hat{\beta} \equiv e^{-\rho(P-S)}\left(\frac{\Theta_{[P,T]}}{\Theta_{[S,P]}}\right)^\sigma$.

Derivation of the Credit Constraints. Consider an individual with ability a, human capital h, and GSL liabilities d_g, contemplating whether to default on private debt d_p at time $t = P$, i.e. when he enters the labor market. If he does not default, he retains access to formal credit markets and is able to optimally smooth consumption. At that point, his net lifetime resources are equal to $\Phi ah^\alpha - d_g - d_p$, and he is able to attain a discounted utility equal to

$$V^R(a, h, d_g + d_p) = \Theta_{[P,T]}\frac{\left[\left(\Phi ah^\alpha - d\right)/\Theta_{[P,T]}\right]^{1-\sigma}}{1-\sigma}, \tag{20}$$

where the superscript R indicates full repayment. On the other hand, if he defaults, he attains utility

$$\begin{aligned} V^D(a,h,d_g,r(\cdot;d_g)) = &\int_P^{P+\pi} e^{-\rho(t-P)}\frac{\left[(1-\gamma)ah^\alpha E(t-P) - r\left(t;d_g\right)\right]^{1-\sigma}}{1-\sigma}dt \\ &+ \frac{e^{-\rho\pi}\Theta_{[P+\pi,T]}}{1-\sigma}\left[\frac{\Phi_{[P+\pi,R]}ah^\alpha - e^{r\pi}\left(d_g - R\left(P+\pi, d_g\right)\right)}{\Theta_{[P+\pi,T]}}\right]^{1-\sigma}. \end{aligned} \tag{21}$$

The first term is the discounted utility during the punishment phase from P to $P+\pi$. During that time, consumption equals earnings net of garnishments (from private lenders) and GSL debt repayments $r(t;d_g)$. The second term reflects the discounted utility acquired after the punishment ends. When entering the post-punishment phase, the individual is cleared of all private debt, but he carries a liability with GSL lenders equal to $e^{r\pi}\left[d_g - R\left(P+\pi, d_g\right)dt\right]$, where $R\left(P+\pi, d_g\right) = \int_P^{P+\pi} e^{-r(t-P)} r\left(t; d_g\right)dt$ represents cumulative repayments to GSL debt throughout the punishment period. With renewed unrestricted access to financial markets, the available resources $\Phi_{[P+\pi,R]}ah^\alpha - e^{r\pi}\left(d_g - R\left(P+\pi, d_g\right)\right)$ are consumed smoothly over the remaining life $[P+\pi, T]$. Here, we define the function $\Phi_{[t_0,t_1]} \equiv \int_{t_0}^{t_1} e^{-r(t-t_0)}E\left(t-P\right)dt$, which generalizes the definition of Φ in the text for any initial and final dates $P \le t_0 < t_1 \le R$. Note that the value of default depends on the actual timing of GSL repayment $r\left(\cdot; d_g\right)$ but not on the amount of private debt d_p. Also, note that the value of repayment does not depend on the timing of GSL repayment, since individuals that do not default can freely borrow and lend after school to fully smooth consumption.

For analytical tractability, we assume that GSL debtors must repay at least a constant fraction δ of their earnings during the punishment period to service their GSL debt. Assume that for a period equal to or longer than the length of default punishment π, repayments to GSL loans are given by $r\left(t, d_g\right) = \delta a h^\alpha E\left(t-P\right)$, i.e. the individual must pay a constant fraction of his earnings. Then, $R\left(P+\pi, d_g\right) = \delta\Phi_{[P,P+\pi]}ah^\alpha$ and the post-punishment balance of GSL debt is $e^{r\pi}\left(d_g - \delta\Phi_{[P,P+\pi]}ah^\alpha\right)$. At one extreme is the "fastest" repayment case when $\delta = \delta_{fast} = d_g / \left(\Phi_{[P,P+\pi]}ah^\alpha\right)$ and all GSL debt must be repaid during the punishment period. This is the most disruptive case and is only relevant if earnings are high enough to cover the debt and leave positive consumption during the punishment period (i.e. $d_g/\Phi_{[P,P+\pi]}ah^\alpha < 1-\gamma$). The attainable utility of a defaulting individual is

$$V_P^D(a,h,d_g,\delta_{fast}) = \Delta\frac{\left[\left(1-\gamma\right)ah^\alpha - d_g/\Phi_{[P,P+\pi]}\right]^{1-\sigma}}{1-\sigma} + e^{-\rho\pi}\left(\Theta_{[P+\pi,T]}\right)^\sigma \frac{\left[\Phi_{[P+\pi,R]}ah^\alpha\right]^{1-\sigma}}{1-\sigma},$$

where $\Delta \equiv \int_P^{P+\pi} e^{-\rho(t-P)}E\left(t-P\right)^{1-\sigma}dt$. At the opposite extreme is the case of "slowest" repayment in which no repayment is made while the individual is being punished, i.e. $\delta = \delta_{slow} = 0$. All GSL debt is rolled-over to the post-punishment period, leading to a balance of $e^{r\pi}d_g$ at time $P+\pi$. This case is relevant only if $\Phi_{[P+\pi,R]}ah^\alpha > e^{r\pi}d_g$. It leads to utility

$$V_P^D(a,h,d_g,\delta_{slow}) = \Delta\frac{\left[\left(1-\gamma\right)ah^\alpha\right]^{1-\sigma}}{1-\sigma} + e^{-\rho\pi}\left(\Theta_{[P+\pi,T]}\right)^\sigma \frac{\left[\Phi_{[P+\pi,R]}ah^\alpha - e^{r\pi}d_g\right]^{1-\sigma}}{1-\sigma},$$

which, in general, is greater than $V_P^D(a,h,d_g,\delta_{fast})$, because repayments are scheduled in a way that minimizes the disruption of consumption smoothing.

In general, for intermediate values of δ, we can use (21) with the condition $V^R \ge V^D$ to obtain a closed form for the constraint on private credit:

$$d_p \le \Phi_{[P,R]}ah^\alpha - d_g - \left[M_0\left(ah^\alpha\right)^{1-\sigma} + M_1\left(M_2 ah^\alpha - e^{r\pi}d_g\right)^{1-\sigma}\right]^{\frac{1}{1-\sigma}},$$

with $M_0 \equiv \Delta\left[1-\gamma-\delta\right]^{1-\sigma} / \left(\Theta_{[P,T]}\right)^\sigma$, $M_1 \equiv e^{-\rho\pi}\left(\frac{\Theta_{[P+\pi,T]}}{\Theta_{[P,T]}}\right)^\sigma$, and $M_2 \equiv \Phi_{[P+\pi,R]} + e^{r\pi}\delta\Phi_{[P,P+\pi]}$. Clearly, private debt limits are positively linked to post-school earnings Φah^α and negatively linked to the amount of GSL debt d_g. However, as expected from its superior enforcement, GSL debt does not lead to a one-to-one reduction in the capacity to borrow from private lenders as captured by the fact that the CES term in the right-hand-side is negatively related to d_g. Thus, in general, an expansion of the GSL credit limit $\bar{d}^G$ leads to an overall expansion in available credit.

For our baseline case, we set $\delta = \delta^* \equiv \left(1-\gamma\right)\frac{e^{r\pi}\left(d_g - \delta\Phi_{[P,P+\pi]}ah^\alpha\right)}{\Phi_{[P+\pi,R]}ah^\alpha}$. In this case, the fraction $\left(1-\gamma-\delta^*\right)$ of income (net of garnishments) available to consume during the punishment period is equal to the fraction $\frac{\Phi_{[P+\pi,R]}ah^\alpha - e^{r\pi}\left(d_g - \delta\Phi_{[P,P+\pi]}ah^\alpha\right)}{\Phi_{[P+\pi,R]}ah^\alpha}$ of the present value of labor earnings (net of GSL debt payments) available for consumption during the post-punishment periods. Imposing this equality, we can write

$$V_P^D(a,h,d_g,\delta^*) = \Delta\frac{\left[\left(1-\gamma\right)\left(ah^\alpha - \frac{d_g}{\Phi - \gamma\Phi_{[P,P+\pi]}}\right)\right]^{1-\sigma}}{1-\sigma} + e^{-\rho\pi}\left(\Theta_{[P+\pi,T]}\right)^\sigma \frac{\left[\Phi_{[P+\pi,R]}\left(ah^\alpha - \frac{d_g}{\Phi - \gamma\Phi_{[P,P+\pi]}}\right)\right]^{1-\sigma}}{1-\sigma},$$

Define $\Theta_D \equiv \Delta(1-\gamma)^{1-\sigma} + e^{-\rho\pi}\left[\frac{\Theta_{[P+\pi,T]}}{\Phi_{[P+\pi,R]}}\right]^{\sigma}\Phi_{[P+\pi,R]}$ and factorize $\left[ah^{\alpha} - \frac{d_g}{\left[\Phi-\gamma\Phi_{[P,P+\pi]}\right]}\right]^{1-\sigma}$, then

$$V_P^D(a,h,d_g,\delta^*) = \Theta_D \frac{\left[ah^{\alpha} - \frac{d_g}{\left[\Phi-\gamma\Phi_{[P,P+\pi]}\right]}\right]^{1-\sigma}}{1-\sigma}.$$

This expression and the condition $V^R \geq V^D$ leads to the formula $d_p \leq \kappa_1 \Phi a h^{\alpha} + \kappa_2 d_g$ in the text, where

$$\kappa_1 \equiv 1 - \frac{\Theta_{[P,T]}}{\Phi}\left(\frac{\Theta_D}{\Theta_{[P,T]}}\right)^{\frac{1}{1-\sigma}} \quad \text{and} \quad \kappa_2 = \left(\frac{\Theta_D}{\Theta_{[P,T]}}\right)^{\frac{1}{1-\sigma}} \frac{\Theta_{[P,T]}}{\Phi - \gamma\Phi_{[P,P+\pi]}} - 1.$$

Direct inspection of these formulas verifies that $0 \leq \kappa_1 \leq 1$, $\kappa_2 > -1$, as well as the other properties stated in the text. Finally, when GSL loans must be repaid within $[P,Q]$ for $P+\pi < Q < R$, the private credit constraint becomes

$$d_p \leq \Phi a h^{\alpha} - d_g - \left(\frac{\Delta}{\left[\Theta_{[P,T]}\right]^{\sigma}}\left[(1-\gamma)ah^{\alpha} - \frac{d_g}{\Phi_{[P,Q]}}\right]^{1-\sigma} + e^{-\rho\pi}\left[\frac{\Theta_{[P+\pi,T]}}{\Theta_{[P,T]}}\right]^{\sigma}\left[\Phi_{[P+\pi,R]}ah^{\alpha} - e^{r\pi}\left(1 - \frac{\Phi_{[P,P+\pi]}}{\Phi_{[P,Q]}}\right)d_g\right]^{1-\sigma}\right)^{\frac{1}{1-\sigma}}.$$

In the text, we consider $M \equiv Q - P = 15$.

Thresholds. With the above characterization of the credit constraints in our baseline model, we can derive explicit formulas for the thresholds of wealth that define the sets of constrained individuals. Let $m^U(a) \equiv \Phi\theta a\left[h^U(a)\right]^{\alpha} + e^{r(P-S)}(1-\theta)h_I^U(a)$. Then, $w^X(a) \equiv e^{-r(P-S)}(1-\theta)^{-1}\left[m^U(a) - \bar{d}^X\right]$, $w^L(a) \equiv e^{-r(P-S)}(1-\theta)^{-1}\left[m^U(a) - \kappa_1\Phi a\left[h^U(a)\right]^{\alpha}\right]$, $w^G(a) \equiv e^{-r(P-S)}(1-\theta)^{-1}\left[m^U(a) - \min\left\{e^{r(P-S)}h^U(a), \bar{d}^G\right\}\right]$, and $w^{G+L}(a) \equiv e^{-r(P-S)}(1-\theta)^{-1}\left[m^U(a) - \kappa_1\Phi a\left[h^U(a)\right]^{\alpha} - \kappa_2\min\left\{e^{r(P-S)}h^U(a), \bar{d}^G\right\}\right]$.

Proof Proposition 3.(Lifecycle Model) All three items in Part 1 and items (i) and (ii) of Part 2 follow virtually the same lines as in Proposition 3 of the two-period case. We proceed to prove item (iii). Our case of interest is when $a > \bar{a}$ and $h > h^* \equiv h_0 + (1+s)\bar{d}^G = \left[\alpha(1+s)\Phi\bar{a}e^{-r(P-S)}\right]^{\frac{1}{1-\alpha}}$. In this case, if the individual is constrained, then $d_g = \bar{d}^G$ and $d_p = \kappa_1\Phi a h^{\alpha} + \kappa_2\bar{d}^G$. As in the two-period model, for $a > \bar{a}$ and $h > h^*$, define $\varrho(a,h)$ as the fraction of life-time labor earnings an individual must pay to cover the maximum debt from the GSL, $\varrho(a,h) \equiv \frac{\bar{d}^G}{\Phi a h^{\alpha}} \leq \varrho(a) \leq \frac{\alpha e^{-r(P-S)}}{1+\frac{h_0}{(1+s)\bar{d}^G}} < \alpha < 1$. To keep notation manageable, define $C_0 = \left(w + e^{-r(P-S)}\left[\kappa_1\Phi a h^{\alpha} + (\kappa_2+1)\bar{d}^G\right] - h_I\right)/\Theta_{[S,P]}$ and $C_1 = \left(\Phi a h^{\alpha}(1-\kappa_1) - (\kappa_2+1)\bar{d}^G\right)/\Theta_{[P,T]}$. The problem for a constrained agent is entirely in terms of h_I :

$$\max_{\{h_I\}}\left\{\Theta_{[S,P]}\frac{C_0^{1-\sigma}}{1-\sigma} + e^{-\rho(P-S)}\Theta_{[P,T]}\frac{C_1^{1-\sigma}}{1-\sigma}\right\}.$$

Since $a > \bar{a}$, the solution is interior and given by the FOC

$$F \equiv C_0^{-\sigma}\left[\alpha\kappa_1\Phi a h^{\alpha-1}(1+s) - 1\right]e^{-r(P-S)} + e^{-\rho(P-S)}C_1^{-\sigma}\alpha\Phi a h^{\alpha-1}(1-\kappa_1)(1+s) = 0.$$

As before, the relationship between ability and investment is given by $\frac{dF}{da}$, which is:

$$\begin{aligned}\frac{\partial F}{\partial a} &= C_0^{-\sigma}\left[\alpha\kappa_1\Phi h^{\alpha-1}\right]e^{-r(P-S)}(1+s) - \sigma C_0^{-(\sigma+1)}\left[\alpha\kappa_1\Phi a h^{\alpha-1}(1+s) - 1\right]e^{-r(P-S)}\frac{\partial C_0}{\partial a} \\ &\quad + e^{-\rho(P-S)}C_1^{-\sigma}\alpha\Phi h^{\alpha-1}(1-\kappa_1)(1+s) - \sigma e^{-\rho(P-S)}C_1^{-\sigma-1}\alpha\Phi a h^{\alpha-1}(1-\kappa_1)(1+s)\frac{\partial C_1}{\partial a}.\end{aligned}$$

Using F=0 and factorizing $\Psi \equiv e^{-\rho(P-S)}C_1^{-\sigma}\alpha\Phi h^{\alpha-1}(1-\kappa_1)(1+s) > 0$,

$$\frac{\partial F}{\partial a} = \Psi\left\{\left[\frac{C_1}{C_0}\right]^{\sigma}e^{(\rho-r)(P-S)}\frac{\kappa_1}{1-\kappa_1} + \sigma\frac{\partial C_0}{\partial a}\frac{a}{C_1}\frac{C_1}{C_0} + 1 - \sigma\frac{\partial C_1}{\partial a}\frac{a}{C_1}\right\},$$

where the second term has been multiplied and divided by C_1. Since the individual is constrained, $C_1/C_0 > e^{\left[\frac{r-\rho}{\sigma}\right](P-S)}$ and $(C_1/C_0)^{\sigma} > e^{[r-\rho](P-S)}$. Using these inequalities in the second and first term, respectively, and then simplifying:

$$\frac{\partial F}{\partial a} > \Psi \left\{ \frac{1}{1-\kappa_1} + \sigma \left(e^{\left[\frac{r-\rho}{\sigma}\right](P-S)} \frac{\partial C_0}{\partial a} \frac{a}{C_1} - \frac{\partial C_1}{\partial a} \frac{a}{C_1} \right) \right\}.$$

Using the definitions of C_0 and C_1, compute $\frac{\partial C_0}{\partial a}\frac{a}{C_1} = \frac{\Theta_{[P,T]}}{\Theta_{[S,P]}} \frac{\kappa_1 e^{-r(P-S)}}{[1-\kappa_1-(\kappa_2+1)\varrho(a,h)]}$ and $\frac{\partial C_1}{\partial a}\frac{a}{C_1} = \frac{1-\kappa_1}{[1-\kappa_1-(\kappa_2+1)\varrho(a,h)]}$. Plug these expressions in and simplify to obtain

$$\frac{\partial F}{\partial a} > \Psi \left[\frac{1}{1-\kappa_1} + \frac{\sigma\left(\kappa_1 \theta^{-1} - 1\right)}{[1-\kappa_1-(\kappa_2+1)\,\varrho\,(a,h)]} \right].$$

The RHS is positive iff

$$\sigma\left(\kappa_1 \theta^{-1} - 1\right) \geq -\frac{[1-\kappa_1-(\kappa_2+1)\,\varrho\,(a,h)]}{1-\kappa_1}, \tag{22}$$

which holds if $\kappa_1 \geq \theta$, i.e. sufficient condition (A), because by construction $1-\kappa_1-(\kappa_2+1)\,\varrho\,(a,h) > 0$ when $a > \bar{a}$ and $h > h^*$. If $\kappa_1\theta^{-1} < 1$, then (22) can be written as $\sigma \leq \left[1 - \frac{(\kappa_2+1)}{1-\kappa_1}\varrho\,(a,h)\right]\left[1-\kappa_1\theta^{-1}\right]^{-1}$. The claim for sufficient condition (B) holds because $\varrho\,(a,h) \leq \varrho\,(a)$. ■

E Calculating Total Educational Expenditures

In calculating total educational expenditures by years of college education (i.e. highest grade completed less 12) and AFQT quartile, we add estimates of foregone earnings and average tuition costs by year of school (see Table E1). We now explain these calculations in some detail.

Foregone earnings. Foregone earnings for each year of college reflect the present value of average earnings relative to someone with the same ability but only 12 years of schooling. Our calculations take into account earnings during college. For someone with AFQT quartile q and $C \geq 1$ years of college completed, foregone earnings are calculated as $FE(q,C) = \sum_{x=0}^{C-1}(1+r)^{1-x}[\bar{y}_{12}(q, 19+x) - \hat{y}_{12+C}(q, 19+x)]$, where $r = .05$, $\bar{y}_{12}(q,j)$ reflects average wage income for men with 12 years of schooling, AFQT quartile q, and age j, and $\hat{y}_{12+C}(q,j)$ reflects predicted earnings for men with C years of completed college, AFQT quartile q, and age j. This prediction is based on a regression of earnings on AFQT quartile indicators, experience (= age -19), and experience-squared using a sample of men that are enrolled in college and whose age is between 19 and 26 (with age not exceeding $18 + C$).

Total costs of schooling and the government subsidy matching rate. Direct expenditures for the first two years of college are based on 2-yr school averages for academic years 1980-81 to 1984-85, while direct expenditures for 3+ years of college are based on 4-yr school averages for academic years 1980-81 to 1989-90.[46] These dates correspond to the years most students in our NLSY79 sample attended college.

To calculate the subsidy rate s, we first compute marginal subsidy rates for each year of college (1-8 years) by AFQT quartile. This is computed as 0.77 × direct expenditures divided by total expenditures, where 0.77 reflects the ratio of current-fund revenue that does not come from tuition and fees averaged over academic years 1980-81 to 1989-90 (Digest of Education Statistics, 2003, Table 333). Since these rates differ somewhat by the number of years of schooling and AFQT quartile, we average over these values using the distribution of completed schooling in our NLSY79 sample. The resulting government subsidy matching rate is $s = 0.799$. In simulating the 'year 2000' economy below, we use a lower subsidy matching rate of $s = 0.710$, consistent with the observed rise in current fund revenue that came from tuition (from 0.23 to 0.28). See the Digest of Education Statistics (2003, Table 333).

[46]Direct expenditures for 3+ year of college are based on 3+ years of average expenditures at 4-year schools, while direct expenditures for 1-2 years are based on 1-2 years at two-year schools. Because average expenditures at 4-year institutions are higher than at 2-year institutions, this generates a noticeable jump in direct expenditures between 2 and 3 years of college.

Table E1: Total Schooling Costs for each year of college by AFQT Quartile (1999 Dollars)

Years of	Direct	Foregone Earnings:				Total Costs:			
College	Expenditures	Quartile 1	Quartile 2	Quartile 3	Quartile 4	Quartile 1	Quartile 2	Quartile 3	Quartile 4
1	6,322	3,604	8,560	6,716	6,841	9,927	14,882	13,038	13,163
2	12,343	8,689	19,446	17,530	15,476	21,032	31,789	29,873	27,819
3	58,275	14,844	30,467	29,257	26,288	73,119	88,742	87,532	84,563
4	75,880	21,222	40,825	40,350	39,106	97,102	116,704	116,229	114,985
5	92,646	26,606	51,201	51,232	51,300	119,252	143,847	143,878	143,947
6	108,615	31,799	60,135	60,509	61,431	140,413	168,750	169,124	170,045
7	123,822	35,531	67,669	69,302	70,733	159,354	191,491	193,125	194,555
8	138,306	36,243	72,981	76,520	78,758	174,549	211,287	214,826	217,064

Notes:

1) Direct expenditures based on average expenditures per student in all colleges and universities. Expenditures for first two years of college are based on 2-yr school averages for school years 1980-81 to 1984-85. Expenditures for grades 15+ are based on 4-yr school averages for school years 1980-81 to 1989-90. Costs are discounted at a 5% annual interest rate back to grade 12. (Source: Digest of Education Statistics, Table 342, 1999.)

2) Foregone earnings reflect the PV of average earnings relative to someone with 12 years of completed schooling, taking into account earnings during college. See text for details.

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