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The Student Loan Consolidation Option

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Abstract

The federal government makes subsidized federal financing for higher education widely available. The extent of the subsidy varies over time with interest rate and credit market conditions. A loan provision that has added considerably to the size and volatility of the subsidy is the consolidation option, which allows students to convert floating rate federal loans to a fixed rate equal to the average floating rate on their outstanding loans. We develop a model to estimate the option's cost and to evaluate its sensitivity to changes in program rules, economic conditions, and borrower behavior. We model borrower behavior using data from the National Student Loan Data System, which provides new insights on the responsiveness of consumers to financial incentives.

1. Introduction

The federal student loan program makes subsidized federal financing for higher education widely available. Such assistance may improve social welfare when imperfections in private credit markets impede efficiency or when there are positive externalities from education,[1] although the effectiveness of these policies is controversial.[2] In any case, one would expect an efficient subsidy to reduce credit rationing or lower the cost of educational investment for target groups. Although the federal student loan program furthers these goals,[3] it also contains costly provisions that are hard to justify on efficiency grounds.

One apparently inefficient provision is the consolidation option, an exotic financial derivative, created by a few paragraphs in the Higher Education Act. This provision allows students to convert their floating rate loans to a fixed rate loan having a rate equal to the average floating rate on their outstanding loans (for some borrowers, it also allows maturity extension).[4] For instance, in 2001 borrowers who consolidated their student loans could lock in an interest rate below 3 percent per annum, a rate that was significantly below prevailing fixed long-term interest rates at that time.

The consolidation option is of interest for several reasons. For one, the option creates a multi-billion dollar public expenditure that has gone largely unmeasured and therefore has

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1 Lochner and Monge-Naranjo (2002) provide a general equilibrium analysis of the interaction between borrowing constraints and human capital formation, and show that government policies that weaken credit constraints can lead to substantial increases in human and physical capital.

2 See, for example, De Fraja (2002), Dynarski (2002), Edlin (1993), Hanushek (1989), and Keane (2002). Gale (1991) points out that many federal credit programs probably have a small real effect on the allocation of credit, in many cases simply crowding out private borrowing and lending.

3 Further, legislation exempting student loans from dismissal in bankruptcy may alleviate market imperfections caused by restrictions on forward contracting of human capital.

4 Recent legislation fixed the rate on new Stafford loans at 6.8 percent starting in July of 2006 and made other changes that significantly lowered the value of the consolidation option on newly originated loans. The option remains valuable for borrowers with outstanding floating rate loans. The options may also be valuable for some fixed rate borrowers who would not otherwise be eligible for longer repayment horizons. Going forward, the option, which remains in effect, could reemerge as a costly benefit if lawmakers reinstate floating rates. They might be motivated to do so, for instance, in response to the budgetary pressures that would arise were the term structure to steepen significantly. More generally, the analysis of this paper offers insight into the cost of and participant response to interest rate incentives in federal programs.

received little scrutiny. It serves as a striking example of how the combination of programmatic complexity and the rules of federal budgeting can make it difficult to infer the full economic cost of federal credit programs. Further, it provides a new setting in which to study how consumers respond to financial incentives, and how their responses vary with the size of an incentive.

In this study, we develop a valuation model to estimate the cost of the consolidation option and to evaluate the sensitivity of that cost to changes in program rules and economic conditions. The model takes into account borrower behavior, program rules, the stochastic properties of interest rates, default, and the market price of risk.

More broadly, the analysis demonstrates how modern financial economics can better inform public policy about program cost and risk exposure.[5,6] Measuring the cost of government programs is hard in general, but when the government program involves complex financial arrangements, the difficulty level soars. Understanding how to evaluate such costs is particularly relevant with the increasing use of credit assistance in lieu of direct grants, for instance under the new federal programs that assist mortgage borrowers using complex government guaranteed loans.

A sample of 700,000 borrower records taken from the Department of Education's student aid database, the National Student Loan Data System (NSLDS), provides evidence on borrower behavior and is used to calibrate the behavioral assumptions embedded in the valuation model. The data suggest that borrowers do respond to the time-varying incentives to consolidate, with

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[5] For example, Falkenheim and Pennacchi (2003), Lucas and McDonald (2006), and Pennacchi and Lewis (1994) also use option pricing methods to determine the value of government liabilities.

[6] A study by the Congressional Budget Office (CBO, 2006) uses a simpler model to examine different but related issues to those discussed here. Technically, our analysis is distinguished by: the use of an interest rate model that better captures rate dynamics and time variation; the use of previously unavailable Student Loan Consolidation data to model consolidation choice; and the integration of the consolidation choice more formally into the valuation model. In addition this paper is the first to consider: whether the high cost of consolidation could have been predicted; the difference between prospective and ex post costs; to what extent student consolidation behavior appears rational; and the sensitivity of consolidation costs to borrower behavior, policy choices, and other model assumptions.

higher rates of consolidation occurring in years when the option is most valuable.[7] Further, interest rate sensitivity increases with the amount of debt outstanding. There is some evidence of learning, as take-up rates have increased steadily over time even after controlling for interest rate incentives. Nevertheless, borrowers do not maximize the value of the option, and we provide estimates of how much money was left on the table at different points in time by comparing observed behavior with predicted behavior in an optimizing model.

We find that the consolidation option has at times been extremely costly to the government. For instance, ex ante, the cost in 2004 was more than $4 per $100 of new loans originated. Although the cost in some years was much lower than in the recent past, the analysis suggests that the possibility of high costs was predictable and hence avoidable.

From a policy perspective, it is doubtful that the subsidy arising from the consolidation option is an efficient mechanism to encourage investment in higher education. Its effect on the perceived cost of borrowing is uncertain, as the option is complicated to value and lenders do not provide borrowers with estimates of its ex ante value. Further, it confers benefits randomly across different cohorts of borrowers ex post, depending on realized interest rate conditions. The largest benefits fall to professional students graduating from high tuition schools, for whom debt relief is not usually a policy priority.

The remainder of the paper is organized as follows: Section 2 briefly describes the federal student loan program and the consolidation option. Section 3 outlines the valuation model and critical modeling assumptions. Probit analysis of data from the NSLDS summarizes consolidation take-up rates and their sensitivity to borrower characteristics and market conditions, and take-up rates are compared to an approximation of optimal behavior. Section 4 reports forward-looking cost estimates and their sensitivity to model assumptions and policy variations.

[7] Although many recent studies have provided evidence of suboptimal financial behavior by individuals, few have reported on whether inefficiency decreases with the amount at stake. An exception is Calvet et al. (2006), who find more efficient portfolio allocations on the part of wealthier households in Swedish data.

Section 5 discusses policy implications and conclusions. Appendices contain more detailed descriptions of the valuation model, supporting assumptions, and the statistical analysis of borrower behavior.

2. The Federal Student Loan Program

The federal student loan program is one of the largest credit programs operated by the U.S. government. In 2004, about 7.7 million students borrowed $55 billion in new federally backed loans, adding to the $432 billion in outstanding federal student loans.[8] This was up from $126 billion in outstanding loans in 1998, reflecting the rapid growth of education costs and strong appetite for consumer debt during that period.

The federal student loan program is complex, and several of its institutional features which have a significant effect on the cost of the consolidation option are taken into account in this analysis. The government makes credit available through two competing student loan programs – the Federal Direct Student Loan (DL, or direct loan) program, and the older Federal Family Education Loan (FFEL, or guaranteed loan) program. Schools have a choice of which program to adopt, and students face similar loan terms in both programs. In the former, the government lends funds directly to qualifying students. In the latter, it guarantees loans originated by private lenders against losses from default and pays lenders any shortfall between the rate charged to students and a promised minimum.[9] Loan terms vary with the purpose of the loan; there are loans for undergraduates, graduate students, various types of professional students, and parent loans. Some students qualify for more highly subsidized loans on the basis of income. The terms offered to a given borrower are generally quite similar under the direct and guaranteed

[8] About 10 percent of the total federal loan portfolio is loans to parents made under the Parent Loans for Undergraduate Students (PLUS) program.

[9] Currently the student rate in repayment is three-month Treasury plus 2.3 percent, whereas the rate guaranteed to lenders is three-month Commercial Paper (CP) plus 2.34 percent.

programs, but the cost to the government is not (Lucas and Moore, 2007). Those loan terms – eligibility, interest rates, amortization, maturity, loan limits, and so forth – are set by statute under the Higher Education Act.

Most loans made under the direct and guaranteed programs are Stafford loans.[10] On Stafford loans originated between October 1998 and July 2006 -- the focus of this analysis -- students pay a variable interest rate based on a three-month Treasury rate that resets annually, plus a spread. The spread varies with the repayment status of the loan: It equals 1.7 percentage points when the student is in school, in the six-month grace period after leaving school, and in periods of deferment; and equals 2.3 percentage points otherwise.[11] The interest rate is capped at 8.25 percent. Loan principal is amortized over the remaining life of a loan, starting when the loan enters repayment.

2.1 The Consolidation Option

The consolidation option has three main features. First, it allows borrowers to combine two or more outstanding loans into a new loan. Since each year's borrowing is counted as a separate loan, the option is available to all students that borrow in more than one academic year, including both their undergraduate and graduate studies. Second, borrowers pay a fixed interest rate on the consolidation loan equal to a weighted average of the rates prevailing on the loans they consolidate. This option to switch a loan from a floating rate to a fixed rate is termed a "swaption" in financial markets. Third, some borrowers have an extension option, which allows

[10] The Stafford loan program includes most types of federally-backed student loans, including direct and guaranteed loans to undergraduate, graduate and professional students. The other major student loan program, PLUS, provides loans to parents of undergraduate students and to graduate students that have exceeded Stafford borrowing limits. Upon consolidation, Stafford and PLUS loans are reclassified as Consolidation loans, but the conversion does not affect the amount of credit outstanding.

[11] For "subsidized" Stafford loans, which account for approximately 50 percent of the value of outstanding Stafford loans, borrowers pay no interest while they are in school, in grace or in other periods of payment deferment. The subsidized share of new loans is declining and is expected to continue to decline.

them to extend the maturity of their loans beyond what is otherwise permitted.[12] Borrowers have the opportunity to consolidate loans for as long as their original Stafford loans remain outstanding. They can also reconsolidate their previously consolidated loans with new Stafford loans, but the new fixed interest rate the borrower obtains is a weighted average of the fixed rates on the previously consolidated loans and the floating rate on the new loans. Thus, borrowers have the valuable right to lock in the floating rate on each original loan only once.[13]

Consolidation also affects program cost through its affect on payments to guaranteed lenders. Consolidation lowers the guaranteed rate of return to lenders by 0.75 percent annually,[14] and lenders pay a small fee to the government at the time of consolidation. The cost saving to the government from lower fees paid to lenders is partially offset by the longer maturity of some consolidation loans and, historically, by the presence of a rate floor on lender payments.

2.2 Benefits to students

The consolidation option provides students with several distinct benefits – an interest rate option, an extension option, and a liquidity benefit (associated with the extension option) of lower monthly payments. Guaranteed lenders also advertise the convenience of a single bill each month.

The greatest benefit to students comes from the interest rate option. For a given maturity,

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[12] Borrowers in the direct program with loans originated between 1998 and 2006 receive little incremental benefit from the extension option, as they can extend loan maturity on their direct Stafford loans with or without consolidating. Many borrowers in the guaranteed program benefit from the extension option, because only borrowers with balances above $30,000 can extend their loans without also consolidating. Consolidation allows all borrowers to extend their repayment period according to the rules governing the direct program. (After June 2006, the extension rules for the direct program are the same as for the guaranteed program.)

[13] Students who have not exhausted their loan limits can lower their borrowing rate further by taking new student loans to pay off older consolidation loans in favorable interest rate conditions. We do not take this possibility into account in estimating the value of the consolidation option.

[14] Lenders receive a special allowance payment (SAP) equal to the rate on three-month commercial paper (CP) + 2.34 for Stafford loans, plus any floor income (a payment when rates fall below a floor on loans issued prior to June 2006). Consolidation lenders get CP + 2.64 - 1.05 = CP + 1.59. The difference is at least 0.75 percent, and may be higher at times due to a rate floor.

the market rate on a floating rate loan is generally lower than the corresponding fixed rate because yield curves tend to slope upward. By allowing students to convert a floating rate into a long-term fixed rate, students can often lock in a favorable spread. Conversely, when floating rates are high, students can choose to defer consolidation. Hence, the floating-to-fixed-rate conversion option has significant value, which diminishes over time as the loan amortizes.

The term extension is also of value for qualifying borrowers in the guaranteed program. Stafford loans allow students to borrow at a below-market rate of interest.[15] The annual interest subsidy can be approximated by the difference between the estimated fair market rate, which is inferred from the private student loan market,[16] and the government rate. By extending the maturity of the loan, the subsidy is received over a longer period, increasing its present value. We refer to this component of government cost as the "extension cost." The extension cost varies with the credit risk of the borrower, and is highest on loans to students with poor credit. This suggests that all else equal, students with relatively poor credit records have a greater incentive to consolidate.[17]

Extending loan maturity also lowers monthly payments. For students facing liquidity constraints, a lower monthly payment might be preferred even if it entails a higher interest rate.[18] Guaranteed lenders marketing consolidation loans emphasize this feature, which lowers monthly payments by about 40 percent. Indeed, extensions motivated by liquidity constraints can mitigate

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[15] This is true even of "unsubsidized" Stafford loans. The government's measure of subsidy cost for budgeting does not include administrative costs or a charge for market risk, causing reported costs to be systematically lower than a market-value based measure of opportunity cost (CBO, 2004, Lucas and Moore, 2007).

[16] The private student loan market developed to serve students that have exceeded federal borrowing limits. Private lenders offer rates that vary with borrower credit score and educational institution. To the extent that private loans are taken disproportionately by professional students (e.g., law, medicine, business), they may not be representative of the average Stafford loan credit quality, although the available evidence suggests similar loss rates on federal and private loans (Lucas and Moore (2007)).

[17] Empirically, the effect of "credit quality" is obscured by the fact that borrowers with high credit ratings tend to attend the most expensive institutions, carry the largest loan balances and, hence, have a large incentive to consolidate.

[18] Gross and Souleles (2002) document the propensity of liquidity-constrained consumers to borrow at high interest rates when credit limits on credit cards are increased.

the extension cost for two reasons: First, lower payments may help some borrowers avoid default and, second, the desire for lower payments may cause some borrowers to consolidate even when interest rate conditions are unfavorable.

The fact that borrowers face liquidity and other constraints complicates the evaluation of whether they are behaving rationally, and of what the option is worth to them. Hence, we are careful to distinguish between the cost to the government, which we assume is only affected by standard financial considerations, and the benefit to students, which we do not attempt to quantify precisely.

3. Modeling Consolidation Value

The cost of the consolidation option to the government, and its sensitivity to program rules, economic conditions, and behavioral assumptions, are evaluated using the valuation model that is described briefly in this section and presented in more technical detail in Appendix B. We begin by discussing the critical factors affecting option value.

3.1 Factors Affecting Costs

3.1.1 Level, Persistence, and Volatility of Interest Rates

The value of the consolidation option depends on the term structure of interest rates, the persistence of interest rates over time, and volatility. Consolidation is most valuable to students and most costly to the government when short-term rates are lower than long-term rates, because then students can lock in a long-term borrowing rate that is below the cost of funding it. Because the term structure is upward-sloping most of the time, consolidation benefits students under normal interest rate conditions. Because interest rates exhibit a high degree of persistence, the value of the option varies with interest rate conditions at origination. As is generally true for options, higher interest rate volatility increases option value.

The well-known model of Cox, Ingersoll, and Ross (CIR) (Cox, et al., 1985) is used to generate a stochastic term structure of interest rates. We employ a two-factor version of the model, following Jagannathan, Kaplin, and Sun (2001) among others. The interest rate model affects both cash flows and discount rates.[19] The model posits a stochastic mean-reverting factor process, with volatility proportional to the square root of each factor. Conveniently, a closed-form solution is available to calculate the implied discount rates for arbitrary maturities. As described in Appendix B, the interest rate model is calibrated to match the historical volatility of the short-term Treasury rate and also, depending on the question at hand, to match the historical or current shape of the yield curve.

3.1.2 Borrower Consolidation Behavior

The cost of consolidation to the government depends on when and whether borrowers choose to consolidate. Theoretically, interest rate conditions are the most important determinant of option value, and observed consolidation rates are in fact correlated with interest rate conditions, as shown in Figure 1.[20] The consolidation option became available in 1994, but it attracted little attention for many years because interest rates were relatively high and often close to the rate cap. In the mid 2000's, as short-term Treasury rates fell to historical lows, the rate of consolidation rose and in some years exceeded 70 percent. The intense marketing of consolidation loans at that time also may have contributed to increased appreciation of the option's value and to rising consolidation rates.

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[19] The results of the analysis would likely be similar under any alternative interest rate model commonly used in valuing interest contingent options.

[20] Defining the consolidation rate consistently over time is complicated by the restriction on in-school consolidations prior to 2005. The restriction implies that a loan originated in a particular year could be ineligible for consolidation for many years while the student completes his or her education. We approximate the loans eligible for consolidation in each year before 2005 as a weighted average of loans from past years, with a linearly increasing weight from 0 percent in the current year to 100 percent in loans 5 years old or older. The graph includes re-consolidations of previously consolidated loans, increasing the reported rates relative to the new consolidations that are the basis of the cost estimates.

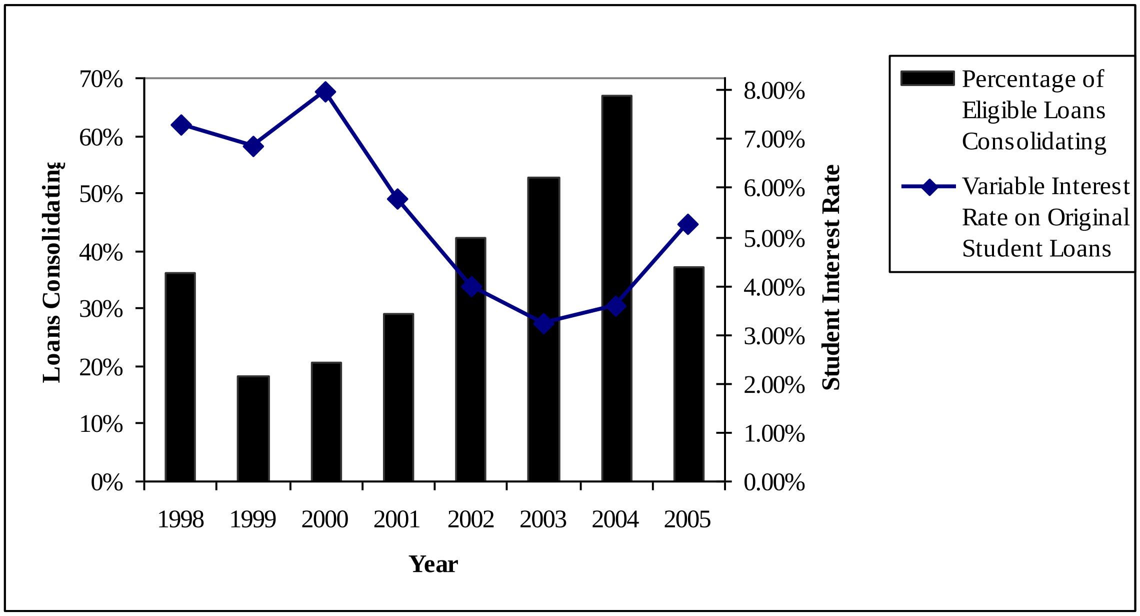
Other options available to students – default, prepayment, deferment and forbearance – all affect the value of consolidation by altering the period of time over which a reduced interest rate is obtained, and so may affect the decision to consolidate. For simplicity, and because in the data we find no evidence to the contrary, the rate of default, prepayment, forbearance, and deferral are assumed to be independent of the consolidation decision. Each is also taken to be i.i.d. over time. Historically, default rates on student loans are low, averaging less than 2.5 percent per year. Consolidation loans in the direct program default at a similar rate to direct and guaranteed Stafford loans, consistent with the assumption of no interaction.[21] We further assume that the effects of prepayment, deferment and forbearance are offsetting, and thereby do not affect the duration of the stream of subsidy benefits. To the extent that these other options are used strategically -- for instance students can forego prepayment or default and seek loan extensions through deferments and forbearance when their consolidated loans are particularly valuable – our estimates of subsidy value will have a conservative bias.

An upper bound on the cost of the consolidation option to the government can be found by assuming that borrowers follow a consolidation strategy that maximizes the theoretical value of the option with respect to interest rates. That is, at every point in time, borrowers compare the present value of the future payments if they consolidate immediately with the present value of all future payments if they consolidate at an optimal time in the future (as in a standard American option valuation). In reality, borrower behavior is more complex. As for home mortgages, the decision to exercise the option is also affected by other factors, such as the desire to lower monthly payments. In addition, borrowers may simply fail to make optimal choices out of ignorance or inertia. Whatever the cause, those suboptimal choices lower the cost to the writer of

[21]. Lucas and Moore (2007) provide more detailed information about default, forbearance, deferral and prepayment, compiled from Department of Education data. Because students on the brink of default in the guaranteed program seem to be encouraged to consolidate into the direct program, consolidation loans made under the guaranteed program have lower default rates. This is an institutional anomaly that does not significantly affect overall program cost, so we do not try to account for the difference.

the option. Hence, we use a behavioral model of consolidation choice, rather than an optimizing model, in calculating government cost.

Figure 1: Student Interest Rate versus Share of Eligible Loans Consolidating



Program rules influence consolidation behavior in several significant ways that are reflected in the behavioral model. Until 2005, borrowers in the guaranteed program could not consolidate loans during the in-school period, creating long and variable lags between origination and consolidation. Historically, the delay could be as short as a year, or could be well over eight years for a loan taken by a freshman who goes on to pursue graduate studies. Temporary relaxation of this rule caused a wave of in-school consolidation activity in 2005.

Two factors encourage borrowers to consolidate their loans early on. For loans issued prior to July 2006, an idiosyncrasy in the law allows loans that are consolidated in school or during a grace period to bear rates that are permanently 0.6 percent lower than if consolidation were postponed to a normal repayment period.[22] Further, early consolidation maximizes the

[22] The grace period applies to students while in school, and in the first six months after leaving school. Allowing extension of the grace-period discount has proven to be costly, as shown in the sensitivity analysis.

principal balance upon which the subsidy is based, because the balance declines as the loan amortizes.

Whether it is optimal for a borrower to consolidate to take advantage of the extension option depends on borrower characteristics such as credit score and liquidity constraints. As for the interest rate option, even if the theoretically optimal policy were apparent, actual exercise behavior is likely more complex. A fundamental difficulty in estimating the cost of the extension option is that the portion of extensions to attribute to consolidation is uncertain. All students under the direct loan program, and students with high loan balances in the guaranteed program, have the extension option independent of whether they consolidate. Yet historically, most extensions occurred with a consolidation. An unobservable but critical quantity is the percentage of loans that would have been extended in the absence of the consolidation option. The base case counterfactual assumption is that a constant fraction of eligible borrowers extend without the option (see Appendix B.3). When consolidation is available, we assume that, consistent with observed behavior, all consolidating borrowers extend to the maximum allowable maturity. In the sensitivity analysis, we consider the possibility that the extension option involves no incremental cost, as the propensity to extend is the same with or without consolidation.

With these considerations in mind, we estimate the effect of interest rate conditions and loan term extension on consolidation rates using a panel of 700,000 randomly selected borrower records taken from the National Student Loan Data System (NSLDS), administered by the Department of Education. The database comprises multiple linked files containing loan characteristics (e.g., loan program, original maturity and amount), limited borrower characteristics (e.g., school attended, whether undergrad/grad/professional, etc.), and time-series information recording changes in the status of each loan since its origination (in school or grace period, defaulted, consolidated, extended, etc.) . Using the data, we constructed a time series for each borrower in the sample that includes consolidation events, total loan amount outstanding (which determines eligibility for term extension), the consolidation interest rate, and other borrower and loan characteristics.

A Probit regression model is used to estimate the probability that a borrower consolidates in any year.[23] This probability is estimated as a function of a 3-tier categorical indicator of the total loan balance outstanding, (BALCAT: < $20,001; $20,001 to $60,000; >$60,000); the variable interest rate on the original loans (RATE), the length of time the loans have been in repayment (YRS IN REPAY),; interactions between these variables; a year dummy (YRDUM) to control for changes in program rules and the availability of information about consolidation; and dummy variables indicating that students borrowed only in the direct program (DIRECT), or that they have loans from both the direct and guaranteed programs (BOTH), controlling for differences in the value of the extension option. The data and variable construction is described in more detail in Appendix A.

Table 1 below reports the results from a simple pooled Probit regression in which the binary dependent variable is whether an eligible borrower consolidates in a given year. We do not include past incentives to consolidate (i.e. past levels of interest rates) in the specification (so-called "burnout effects"), although necessarily fewer borrowers will be left to consolidate if past circumstances have been favorable to consolidation. We have not attempted to adjust the standard errors for the possibility of individual specific random effects in the consolidation results, or for uncontrolled temporal dependence in individual choices.[24]

Parameter estimates have the expected sign and magnitude. Consistent with optimizing behavior in the presence of fixed costs, interest rates interact with balance size to produce larger consolidation probabilities among borrowers with larger balances and when rates are lower. For

[23] The properties of parameter estimates on markov chain, duration or cross-sectional binary time series data is well-documented (see for example, chapter 11 of Amemiya, 1985 and Maddala, 1987). Knapp and Seaks (1992) use a Probit model similar to ours to analyze the related question of what determines student loan default behavior.

[24] An example of a controllable temporal dependence that has been excluded from the specification is the previous year's incentive to consolidate.

instance, at a variable interest rate of 8 percent, a borrower with a balance of $60,000 or more has a 4 percent probability of consolidating in the year he or she enters repayment, versus a 2 percent probability for a borrower with a balance under $20,000. At a 4 percent interest rate, those probabilities increase to 39 percent and 4 percent, respectively. Figure 2 reports the estimated annual consolidation probability for borrowers with particular characteristics for various interest rates, balance sizes, and time in repayment.

An alternative for estimating the probability of consolidation would be to use a duration model, such as the Cox proportional hazards framework, as in Deng, Quigley and Van Order (2000). The continuous time Cox model can be shown to be the limiting case of a discrete time two-state markov model analogous to our Probit specification (see Amemiya, 1985, pp 433 - 435).[25] Deng et al use a competing hazards specification of the Cox model in order to jointly estimate the intensity of default and prepayment. A more complete specification of our model would capture the transition between various states including consolidation, prepayment, default and forbearance. However, as the Deng et al results show, parameter estimates are relatively robust to separate estimation of individual hazards and ignoring individual level heterogeneity.[26]

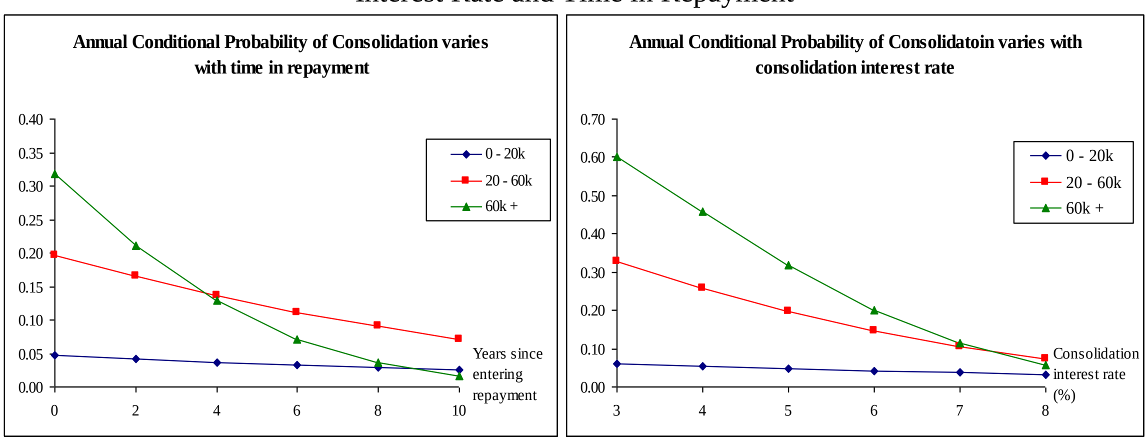
[25] The Cox proportional hazard model is the limiting case of a binary logit model on panel data. We selected a probit instead of logit model because we considered it to be a better fit for our data. Han and Hausman (1990) use discrete time probit and logit approximations interchangeably in their exposition of competing hazards estimation and the two model generally produce similar estimates.

[26] In the absence of individual level heterogeneity, parameter separately estimated from each hazard are identical to parameter estimates from a jointly estimated competing hazard model.

Table 1: Parameters Estimates and Standard Errors from a Probit Regression of Consolidation Decisions on Covariates.

Parameter	Estimate	Standard Error
Intercept	-1.046	0.0292
HAS DIRECT	-0.085	0.0035
HAS BOTH	0.373	0.0051
BALCAT1 (20 – 60k)	1.146	0.0624
BALCAT2 (60k+)	2.856	0.3533
RATE	-0.123	0.0032
RATE*BALCAT1	-0.043	0.0075
RATE*BALCAT2	-0.210	0.0427
YRS IN REPAY	-0.028	0.0006
YRS IN REPAY*BALCAT1	-0.032	0.0013
YRS IN REPAY*BALCAT2	-0.032	0.0044
YRDUM (POST 2002)	0.695	0.0048
YRDUM (1998 – 2002)	0.315	0.0044

Figure 2: Estimated Consolidation Probabilities by Outstanding Balance, the Consolidation Interest Rate and Time in Repayment



In the prospective cost estimates, we use the Probit model specification as our model of consolidation behavior. Thus, in repayment, the probability of consolidation is positively related to the current interest rate advantage. Even when interest rate conditions are unfavorable, however, a strictly positive fraction of borrowers consolidate their loans. The sensitivity analysis explores alternative behavioral parameterizations with respect to interest rate sensitivity. The typical consolidation behavior summarized by the Probit model also is compared to the implications of an optimizing model to evaluate how much of the potential value of the option is exploited by borrowers.

3.1.3 Federal Borrower Interest Rates versus Market Interest Rates

As discussed above, the greater the difference between the interest rate on federal loans and private interest rates, the greater the extension cost. The extension cost estimate is based on the typical spread between private and government loan rates and on default and recovery rates estimated from information from the Department of Education, as summarized in Lucas and Moore (2007).

The rate on all federal floating rate student loans is capped at 8.25 percent. The cap affects option value and is reflected in the valuation model. The cap effectively increases the value of the original variable rate loans, particularly in high interest rate periods. At the same time, it reduces the interest rate benefit of a fixed rate consolidation, because it limits the worst-case interest rate a borrower would pay without consolidating.

3.2 Valuation Model

The cost of the consolidation option is estimated using an arbitrage pricing approach, implemented numerically with Monte Carlo simulation. Projected cash flows are discounted along each sample path at risk-adjusted rates. Cost is measured at the time of loan origination. The marginal cost of the option is measured by taking the difference between the present value of net government cash flows on loans with and without the consolidation option.

Cash flows depend on the stochastic path of future interest rates, the program rules, and the behavioral decision rules. Each month a random draw determines the innovation in the short-term interest rate, and the corresponding term structure is derived from the CIR model described earlier. For the direct program, monthly cash flows to the government depend on whether the student is in school, the current short-term rate if the loan has not yet been consolidated, and a fixed rate based on the short-term rate at the time of conversion if the loan has been consolidated, whether the interest rate cap on the student loan is binding, the average rate of default and prepayment, and an administrative charge.[27] For the guaranteed program, the relevant cash flows are between the government and guaranteed lenders. These cash flows consist of special allowance payments (SAP) that vary with consolidation status, and of compensation to lenders for credit losses.

The discount rates reflect the opportunity cost of providing loan capital to students. There are at least two potential sources of priced risk that affect the discount rate – interest rate risk and credit risk. The approach taken here is to capture the price of interest rate risk by using a risk-neutral representation of the CIR model. For simplicity, we assume that credit risk is orthogonal to interest rate risk.[28] The credit risk premium is represented by a spread over risk-neutral rates. The credit risk premium cannot be directly extracted from available market price data, but as described in detail in Lucas and Moore (2007) the credit risk premium can be inferred from the spread between private and federal student loans by adjusting for expected losses and administrative charges. These adjustments yield a risk premium of approximately 2 percent per annum over the Treasury rate.

4. Results

The main focus of the analysis is on prospective, or ex ante, estimates of the cost to the government. These ex ante estimates measure the lifetime cost imposed on taxpayers in the year a loan is originated, and hence are the relevant quantity for program evaluation. We estimate these costs for historical interest rate conditions, and examine the sensitivity of the results to

[27] The administrative charge on direct loans cancels out when pricing the option as long as the old and new loans have similar maturity.

[28] In our data, we find default rates on student loans to have a low correlation with interest rates. Empirical evidence (e.g., Collin-Dufresne and Solnik (2001)) find a negative correlation between default-free interest rates and the likelihood of corporate defaults, but we are unaware of published evidence on the correlation for consumer credit.

parametric and program variations. We also report on ex post realized costs, both as a plausibility check on the more subtle ex ante calculations, and to be able to explore the question of to what extent the high ex post costs could have been anticipated by policy analysts.

4.1 Prospective Costs

Our prospective cost estimates take into account consolidation behavior and program rules that affect the time between origination and consolidation and whether a consolidation occurs at all. Under the base-case assumptions, and under interest rate conditions prevailing in each year from 1998 to 2005, the cost of the consolidation option per $100 of Stafford loans originated is shown in Table 2. The table also shows the current three-month bill and ten-year bond rates, which explain much of the difference in cost across years.

Table 2: Prospective Consolidation Costs and Interest Rates (1998 – 2005)

Origination Year	Three-month T-Bill Rate	Ten-year T-bond Rate	Consolidation Cost ($ per $100)
1998	4.98%	5.50%	1.33
1999	4.57%	5.90%	1.22
2000	5.69%	6.10%	0.80
2001	3.49%	5.28%	2.05
2002	1.70%	4.93%	3.54
2003	0.92%	3.33%	6.44
2004	1.27%	4.73%	4.21
2005	2.97%	4.00%	3.70

Looking forward, these costs will likely be much lower. Legislation reauthorizing student loans through 2012 was passed in early 2006. Interest rates on Stafford loans originated after July 2006 revert to a fixed 6.8 percent, with consolidation interest rates on those loans as high as 6.875 percent. This legislation also eliminates the possibility of consolidation while in school. Although there was little discussion of the effect on the value of the consolidation option, these changes effectively eliminate the option’s value for loans originated after 2005. Even so, understanding the cost of this option is useful. By mandating a fixed lending rate independent of market conditions, the legislation imposes several new risks on the government that could trigger a return to floating rates in the future. Most obviously, the new rules create the risk of much higher program costs were interest rates to increase significantly. Further, because guaranteed lenders are reimbursed according to a formula based on a floating rate, it also creates a maturity mismatch for the government (fixed rate assets and floating rate liabilities) adding volatility to program cost.

4.2 Sensitivity analysis

We examine the sensitivity of the prospective cost estimate for loans originated in 2005 to several of the key economic, behavioral, and programmatic parameters: initial interest rate conditions, the volatility of interest rates, the risk spread in the discount rate, consolidation behavior, and extension behavior. We also estimate the cost of the grace-period loophole, which allows borrowers a permanent rate decrease of 0.6 percent if they consolidate during the grace period, and the cost of allowing immediate consolidation instead of forcing delay until leaving school. All costs are expressed as a present value per \$100 of original Stafford loans at origination.

4.2.1 Interest Rates and Spreads

As discussed earlier, the cost of the consolidation option increases with a reduction in short-term interest rates and an increase in the volatility of interest rates. The cost also is affected by the credit spread, but there are two partially offsetting effects – a higher credit spread makes the extension option more valuable but raises the discount rate, which decreases the present value. Table 3 shows that moderate changes in the interest rate model, relative to a base-case discount rate of Treasury plus 2 percent, have only small effects on prospective costs. The column labeled "High Volatility" reports the results of increasing the interest rate volatility parameter of the first factor, σ_1, from 0.2 to 0.3, approximately corresponding to a 50 percent increase in the volatility of the instantaneous interest rate.

Table 3: Prospective Cost Sensitivity to Interest Rate Model

Origination Year	Discount Rate T+1%	Discount Rate T+3%	High Volatility
	($ per $100 of Stafford originations)		
1998	1.13	1.46	2.17
1999	1.09	1.29	1.99
2000	0.66	0.89	1.49
2001	1.89	2.14	2.88
2002	3.45	3.56	4.17
2003	6.15	6.57	7.41
2004	4.13	4.22	4.88
2005	3.32	3.93	4.63

4.2.2 Borrower Behavior

The extent to which borrowers respond to the economic value of the option is estimated with considerable uncertainty. We compare the results of the base case, in which consolidation rates are quite sensitive to interest rates, with the alternative of fixing the consolidation rate at close to the historical average of 45 percent of eligible borrowers. Comparing the first two columns of Table 4, a fixed consolidation rate increases the cost to the government in some years and decreases it in others. The reason that a fixed consolidation rate is not uniformly less costly is that the benchmark rule was not chosen to be optimal but rather to match observed behavior.

Another uncertainty is the extent to which the option to consolidate increases the propensity to extend loan maturity. As mentioned earlier, even without consolidation, all borrowers under the direct loan program, and borrowers with combined balances in excess of $30,000 in the guaranteed program, can request a term extension without consolidating. However, consolidation increases the incentive to extend maturity, because the rate reduction obtained with consolidation increases the value of extension. The empirical evidence does not offer much guidance on the size of this effect, because the legislation that enabled borrowers to extend loan maturity took effect at about the same time that the consolidation option became highly valuable. In the base case we assume, in the counterfactual scenario where extension is available but consolidation is not, that each year borrowers will extend the term of their loans with a constant probability.[29] An alternative set of cost estimates, shown in column 3 of table 4 is more conservative in that we assume borrowers extend their loans in the counterfactual case as frequently as they consolidate them (where the rules governing term extension allow this). Comparison of the first and third column of Table 4 shows that raising the probability of extensions in the absence of consolidation lowers the cost by approximately 30 percent.

Table 4: Prospective Cost Sensitivity to Behavioral Variations in Consolidation Rates ($ per $100 of Stafford Originations)

Origination Year	Benchmark	No Sensitivity of Consolidation to Interest Rates	Optimal Consolidation	More Frequent Extensions in the Counterfactual Case
1998	1.33	2.06	4.3	0.89
1999	1.22	2.24	5.3	0.92
2000	0.80	1.96	2.8	0.71
2001	2.05	2.48	7.8	1.43
2002	3.54	2.97	11.9	2.52
2003	6.44	3.23	13.9	4.35
2004	4.21	3.09	13.0	3.07
2005	3.70	2.55	8.5	2.20

Borrower behavior deviates substantially from what is optimal in a pricing model where interest rates are the only consideration and there are no transactions costs. For all of the years in our sample, immediate consolidation (i.e. consolidation as soon as legislation permits) is very

[29] This is equal to the estimated base rate of consolidation in the Probit model when the borrower interest rate is at the 8.25 percent cap, and so might reflect the proportion of students who are anxious to extend maturity.

close to the naively optimal strategy.[30] The cost of consolidation under this rule is two to three times that cost under the empirical consolidation rule (Table 4, column 3).[31]

4.2.3 Effects of Related Policies on Consolidation

The cost of the grace-period rate reduction on Stafford loans is limited by the six-month length of the grace period after graduation, but nevertheless is quite costly.[32] Relative to a hypothetical consolidation loan with a discount that ends with the grace period, the incremental cost of this loophole has averaged $0.78 per $100 of Stafford loans originated.

In May 2005, the Department of Education appeared to change its position on in-school consolidations in the guaranteed program, writing that, contrary to popular opinion, in-school consolidation is permissible under current law.[33] Together with favorable interest rate conditions that were expected to be transitory, the decision triggered a wave of consolidation activity. Had in-school consolidation been a common practice over the entire period of 1998 to 2005, we estimate that it would have increased the prospective cost of the consolidation option by an average of $0.55 per $100 of Stafford loans originated.[34]

Consolidations of troubled loans are much more likely to occur under the less expensive direct loan program. This is perhaps because guaranteed lenders can avoid dealing with some troubled loans by declining to consolidate them, causing those students to turn to the direct

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[30] The naïve rule of consolidating immediately is usually optimal because deferring is only valuable when the term structure is downward sloping, or when there is no extension benefit.

[31] The difference between the base case and optimal cost cannot be interpreted as the money left on the table by borrowers, since part of the difference is due to the affect of behavior on the amount paid to guaranteed lenders.

[32] The grace period can be reactivated by returning to school for additional education, increasing the grace period's average value.

[33] The ruling was not a response to any legislative change, but was rather a response to inquiries by guaranteed lenders. In-school consolidation was already available to direct program borrowers at that time.

[34] We assume that in-school consolidation also is sensitive to current interest rate conditions, but that, for any level of interest rates, in-school consolidation is less likely than after graduation.

program for consolidation. We do not try to quantify the effect on cost, however, because it would require considerable additional modeling and assumptions.

4.3 Realized costs

In this section we report the realized annual cost to the government from 1998 to 2005. The calculation is based on the intrinsic value of realized consolidation volume in a given year. The intrinsic value is the difference between the present value of cash flows for a consolidated loan and the present value of cash flows for an unconsolidated loan from the date that the consolidation occurs.[35] Table 5 shows that the costs vary considerably from year to year, ranging from a gain to the government of $5.44 per $100 of loans consolidated in 2000 to a loss of $18.73 per $100 of loans consolidated in 2003. In total dollar terms, the costs peaked in 2004, at $8.8 billion. The wide variation in cost over time reflects changes in interest rate conditions and borrower behavior. The cost varies more over time than in the prospective cost calculations (Table 2) because for the prospective costs the delay between origination and consolidation provides time for interest rates to revert toward their long-run average, making initial interest rate conditions less important.[36] Also, the prospective cost is lower on average because it includes loans that never consolidate or that have a long delay until consolidation. The scatter plot in Figure 3 shows that borrowers consolidate at higher rates when the subsidies they receive from the government are greater, although a considerable number consolidate even when it results in cost savings to the government.

[35] In each case, we assume that borrowers take the maximum term extension available to them. For each program year, we take as a starting point the interest rate conditions in that year, and simulate the loan cash flows and discount rates over the life of the representative set of loans. The behavioral parameters are not relevant, however, because consolidation is assumed to have occurred in that year.

[36] The interest model implies that in the steady state the 3-month yield is 3.5 percent, the 10-year yield is 4 percent, the annual volatility of the 3-month yield is .8 percent and the annual volatility of the 10-year yield is .5 percent. In 2003, when the 3-month yield was .92 percent, the half life to the steady state is 15 months.

4.3.1 Were High Costs Predictable?

An interesting question is to what extent could the recent high costs of this policy have been predicted? In other words, was there just a particularly unlucky draw of interest rates, or could the magnitude of realized costs have been anticipated by lawmakers?

A simple back-of-the-envelope calculation suggests the possibility that large costs should have been easy to predict. Historically a 2 percent yield spread between 3-month Treasury and 10 year bond rates has prevailed at numerous points in history. Locking in a two-percent annual interest rate advantage on a 20-year amortizing loan produces a cost estimate of $12 per $100 consolidated, which is in line with the predictions of our more complicated option-pricing model. To establish this more rigorously, and to demonstrate the plausible range of variation of future costs, the CIR model, calibrated to interest rate conditions in 1998 and to historic interest rate volatility, is used to generate the distribution of the term structure of interest rates and corresponding consolidation costs for subsequent years. Figure 4 shows the distribution of cost per $100 consolidated, at the time of consolidation, based on 2000 draws of the forward distributions of term structures. The calculations suggest that, although the very high costs realized in 2003 and 2004 were unlikely ex ante, at the end of four years, there was a 23 percent chance of costs in excess of $8 per $100 of loans consolidated. Table 6 contains descriptive statistics for the distribution of intrinsic values at other terminal horizons, in each case starting from initial interest rate conditions in 1998. Clearly, this risk exposure could have been avoided, while still allowing conversion to a fixed rate, by changing the law to index the consolidation rate to current long-term fixed rates rather than to current short-term rates.

Table 5: Consolidation Volume and Estimated Cost (1998 – 2005)			
Consolidation Year	Consolidation Volume (millions of $)	Consolidation Cost (millions of $)	Consolidation Cost ($ per $100)
1998	12,312	21	0.17
1999	10,217	409	4.01
2000	15,466	-841	-5.44
2001	26,411	1,099	4.16
2002	39,283	3,480	8.86
2003	43,770	8,198	18.73
2004	55,272	8,800	15.92
2005	29,203	5,832	7.60

Figure 3: Proportion of Eligible Loan Consolidating versus Government Consolidation Cost, 1998 – 2005.

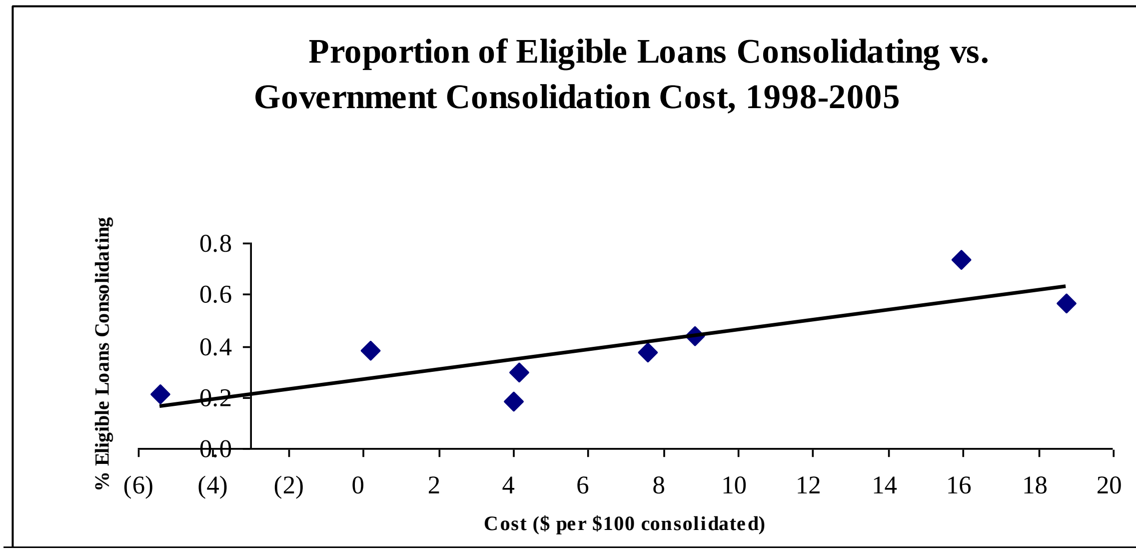

Figure 4: Simulated Distribution of Intrinsic Value, Simulated Four Years Forward From Interest Rate Conditions in 1998

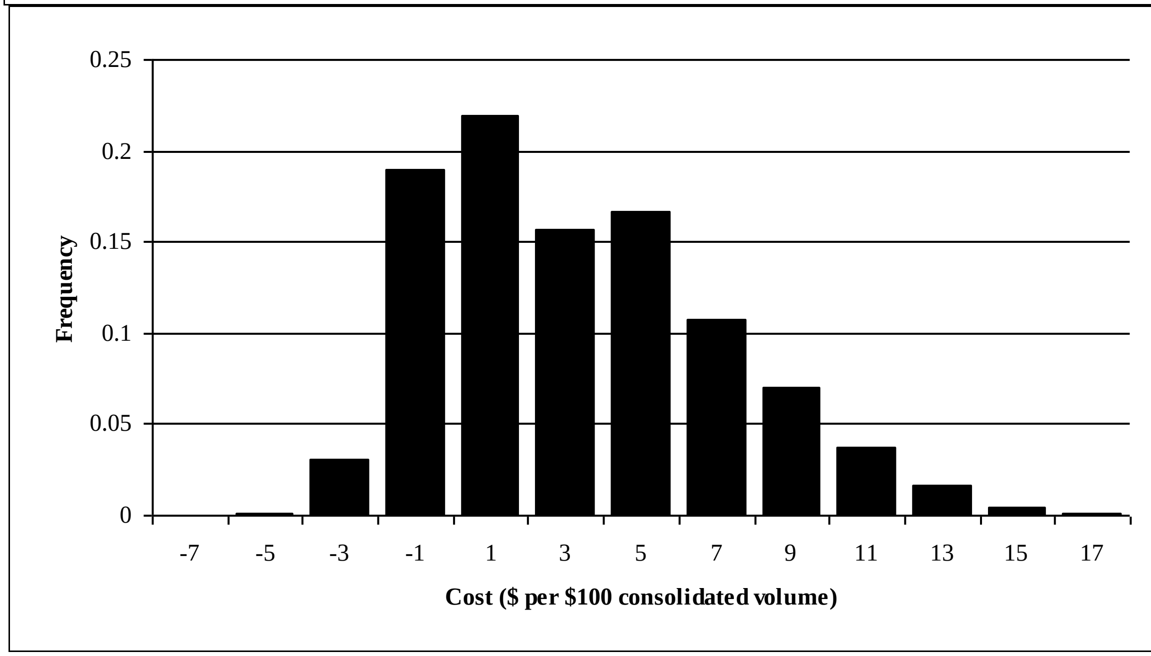

Table 6: Intrinsic Cost of Consolidation ($ per $100 loans consolidated)

	Number of Years				
	2	4	6	8	10
Mean	3.33	3.51	3.58	3.56	3.66
Median	3.01	2.95	2.90	3.01	3.10
Standard Deviation	3.40	3.77	3.97	3.92	3.96
Relative Skewness	0.61	0.70	0.71	0.68	0.65

Notes: Initial interest rate conditions are set at 1998 levels. Interest rates are simulated the set number of years forward indicated.

5. Discussion and Conclusions

In this study, we develop a model to value the student loan consolidation option, and use it to estimate the cost to the government of this provision under a variety of assumptions about economic conditions and student behavior. The analysis implies that, between 1998 and 2005, the option had an ex post cumulative cost to the government of about $27 billion. Although this estimate is sensitive to model assumptions, the estimate remains strikingly large for a wide range of modeling assumptions. The subsidy appears to be an inefficient means of supporting investment in higher education. It confers the greatest benefit on those cohorts who happen to graduate when the yield curve is steep and rates are low, to professional students with the largest loan balances, and to borrowers with the sophistication to manage their loans efficiently.

Why then, has this subsidy persisted? We believe that several factors make its cost relatively easy to overlook or, at least, to ignore. Clearly, repealing a popular subsidy is not easy. Like many other subsidies involving an opportunity cost but not a direct transfer of funds, there is no cash trail. The current budget treatment of student loans obscures the cost.[37] The Office of Management and Budget treats consolidation loans as new loans, resulting in a zero budget cost for future consolidations associated with newly originated Stafford loans. CBO does incorporate expected future consolidation costs into its cost estimates for new Stafford loans, but neither budget agency breaks out the cost of anticipated future consolidations associated with current Stafford loan originations.

Perhaps a more encouraging finding that emerges from the analysis is the high take-up rate by borrowers for whom the option is most valuable. Comparing observed consolidation behavior with an optimizing strategy suggests that many borrowers had sufficient understanding of the option's value to act upon it. The extent to which this was due to a fundamental understanding of the value of the swaption, or rather to the intense marketing efforts of consolidators or to the side-effect of lower payments through maturity extension, however, remains an open question.

[37] Student loans, like other federal credit programs, are accounted for under the Credit Reform Act. See CBO (2004) for a summary of those rules.

Appendix A: The National Student Loan Database

The National Student Loan Database System (NSLDS) is maintained by the Department of Education. For each borrower, the database contains a record of all current and previous loans, which avoids any potential survival bias. Each loan record contains the loan amount disbursed, the amount outstanding at January 2006, the type of loan, and current status of the loan. In addition, each loan record contains a full history of loan status changes and associated dates. We simplified the complicated loan status variable into seven categories, four of them open and three closed. The open status categories, in which the loan is still open and collectible, are:

- G: In school or in the six-month grace-period. Usually the borrower does not make payments on a loan in this status.
- F: In forbearance or deferment. The borrower does not make payments in this status.
- R: In repayment.
- d: In default. A borrower who fails to make the prescribed minimum repayments for more than 270 days is in default.

The closed status categories are:

- D: Collected, negotiated, or written off.
- P: Paid in full.
- C: Consolidated. Note that some loans that were coded as paid in full in the data were actually consolidated. (We have corrected this error where possible by linking paid-in-full records to the corresponding origination of a consolidated loan for that borrower.)

We analyzed the transition of loans from the repayment status (R) to consolidation status (C) at the borrower level by constructing a time series of consolidation events and associated controls for each borrower. This gives us approximately 3 million pooled observations on the following variables:

1. CONSOL: Consolidation indicator variable taking the value 1 if the borrower consolidates currently outstanding loans at a particular point in time, 0 otherwise. This is the endogenous variable in the Probit regression model.
2. BALCATx: Dummy variable indicating the estimated total value of the borrower's outstanding original loans. The categories are as follows: x = 0, balances of $0 to $20,000; x = 1, balances of $20,001 to $60,000; and x = 2, balances exceeding $60,000. Because we do not observe the principal outstanding for every loan in every year, we estimated it from the disbursed amount of each loan under assumed interest rates and an assumed amortization schedule for loans in repayment.
3. RATE: The weighted average interest rate on the borrower's outstanding original loans. This is the rate a borrower would lock in if he or she consolidated (ignoring any rate incentives he or she may have received from a guaranteed lender).
4. YRS IN REPAY: Weighted average time the loans have been in repayment.
5. Dummy variables for the program the loans originated in. Loan consolidation rates appear to be higher in the guaranteed than in the direct program. Moreover, borrowers with loans from both programs are more likely to consolidate than those with loans from a single program.
6. Dummy variables for the year of the decision, proxying for the changes over time in the information borrowers receive about consolidation.

Appendix B: Model and Methodology

To estimate the cost of the consolidation option we:

1. Use the Cox-Ingersoll-Ross model to simulate a risk-neutral path of interest rates for various maturities. It is the basis for discounting credit-risk-free nominal cash flows in the model;
2. Compute cash flows: In the prospective cost estimates, the government's stream of cash flows is computed across each interest rates path, the default state of the loan, and stochastic consolidation and extension behavior. In the intrinsic value calculations, cash flows across each interest rate path and the default state are computed as in the prospective case. However, the date of consolidation, and hence current interest rate conditions, are taken as known and given. Consolidation occurs at the beginning of repayment, and the maximum allowable extension is assumed.
3. Discount the government's default contingent cash flows using a state price deflator to account for default risk and the simulated risk free interest rate to account for time value of money;
4. Compute the cost of consolidation to the government for each loan type (direct or guaranteed) by taking the difference between the present value of cash flows with and without the option to consolidate, averaging across the simulated risk-neutral interest rates and stochastic consolidation and extension behavior; and
5. Aggregate cash flows for a representative set of loans made under the two lending programs.

B.1 Interest Rates

We adopt the Cox-Ingersoll-Ross (CIR) model to simulate future paths of Treasury rates, and we adjust rates upward for the systematic component of default risk, as described in section

B.4. In the CIR model, the instantaneous interest rate, $R(t)$, is the sum of a constant and n factors, $z_i(t)$, for $i = 1,\ldots,n$, the state variables in the model:

$$R(t) = \bar{R} + \sum_{i=1}^{n} z_i(t) \tag{1}$$

The constant $\bar{R}$ takes care of the well-known difficulty of the standard CIR model (without constant) to fit all term structure shapes with strictly positive factors. Each factor obeys a mean reverting square root process:

$$dz_i(t) = \kappa_i\left[\theta_i - z_i(t)\right]dt + \sigma_i\sqrt{z_i(t)}\,dZ_i(t) \tag{2}$$

where θ_i is the mean reverting rate, κ_i is the speed of mean reversion, σ_i is the volatility parameter, and $dZ_i(t)$ is a standard Weiner process independent across factors.

Under the risk neutral (or equivalent martingale) measure:

$$dz_i(t) = \bar{\kappa}_i\left[\bar{\theta}_i - z_i(t)\right]dt + \sigma_i\sqrt{z_i(t)}\,dZ_i(t) \tag{3}$$

where

$$\bar{\kappa}_i = \kappa_i + \lambda_i \tag{4}$$

and

$$\bar{\theta}_i = \frac{\kappa_i\theta}{\kappa_i + \lambda_i} \tag{5}$$

λ_i is the constant market price of risk for factor, i. The price at time t of a zero coupon bond with unit coupon and expiry at time T is:

$$p(t,T) = e^{-\bar{R}(T-t)}\prod_{i=1}^{n} A_i(t,T)\,e^{-B_i(t,T)y_i(t)} \tag{6}$$

where

$$A_i(t,T) = \left[\frac{2\gamma_i \exp\left[(\gamma_i + \bar{\kappa}_i)(T-t)/2\right]}{(\gamma_i + \bar{\kappa}_i)\left[\exp\left[\gamma_i(T-t)\right]-1\right] + 2\gamma_i}\right]^{2\bar{\kappa}_i\bar{\theta}_i/\sigma_i^2} \tag{7}$$

$$B_i(t,T) = \frac{2\exp\left[\gamma_i(T-t)-1\right]}{(\gamma_i+\bar{\kappa}_i)\left[\exp\left[\gamma_i(T-t)\right]-1\right]+2\gamma_i} \tag{8}$$

and

$$\gamma_i = \sqrt{\bar{\kappa}_i^{\,2}+\sigma_i^{\,2}} \tag{9}$$

The yield to maturity, *y*, of a zero-coupon bond maturing at *T* is

$$y(t,T) = \frac{-\ln p(t,T)}{T-t} \tag{10}$$

Jagannathan, Kaplin and Sun (2001) estimate the factors from the two-factor model, using weekly LIBOR rates of various maturities from 1995 through 1999 as shown in Table B-1.

Table B1: Parameters for the Cox-Ingersoll-Ross Two-factor Interest Rate Model

$\bar{R}$ = -0.229

Factor	κ	θ	σ	λ
1	0.392	0.272	0.0153	-0.00038
2	0.0532	0.0162	0.0430	-0.0592

Under these parameters, factor 1, with the stronger degree of mean reversion, drives the gap between long- and short-term rates, and factor 2 determines long-term rates. We subtract 20 basis points from $\bar{R}$ to reflect the average spread between three-month Treasury and LIBOR yields.

For each Monte Carlo run, initial levels of the state variables are calibrated to fairly price an initial three-month Treasury bill and ten-year Treasury bond. For each simulation, the instantaneous rate is sampled monthly for as many months as the maximum maturity of the student loan, using a discrete Euler approximation of the risk-neutral process in equation (3).[38]

B.2 Government Cash Flows in the Direct and Guaranteed Programs

The cost of consolidation for a particular loan is the difference in the present value of government cash flows when a student has access to the consolidation option and when the student does not have access. The cash flows themselves depend on the evolution of the principal balance and interest charged on the student loans, which we outline now.

Loans originate at time 0, begin repayment at time T^R, and have a maturity of T^M, so the loan is repaid in T^R+T^M months. T^M depends on whether the consolidation option is exercised or, in the counterfactual case, the loan term is extended. $T^{C,j}$ is the month that borrower consolidates in simulation j ($T^{C,j} > T^R+T^M$ if the borrower never consolidates). The original maturity of Stafford loans is 10 years. Section B.3 describes the stochastic rules governing consolidation and extension.

Interest accrues on outstanding principal every month. The interest rate prior to consolidation is linked to the yield on the three-month Treasury bill on May 30 each year, and is fixed for a year. Thus the reference rate, $\tilde{R}^j_{12k+i}$, in month i of year k is:

$$\tilde{R}^j_{12k+i} = \exp\left[y^j\left(k, k+3/12\right)\right] - 1, \forall i = 1,\ldots,12,\ k = 0,1,2,\ldots \tag{11}$$

The borrower pays an interest rate that depends on whether he or she is in repayment and whether he or she has consolidated. Extension of loan term without consolidation does not affect the interest rate. Before consolidation and before entering repayment the student rate, $R^j_{S,t}$, is the lower of the reference rate plus 1.7 percent and the interest rate cap of 8.25 percent,

[38] The evolution of each rate factor over a discrete interval of time obeys a noncentral chi-squared distribution. The standard Euler approximation with random draws taken from a normal distribution and truncated to prevent non-positive factor values produced only negligible bias for our interest rate parameters (truncation occurred in less than 0.1 percent of draws) and was ten times faster to calculate.

$$R^{j}_{S,t} = \min\left[\tilde{R}^{j}_{t} + 1.7\%, 8.25\%\right],\ \forall t < T^{R}\ and\ t \leq T^{C,j} \tag{12}$$

After entering repayment but prior to consolidation, the student rate is the lower of the reference rate plus 2.3 percent and the interest rate cap:

$$R^{j}_{S,t} = \min\left[\tilde{R}^{j}_{t} + 2.3\%, 8.25\%\right],\ \forall t \geq T^{R}\ and\ t \leq T^{C,j} \tag{13}$$

After consolidation, the student rate is fixed at the rate prevailing at the time of consolidation, $T^{C,j}$:

$$R^{j}_{S,T^{c}+k} = R^{j}_{S,T^{c}} \quad \forall k > 0 \tag{14}$$

The monthly compounding student rate, $r^{j}_{S,t}$, is:

$$r^{j}_{S,t} = \left(1 + R^{j}_{S,t}\right)^{\frac{1}{12}} - 1 \tag{15}$$

Let P^{j}_{t} denote the evolution of principal (prior to default) over month t in each simulation j. Given an initial principal of $P^{j}_{0} = P_{0}$, monthly principal evolves according to:

$$P^{j}_{t+1} = P^{j}_{t}\left[1 + r^{j}_{S,t}\right] - A^{j}_{t+1} \tag{16}$$

The prescribed monthly payment, A^{j}_{t}, depends on the loan status, and is based on amortizing the principal at the current interest rate over the remaining life of the loan:

$$A^{j}_{t+1} = \begin{cases} \dfrac{P^{j}_{t}\, r^{j}_{S,t}}{1 - \left(1 + r^{j}_{S,t}\right)^{T^{R} + T^{M} - t}}, & t \geq T^{R} \\ 0, & t < T^{R} \end{cases} \tag{17}$$

In the direct program, the government's cash flows on performing loans are the student loan payments less any administrative fees:

$$A^{j}_{t} - fP^{j}_{t} \tag{18}$$

where f is the proportional administrative fee. The fee is 0.75 percent per annum in the benchmark calibration, reflecting typical servicing and other administrative costs. In default, the government recovers in proportion to the present value of remaining payments.

In the guaranteed lending program, the government cash flows are the quarterly payments to lenders – the SAP less any consolidation fee paid by lenders to the government – while the loan is in good standing, and the lump sum payment of outstanding principal and accrued interest in the event of default. We ignore administrative costs because they are largely borne by the guaranteed lender.

The quarterly SAP is the difference between the student rate and the three-month commercial paper rate plus a spread, but has a floor of zero. We assume the annualized three-month commercial paper rate, R_C, tracks the T-bill rate with a 20 basis point spread:

$$R_{C,t}^{j} = \exp\left[y^{j}\left(t, t+3/12\right)\right] + .002 - 1, \forall t = 1,2,...,T \tag{19}$$

Absent default, the government cash flow in each month is the SAP less any consolidation fee paid from lenders to the government, or 1.05% of principal. We denote the net guarantee payment from the government while the loan is in good standing by G:

$$G_t^j = \begin{cases} -P_{3k}^j \max\left[R_{C,3k}^j + 1.74\% - R_{S,3k}^j, 0\right]/4, & 3k < T^R \text{ and } 3k < T^{C,j} \ \forall k = 0,1,2,... \\ -P_{3k}^j \max\left[R_{C,3k}^j + 2.34\% - R_{S,3k}^j, 0\right]/4, & 3k \geq T^R \text{ and } 3k < T^{C,j} \ \forall k = 0,1,2,... \\ -P_{3k}^j \left(\max\left[R_{C,3k}^j + 2.64\% - R_{S,3k}^j, 0\right] - 1.05\%\right)/4, & 3k \geq T^{C,j} \ \forall k = 0,1,2,... \\ 0, & otherwise \end{cases} \tag{20}$$

In default, the government pays the outstanding principal, P_t^j, to the lender, assumes the loan, and recovers in proportion to the present value of the remaining outstanding payments. The default and recovery rates used in the calibration are described in Section B.5.

Several additional factors affect the timing and magnitude of student loan cash flows but we omit them from the analysis because we expect their impact on consolidation cost to be small. Voluntary prepayment has the effect of reducing the cost to the government by shortening effective loan maturity, but deferment and forbearance, by delaying payment, increase it.[39]

[39] Although borrowers should theoretically never exercise the option to prepay a loan when the interest on the loan is below the rate on their best alternative, in fact they frequently do. Offering prepayment thus can be a source of cost savings for the federal government.

Subsidized Stafford loans offer a higher subsidy than so-called unsubsidized Stafford loans, because interest does not accrue while the student is in school. In the prospective cost estimates, this affects the principal balance at the time repayment begins, making it smaller than it otherwise would be. In the analysis, all loans are unsubsidized loans.

B.3 Timing of Consolidation and Extension

We use a Probit rule for the intensity of consolidation of a given loan type, using the coefficient point estimates from Appendix A. Specifically, consolidation is decreasing in the student interest rate and decreasing in the time since repayment begins. We assume borrowers consolidate loans during the grace period consistent with the rule for consolidation at other times, but cannot consolidate at all while they are in school.[40] Thus, the annualized probability of consolidation, $q_{C,t}$, at month t is

$$q_{C,t}^{j} = \begin{cases} 0, & t < T^{R} - 6 \\ \Phi\left(\beta_1 + \beta_2 R_{S,t}^{j}\right), & T^{R} - 6 \leq t < T^{R} \\ \Phi\left(\beta_1 + \beta_2 R_{S,t}^{j} + \beta_3 \max\left(\left[t - T^{R}\right]/12, 0\right)\right), & t \geq T^{R} \end{cases} \tag{21}$$

where Φ is the cumulative standard normal distribution function and β_1 through β_3 are the loan type specific parameters reported in Table B-2 below (computed from the Probit estimates in Appendix A after substituting all static variables). We also apply the rule for consolidation to capture extension behavior in the counterfactual case with no consolidation. In the benchmark simulation without consolidation, we assume a constant rate of extension equal to the rate of consolidation when the interest rate is at the 8.25 percent cap. This implies the rate of extension is invariant to the level of the student rate. If anything, standard theory would predict that the sensitivity of extension behavior to short-term interest rates should be positive because, relative to market rates, the present value of an extension increases when interest rates are closer to the cap.

40 For the 2006 academic year, borrowers were allowed to consolidate during their in-school period.

Consolidation allows students to extend their original 10-year loans to as long as 30 years. To estimate the cost of the option to the government, several factors relating to the maturity change are accounted for. One such factor is that annual administrative expenses such as servicing and collection will be incurred for a longer period. Losses from defaults also must be treated consistently. The performance history of consolidated and unconsolidated loans suggests a higher cumulative default rate for consolidation loans. On a per annum basis, the default rates are similar, so we assume the same 2 percent per annum default rate for all loans.

Table B-2: Parameters Determining the Annual Frequency of Consolidation

	β_1	β_2	β_3
Consolidation: Eligible for 10-year term (balance $0-$20k)	-1.56	-0.06	-0.03
Consolidation: Eligible for 20-year term (balance $20k - $60k)	0.00	-0.20	-0.06
Consolidation: Eligible for 30-year term (balance exceeds $60k)	1.20	-0.37	-0.17
No Consolidation: Eligible for 10-year term (balance $0-$20k)	-1.79	0.00	-0.03
No Consolidation: Eligible for 20-year term (balance $20k - $60k)	-0.82	0.00	-0.06
No Consolidation: Eligible for 25- or 30-year term(balance > $60k)	-0.27	0.00	-0.17

B.4 Aggregate Cost of Consolidation

The forward-looking measure of consolidation cost is the sum of the cost of consolidation at origination for a representative cohort of loans issued in each year of the program. The cost of consolidation at origination of any individual loan in a cohort is:

- The present value of government cash flows under a loan originated when the borrower is eligible to consolidate; less
- The present value of cash flows under a loan originated where the borrower is not eligible to consolidate but may be eligible for term extension.

Each year's cohort of originated loans comprises direct and guaranteed loans, loans from borrowers with different total balances (determining their eligibility for various loan maturities under consolidation and term extension), and loans with different repayment start times based on how close to graduation the borrower is. In each year and roughly consistent with data, we assume that the fraction of volume originated in the guaranteed program is 75 percent and the fraction in the direct loan program is 25 percent. For guaranteed loans, the assumed distribution of loan balances is such that:

- 30% of originated volume is not eligible for any term extension. The borrower must pay off the loan over the original 10-year term.
- 30% of originated volume is eligible for 20-year term extension if consolidation is available. In the counterfactual where consolidation is not available, borrowers are not eligible for term extension.
- 40% of originated volume is eligible for 30-year term extension if consolidation is available. In the counterfactual, a term extension of 25 years is available.

For the direct program, borrowers have a symmetric opportunity to extend the maturity of their loans with and without consolidation:

- 30% of originated volume is not eligible for any term extension.
- 30% of originated volume is eligible for 20-year term extension.
- 40% of originated volume is eligible for 30-year term extension.

In a given year, repayment of each loan originated begins only after the borrower finishes school. We assume that the loans originated have repayment times uniformly distributed between one and five years. The aggregate cost of consolidation is the sum of the individual costs of consolidation across both direct and guaranteed programs, categories of maturity extension eligibility, and repayment start times.

For the intrinsic value calculations, we compute the cost of consolidation at the time the loans are consolidated. For simplicity, we assume the distribution of loan amounts and eligibility for term extension is the same as for the forward-looking estimates. On the other hand, we no longer treat term extension in the absence of consolidation as stochastic, and simply assume that borrowers take advantage of any term extension available, both in fact and in the counterfactual.

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