# The Economics of the Family and the Economics of Child Development: Becker and Tomes and Its Aftermath

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INET Summer Workshop July 12, 2012 • Becker and Tomes (1979, 1986)

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- While there were many other models of intergenerational inheritance of physical wealth, none had examined the intergenerational transmission of human capital.

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  - Scalar measure of capability "ability" or "human capital"
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  - Role of the parent is through active investment and through dealing with credit markets to secure investment in the child.

 $\theta_g$ : Capability of children in generation g

 $I_{g-1}$ : **Investment** in children of generation g by parents of generation g-1

 $G_{g-1}$ : Investment in children of generation g by schooling (and other public goods)

 $e_g$ : Endowment of children at birth

Capabilities are created by investment and endowments.

$$\theta_{\mathsf{g}} = \phi(\mathsf{I}_{\mathsf{g}-1}, \mathsf{G}_{\mathsf{g}-1}, \mathsf{e}_{\mathsf{g}})$$

A deterministic relationship.

**Endowments**: exogenous and subject to shocks  $u_g$ :

$$e_g = \lambda_0 + \lambda_1 e_{g-1} + u_g$$

• No direct effect of parents on transmission of endowments.

**1** Labor market: rewards human capital  $\theta_g$   $W_g$ : Reward in generation g (payment per unit human capital)  $L_g$ : "luck" in g (out of the control of the agent):

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     Perfect markets (parents can lend and borrow and commit debt to future generation)
  - Generalized in Becker-Tomes (1986) to allow for imperfect markets across generations. (Parents cannot commit debt to future generations.)

Preferences:

Parental utility for generation  $g: U_g$  $Z_{\varphi}$  is parental consumption

$$U_g = \eta(Z_g) + \underbrace{\delta}_{\mathsf{altruism}} U_{g+1}$$

Dynastic form of the utility function:

$$U_{g} = \sum_{j=0}^{\infty} \delta^{j} \eta(Z_{g+j})$$

#### Parents' Problem:

Parents allocate resources between adult consumption  $Z_{g}$  and investment in the child  $I_{g-1}$  under different market settings.

Endogenous altruism

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- Assortative mating

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- Fertility

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- Fertility
- Multiple child families and interactions between children and parents and children

These papers are the basis of an active literature on intergenerational transmission (see, e.g., Bjorklund, Jantti, Mazumbder, Solon, Haider, and Solon). Good recent summary by Black and Devereux (2011)

# Intergenerational Correlations of Earnings and Education

•  $Y_1$  is income in generation "1";  $Y_0$  is income in generation "0"

$$\underbrace{\ln(Y_1)}_{\substack{\text{child} \\ \text{permanent} \\ \text{earnings}}} = \omega + \beta \underbrace{\log(Y_0)}_{\substack{\text{parent} \\ \text{permanent} \\ \text{earnings}}} + \underbrace{L_1}_{\substack{\text{``Luck''}}}$$
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- $\beta$ : the intergenerational elasticity (IGE)
- $(1 \beta)$ : measure of intergenerational mobility

• Intergenerational correlation ( $\rho$ ): an alternative to  $\beta$ 

$$\rho = (\sigma_0/\sigma_1)\beta \tag{2}$$

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Intro

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- $\bullet$   $\sigma$  is the standard deviation of log earnings.
- Factors out the cross-sectional dispersion of log earnings in the two generations.
- $\beta$  can be higher in one society than in another simply because the variance of log earnings in the child's generation is higher in that society.

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  - Classical measurement error
  - Alignment error (ages of father and son)

Table: Elasticity and correlations from Jäntti et al. (2006)

Country	Elasticity $(\beta)$	Correlation $( ho)$
Men		
Denmark	0.071	0.089
	[0.064, 0.079]	[0.079, 0.099]
Finland	0.173	0.157
	[0.135, 0.211]	[0.128, 0.186]
Norway	0.155	0.138
	[0.137, 0.174]	[0.123, 0.152]
Sweden	0.258	0.141
	[0.234, 0.281]	[0.129, 0.152]
UK	0.306	0.198
	[0.242, 0.370]	[0.156, 0.240]
US	0.517	0.357
	[0.444, 0.590]	[0.306, 0.409]

Numbers in brackets below the point estimates show the bias corrected 95% bootstrap confidence interval.

Source: This reproduces much of Table 2 from Jäntti et al. (2006).



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$$\underbrace{\theta_1}_{\substack{\text{human capital} \\ \text{of child}}} = \underbrace{\psi}_{\substack{\text{productivity} \\ \text{of the} \\ \text{transmission} \\ \text{process}}} \log(I_0 + \underbrace{G_0}_{\substack{\text{governmental} \\ \text{investment}}}) + \underbrace{e_1}_{\substack{\text{child initial} \\ \text{endowment}}} \tag{4}$$

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• Observe  $I_0$  and  $G_0$  are perfect substitutes.

• Child endowments follow AR(1) process:

$$e_1 = \lambda_0 + \lambda_1 e_0 + \tau_1, \tag{5}$$

where  $\lambda$  is between 0 and 1 and  $\tau_1$  is white noise.

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• p is the return to a unit of human capital.

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- By maximizing the utility function with respect to parental investment and collecting terms, one arrives at

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•  $e_1$  correlated with  $ln(Y_0)$  through common shock  $e_0$ .

$$\beta = \frac{(1 - \gamma)\tau p + \lambda_1}{1 + (1 - \gamma)\tau p \lambda_1} \quad \uparrow \text{ as } \lambda_1 \uparrow, \tau \uparrow, p \uparrow, \gamma \downarrow.$$

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  - ${f 2}$  au is higher so that the human capital accumulation process is more efficient,
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  - $oldsymbol{0}$  governmental investment in human capital is less progressive so  $\gamma$  is smaller.

• Cross section variance of  $log Y_1$  (steady state)

$$\mathsf{Var}(\mathsf{In}\; Y) = \frac{[1 + (1-\gamma)\tau p\lambda_1]p^2\,\mathsf{Var}(u)}{[1 + (1-\gamma)\tau p\lambda_1](1-\lambda_1^2)[1 - (1-\gamma)\tau p]^2}$$

$$\uparrow$$
 in  $\lambda_1, \tau, p, 1 - \gamma$ 

New term not in  $\beta$  is Var(u)

Can show that out of steady state as income inequality  $\uparrow$ ,  $\beta \uparrow$ 

## Extensions of the Models in the *Treatise*

Recent research in the economics of the family (Cunha et al. 2006, 2007, 2008, 2009, 2010; Moon, 2008; Bernal and Keane, 2009; Todd and Wolpin, 2007; Del Boca, Flinn and Wiswall, 2010; Tartari, 2010; Conti et al., 2010; Akabayashi, 1995, 2000; Weinberg, 2006; Cosconati, 2009; Caucutt and Lochner, 2011) and research underway builds on earlier work by Becker and Tomes (1986) in the following ways:

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  - Multiple capabilities (cognitive, noncognitive, and biological capabilities)
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  - Interactions between child and parents in shaping investment (principle-agent problems)

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• Each component may be a vector.

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- The individual dies at the end of the period in which he is 2T years-old, just before his child's child is born.

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- The output of the investment process is a skill vector.

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- The technology of production of skill when the child is t years-old:

$$\theta_{t+1} = f_t(h, \theta_t, I_t), \qquad t = 1, \dots, T.$$
 (8)

New idea: parental environmental variables affect productivity of investment

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- $f_t$  is neoclassical: strictly increasing, strictly concave, and twice continuously differentiable in  $I_t$ .
- Solve recursively to obtain:

$$\theta_{t+1} = m_t (h, \theta_1, I_1, \dots, I_t). \tag{9}$$

$$\frac{\partial^{2} f_{t}\left(h, \theta_{t}, I_{t}\right)}{\partial \theta_{t} \partial I_{t}'} > 0.$$

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- Two distinct ideas:
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  - 2 Investment today raises the stock of skills in future periods and raises the productivity of future investment.

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This includes own and cross effects.
 (Cross complementarity of capabilities)

• This technology describes learning in rodents and macaques as documented, respectively, by Meaney (2001), Cameron (2004), and Knudsen (2006).

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- Early parental emotional environments encourage the animals to explore (and learn) more.
- This technology also captures the critical and sensitive periods in humans and animals.

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Intro

$$\frac{\partial f_t(h,\theta_t,I_t,)}{\partial I_t} = 0 \qquad \text{for } t \neq t^*$$

 $t^*$  is the critical period for that investment.

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then t is a sensitive period, where "·" is a common point of evaluation.

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- Dynamic complementarity explains why investment in more adults is more productive than for the less able.

Parental preferences for child outcomes

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  - Alternative: merit goods: Parents value specific outcomes, not necessarily child utility.

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- $c_1$  and  $c_2$  denote the consumption of the household in the first and second period of the life cycle of the child.
- The budget constraint is:

$$c_1 + l_1 + \frac{c_2 + l_2}{(1+r)} + \frac{b'}{(1+r)^2} = wh + \frac{wh}{(1+r)} + b.$$
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- Problem of the parent:

$$V(h, b, \theta_1) = \max \left\{ \eta(c_1) + \beta \eta(c_2) + \beta^2 \delta E\left[V(h', b', \theta_1')\right] \right\}. \tag{11}$$

### **A Special Case**

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- Adult stocks of skills critically depend on how investments are distributed over time.
- If investments in period one are zero,  $I_1 = 0$ , then it does not pay to invest in period two.
- If late investments are zero,  $I_2 = 0$ , it does not pay to invest early.

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- It is essential to invest early to get satisfactory adult outcomes.
- But it is also essential to invest late to harvest the fruits of the early investment.

$$h' = m_2 \left( h, \theta_1, \left[ \gamma \left( I_1 \right)^{\phi} + \left( 1 - \gamma \right) \left( I_2 \right)^{\phi} \right]^{\frac{1}{\phi}} \right), \qquad (15)$$

for  $\phi < 1$  and  $0 < \gamma < 1$ .

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- Thus  $I_1$  directly increases  $\theta_2$  which in turn affects the productivity of  $I_2$  in forming h'.
- $\gamma$  captures the net effect of  $I_1$  on h' through both self-productivity and direct complementarity.

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- The other face of CES complementarity is that when  $\phi$  is small, high early investments should be followed with high late investments if the early investments are to be harvested.
- In the extreme case when  $\phi \to -\infty$ , (15) converges to (14).

• This technology explains — why returns to education are low in the adolescent years for disadvantaged (low h, low  $l_1$ , low  $\theta_2$ ) adolescents but are high in the early years.

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- In the one-period model of childhood, inputs at any stage of childhood are perfect substitutes.
- Application of the one period model supports the widely held but empirically unsupported intuition that diminishing returns make investment in less advantaged adolescents *more* productive.

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- Productivity of early investment:  $\gamma$ ; late investment  $(1 \gamma)$ .
- Invest early if  $\gamma > (1-\gamma)(1+r)$

### **General Case**

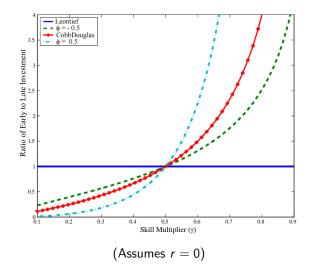
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- For  $-\infty < \phi < 1$ , the first-order conditions are necessary and sufficient given concavity of the technology in terms of  $I_1$  and  $I_2$ .
- $-\infty < \phi < 1$ :

$$\frac{I_1}{I_2} = \left[\frac{\gamma}{(1-\gamma)(1+r)}\right]^{\frac{1}{1-\phi}}.$$
 (16)

# The Ratio of Early to Late Investment in Human Capital As a Function of the Skill Multiplier for Different Values of Complementarity



Source: Cunha et al. (2007, 2009).

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- Even in this "perfect" credit market setting, parental investments depend on parental skills, h, because these characteristics affect the returns to investment.
- (But not other features of the model.)
- This generalizes Becker-Tomes.
- From the point of view of the child, this is a market failure due to the accident of birth.

### **Constraints on Borrowing Across Generations**

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### **Constraints on Borrowing Across Generations**

- Suppose parents cannot borrow against child's future earnings. (Becker-Tomes, 1986)
- A second credit constraint: the parental bequests must be non-negative and parents only have access to of a risk-free bond, and not to contingent claims.
- The problem of the parent is to maximize (11) subject to (10), the technology (15), and the liquidity constraint:

$$b' \ge 0. \tag{17}$$

• If binding, realized investment  $\hat{l}_j$  less than optimal  $l_j^*$   $\hat{l}_1 \leq l_1^*$  (unconstrained),  $\hat{l}_2 \leq l_2^*$  (unconstrained)

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- Under this formulation of market incompleteness, underinvestment in skills starts at early ages and continues throughout the life cycle of the child.
- Lower investment in both periods  $does \ not$  affect ratio of investments  $(I_1/I_2)$ .

• Both early and late investments depend on parental initial wealth *b* for the families for whom the constraint (17) binds.

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- Children who come from constrained families with lower *b* will have lower early *and* late investments.
- Interventions that occur at early stages would exhibit high returns, especially if they are followed up with resources to supplement late investments.

## Parents Themselves Face Lifetime Liquidity Constraints

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### Parents Themselves Face Lifetime Liquidity Constraints

- Cunha and Heckman (2007).
- Parents are subject to lifetime liquidity constraints and constraints that prevent the parents from borrowing against their own future labor income, which may affect their ability to finance investments in the child's early years.
- Assume that parents' productivity grows exogenously at rate  $\alpha$ .

• s: parental savings.

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- Parents face a sequence of constraints at each stage of the life cycle of the child:

$$c_1 + l_1 + \frac{s}{(1+r)} = wh + b$$
 (18)

$$c_2 + l_2 + \frac{b'}{(1+r)} = w(1+\alpha)h + s,$$
 (19)

 $s \ge 0$  and  $b' \ge 0$ .

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- $\bullet$  Some parents may be willing to do this, especially when  $\alpha$  is high.
- In the case when  $s \ge 0$  and  $b' \ge 0$  bind, and investments are not perfect substitutes, early income matters.

• Suppose  $\eta(c) = (c^{\lambda} - 1)/\lambda$ :

$$\frac{I_1}{I_2} = \left[\frac{\gamma}{(1-\gamma)(1+r)}\right]^{\frac{1}{1-\phi}} \underbrace{\left[\frac{(wh+b-I_1)}{\beta((1+\alpha)wh-I_2)}\right]^{\frac{1-\lambda}{1-\phi}}}_{(1-\gamma)(1+r)}.$$

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- $(1-\lambda)/(1-\phi) \in [0.8\bar{3},1.\bar{3}]$ . Family resource influence on investment.

 $\bullet$  This analysis of credit constrained families joined with a low value of  $\phi$  interprets the fact that the timing of family income in the early stages of childhood apparently affects the level of ability and achievement of the children, although there is still some controversy about the empirical importance of this effect.

### **Estimating and Interpreting the Estimates of the Technology** of Skill Formation

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- Cunha and Heckman (2008) and Cunha, Heckman, and Schennach (2010) estimate versions of the technology of skill formation. (Dynamic state space models)
- Can identify the technology under many different credit market structures.

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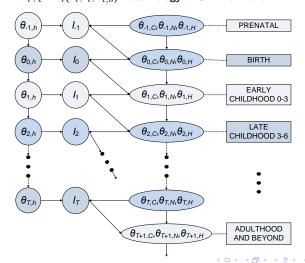
Fst

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  - Output as measured by test scores is meaningless.

A Life Cycle Framework for Organizing Studies and Integrating Evidence  $\theta_t = (\theta_C, \theta_N, \theta_H)$  capacities at t

 $\theta_{t,h}$ : parental traits at t

 $I_t$ : investment at t  $\theta_{t+1} = f_t(\theta_t, I_t, \theta_{t,h})$ : Technology of Skill Formation



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  - ② Self-productivity becomes stronger as children become older, for both cognitive and noncognitive skill formation (i.e.,  $\frac{\partial \theta_{t+1}}{\partial \theta_{*}} \uparrow t$ ).
  - Omplementarity between cognitive skills and investment becomes stronger as children become older. The elasticity of substitution for cognition is *smaller* in second stage production.

 $(\sigma_C \doteq 0.3)$  It is more difficult to compensate for the effects of adverse environments on cognitive endowments at later ages than it is at earlier ages. This pattern of the estimates helps to explain the evidence on ineffective cognitive remediation strategies for disadvantaged adolescents reported in Cunha et al. (2006).

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- Complementarity between noncognitive skills and investments becomes slightly weaker as children become older.

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- Noncognitive traits promote the accumulation of cognitive traits (but not vice versa).
- This econometric evidence is consistent with a broad array of evidence from interventions studies on life cycle profile of rates of return.

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- 16% is due to adolescent cognitive capabilities.
- 12% is due to adolescent noncognitive capabilities.
- Measured parental investments account for 15% of the variation in educational attainment.
- These estimates suggest that the measures of cognitive and noncognitive capabilities are powerful, but not exclusive, determinants of educational attainment and that other factors, besides the measures of family investment that we use, are at work in explaining variation in educational attainment.

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- 50-60% of the variance in lifetime income determined by factors present at the time college-going decisions are being made (Cunha et al., 2005; Hoffman, 2010; Yaron et al., 2010)

# Some Implications for Policy

Targeted strategies

#### Some Implications for Policy

- Targeted strategies
- Arises because compensation for adversity in noncognitive skills is somewhat less costly in the second period, and because of discounting of costs and concavity of the technology, it is efficient to invest relatively more in noncognitive traits in the second period.

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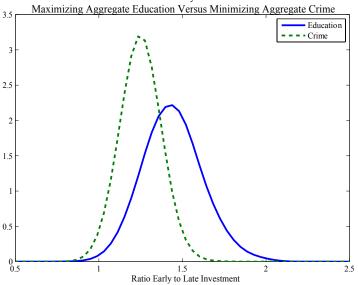
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Intro

- Arises because compensation for adversity in noncognitive skills is somewhat less costly in the second period, and because of discounting of costs and concavity of the technology, it is efficient to invest relatively more in noncognitive traits in the second period.
- The opposite is true for cognitive skills.

#### Densities of Ratio of Early to Late Investments



# **Integrating Family Intervention Studies With Family Influence Studies**

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- Many experiments that target early childhood—some long running (e.g., Perry Preschool)
- Evidence that they are effective (rate of return is 7–10%), and a primary channel of influence is through noncognitive skills personality (Heckman, Malofeeva, et al., 2008; revised 2011).

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  - Oan model interaction of parental investment with governmental investments: components may be perfect substitutes or not.
  - Identify different technologies (public and private) that both produce the same  $\theta_t$ (may use both)

•  $I_t^G$ : government investment

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- Can establish the channels through which government (external) investment promotes capabilities.

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- Experimental interventions that supplement family life show that we can boost  $\theta_t$  by interventions (surrogate parenting)

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- Skills are multidimensional.
- They produce inequality in education, wages, health, crime, and determine a host of important outcomes.

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Gaps in skills between advantaged and disadvantaged children

- Family life plays an enormously important role in shaping skills.
- Progress in the economics of the family is essential in shaping understanding of the origins of inequality.
- Much recent work shows the importance of the early years in shaping skills.

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- Schools matter, but what schools do depends on what parents send them, and how parents support the children in school.

#### **Appendix**

## Interpreting the Estimates of Cunha, Heckman, and Schennach

• The promise and limitations of the literature

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- To examine the implications of these estimates, analyze a standard social planning problem that can be solved solely from knowledge of the technology of skill formation and without knowledge of parental preferences and parental access to lending markets.

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- To examine the implications of these estimates, analyze a standard social planning problem that can be solved solely from knowledge of the technology of skill formation and without knowledge of parental preferences and parental access to lending markets.
- Determine optimal allocations of investments from a fixed budget to maximize aggregate schooling for a cohort of children.

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- May understate or overstate the parental response.
- These simulations produce a measure of the investment that is needed from whatever source to achieve the specified target.

• Agent heterogeneity in endowments and parental environments.

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- Optimal ratio of  $I_1/I_2$  depends on initial conditions.

• Even though there is static complementarity in each period

$$\frac{\partial^2 f_1(\theta_1, I_1, h)}{\partial I_1 \partial \theta_1} > 0,$$

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- The optimal ratio of early-to-late investment depends on the desired outcome, the endowments of children and the budget.
- Crime is more intensive in noncognitive skill than educational attainment, which depends much more strongly on cognitive skills.

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- Its efficient production begins before formal schooling begins.
- Education plays an important role, but early life factors create education and play independent roles beyond their effects on education.
- Human capital policy, broadly defined, has important implications for social policy about health, crime, wage inequality, teenage pregnancy.

