The Economics of the Family and the Economics of Child Development: Becker and Tomes and Its Aftermath

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• Becker and Tomes (1979, 1986)
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While there were many other models of intergenerational inheritance of physical wealth, none had examined the intergenerational transmission of human capital.

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**OLG Model**

1. One period of childhood
2. Scalar measure of capability “ability” or “human capital”
3. Scalar measure of investment (schooling, etc.)
4. Role of the parent is through active investment and through dealing with credit markets to secure investment in the child.
\[ \theta_g: \textbf{Capability} \text{ of children in generation } g \]

\[ l_{g-1}: \textbf{Investment} \text{ in children of generation } g \text{ by parents of generation } g - 1 \]

\[ G_{g-1}: \text{Investment in children of generation } g \text{ by schooling (and other public goods)} \]

\[ e_g: \text{Endowment of children at birth} \]
**Capabilities** are created by investment and endowments.

\[ \theta_g = \phi(l_{g-1}, G_{g-1}, e_g) \]

- A deterministic relationship.

**Endowments**: exogenous and subject to shocks \( u_g \):

\[ e_g = \lambda_0 + \lambda_1 e_{g-1} + u_g \]

- No direct effect of parents on transmission of endowments.
Markets:

- Labor market: rewards human capital $\theta_g$
  
  $W_g$: Reward in generation $g$ (payment per unit human capital)
  
  $L_g$: “luck” in $g$ (out of the control of the agent):

  $$Y_g = W_g \theta_g + L_g$$
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b) Credit market in which agents (parents) can lend and borrow

   Perfect markets (parents can lend and borrow and commit debt to future generation)

2. Generalized in Becker-Tomes (1986) to allow for imperfect markets across generations. (Parents cannot commit debt to future generations.)
Preferences:
Parental utility for generation $g$: $U_g$

$Z_g$ is parental consumption

$$U_g = \eta(Z_g) + \delta U_{g+1}$$

altruism

Dynastic form of the utility function:

$$U_g = \sum_{j=0}^{\infty} \delta^j \eta(Z_{g+j})$$

Parents’ Problem:
Parents allocate resources between adult consumption $Z_g$ and investment in the child $I_{g-1}$ under different market settings.
Papers in *Treatise* discuss, but do not fully characterize (by author’s own admission)

- Endogenous altruism
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- **a** Endogenous altruism
- **b** Assortative mating
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- **a** Endogenous altruism
- **b** Assortative mating
- **c** Fertility
- **d** Multiple child families and interactions between children and parents and children
These papers are the basis of an active literature on intergenerational transmission (see, e.g., Bjorklund, Jantti, Mazumbder, Solon, Haider, and Solon). Good recent summary by Black and Devereux (2011)
Intergenerational Correlations of Earnings and Education

- $Y_1$ is income in generation “1”; $Y_0$ is income in generation “0”

$$\ln(Y_1) = \omega + \beta \log(Y_0) + L_1$$  \hspace{1cm} (1)

- $\beta$: the intergenerational elasticity (IGE)
- $(1 - \beta)$: measure of intergenerational mobility
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Intergenerational correlation ($\rho$): an alternative to $\beta$

$$\rho = (\sigma_0/\sigma_1)\beta$$

(2)
• Intergenerational correlation ($\rho$): an alternative to $\beta$

$$\rho = \left( \frac{\sigma_0}{\sigma_1} \right) \beta$$  \hspace{1cm} (2)

• $\sigma$ is the standard deviation of log earnings.
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- Factors out the cross-sectional dispersion of log earnings in the two generations.
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- $\sigma$ is the standard deviation of log earnings.
- Factors out the cross-sectional dispersion of log earnings in the two generations.
- $\beta$ can be higher in one society than in another simply because the variance of log earnings in the child’s generation is higher in that society.
Issues in estimating the intergenerational elasticity of earnings

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Intro

Optimal Investment

Est

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  - Classical measurement error
  - Alignment error (ages of father and son)
Table: Elasticity and correlations from Jäntti et al. (2006)

<table>
<thead>
<tr>
<th>Country</th>
<th>Elasticity ($\beta$)</th>
<th>Correlation ($\rho$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Men</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Denmark</td>
<td>0.071</td>
<td>0.089</td>
</tr>
<tr>
<td></td>
<td>[0.064, 0.079]</td>
<td>[0.079, 0.099]</td>
</tr>
<tr>
<td>Finland</td>
<td>0.173</td>
<td>0.157</td>
</tr>
<tr>
<td></td>
<td>[0.135, 0.211]</td>
<td>[0.128, 0.186]</td>
</tr>
<tr>
<td>Norway</td>
<td>0.155</td>
<td>0.138</td>
</tr>
<tr>
<td></td>
<td>[0.137, 0.174]</td>
<td>[0.123, 0.152]</td>
</tr>
<tr>
<td>Sweden</td>
<td>0.258</td>
<td>0.141</td>
</tr>
<tr>
<td></td>
<td>[0.234, 0.281]</td>
<td>[0.129, 0.152]</td>
</tr>
<tr>
<td>UK</td>
<td>0.306</td>
<td>0.198</td>
</tr>
<tr>
<td></td>
<td>[0.242, 0.370]</td>
<td>[0.156, 0.240]</td>
</tr>
<tr>
<td>US</td>
<td>0.517</td>
<td>0.357</td>
</tr>
<tr>
<td></td>
<td>[0.444, 0.590]</td>
<td>[0.306, 0.409]</td>
</tr>
</tbody>
</table>

Numbers in brackets below the point estimates show the bias corrected 95% bootstrap confidence interval.

Source: This reproduces much of Table 2 from Jäntti et al. (2006).
Why the IGE may differ across countries and over time

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- First, the budget constraint assumes families must allocate all after-tax lifetime income to either parental consumption ($Z_0$) or investment in the child ($I_0$):

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- Human capital of the child \( \theta_1 \) is produced by a semi-log production function:

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- Observe $I_0$ and $G_0$ are perfect substitutes.
Child endowments follow AR(1) process:

$$e_1 = \lambda_0 + \lambda_1 e_0 + \tau_1, \quad (5)$$

where $\lambda$ is between 0 and 1 and $\tau_1$ is white noise.
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Earnings equation:

\[ \log(Y_1) = \mu + p\theta_1 \]  \hspace{1cm} (6)
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Earnings equation:

\[ \log(Y_1) = \mu + p\theta_1 \]  

\( p \) is the return to a unit of human capital.
The family maximizes

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By maximizing the utility function with respect to parental investment and collecting terms, one arrives at
\[ \log(Y_1) = \mu^* + [(1 - \gamma) \psi p] \log(Y_0) + pe_1 \quad (7) \]
which is the form of the standard IGE regression.
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\( e_1 \) correlated with \( \ln(Y_0) \) through common shock \( e_0 \).
In steady state, $\sigma_0 = \sigma_1$

$$\beta = \frac{(1 - \gamma)\tau p + \lambda_1}{1 + (1 - \gamma)\tau p \lambda_1} \uparrow \text{ as } \lambda_1 \uparrow, \tau \uparrow, p \uparrow, \gamma \downarrow.$$
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1. the heritability coefficient $\lambda$ is higher so ability is more highly correlated across generations,
2. $\tau$ is higher so that the human capital accumulation process is more efficient,
3. earnings returns to human capital are higher so $p$ is larger, or
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Cross section variance of log $Y_1$ (steady state)

$$Var(\ln Y) = \frac{[1 + (1 - \gamma)\tau p\lambda_1]p^2 Var(u)}{[1 + (1 - \gamma)\tau p\lambda_1](1 - \lambda_1^2)[1 - (1 - \gamma)\tau p]^2}$$

↑ in $\lambda_1, \tau, p, 1 - \gamma$

New term not in $\beta$ is $Var(u)$

Can show that out of steady state as income inequality ↑, $\beta$ ↑
Extensions of the Models in the *Treatise*

Recent research in the economics of the family (Cunha et al. 2006, 2007, 2008, 2009, 2010; Moon, 2008; Bernal and Keane, 2009; Todd and Wolpin, 2007; Del Boca, Flinn and Wiswall, 2010; Tartari, 2010; Conti et al., 2010; Akabayashi, 1995, 2000; Weinberg, 2006; Cosconati, 2009; Caucutt and Lochner, 2011) and research underway builds on earlier work by Becker and Tomes (1986) in the following ways:
Many expand the original framework to recognize:

- Multiple stages of childhood and adulthood
- Moves beyond "schooling" as investment to allow economists to address the benefits and costs of different types of investments
  - Schooling
  - Training
  - Preschool and early childhood investments
- Recognizes the modern literature on the biology and psychology of skill formation and the literature on critical and sensitive periods in development
- Multiple capabilities (cognitive, noncognitive, and biological capabilities)
- Child preference formation and emergence of decision making (transition from child to adult)
- Interactions between child and parents in shaping investment (principle-agent problems)
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Children possess a vector of capabilities at each age $t$. 
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$$\theta_t = (\theta_t^C, \theta_t^N, \theta_t^H)$$
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Each component may be a vector.
Draw on my work with Cunha (2007, 2009)

- Individual lives $2T$ years. ($T \geq 2$)
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- The first $T$ years, the individual is a child of an adult parent.
- From age $T + 1$ to $2T$ the individual lives as an adult and is the parent of a child.
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- Individual lives $2T$ years. ($T \geq 2$)
- The first $T$ years, the individual is a child of an adult parent.
- From age $T + 1$ to $2T$ the individual lives as an adult and is the parent of a child.
- The individual dies at the end of the period in which he is $2T$ years-old, just before his child’s child is born.
A household consists of an adult parent and his child.
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• Parents invest in their children because of altruism.
• $l_t$: parental investments in child skill when the child is $t$ years-old, where $t = 1, 2, \ldots, T$.
• The output of the investment process is a skill vector.
Agent born with initial conditions: $\theta_0$. 

$\theta_t$ is the vector of capabilities. The technology of production of skill when the child is $t$ years-old:

$$\theta_{t+1} = f_t(h, \theta_t, I_t), t = 1, \ldots, T.$$ 

New idea: parental environmental variables affect productivity of investment $f_t$ is neoclassical: strictly increasing, strictly concave, and twice continuously differentiable in $I_t$. 

Solve recursively to obtain:

$$\theta_{t+1} = m_t(h, \theta_1, I_1, \ldots, I_t).$$
• Agent born with initial conditions: $\theta_0$.
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This can be influenced by family investment (also has genetic component).

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The **technology of production of skill** when the child is $t$ years-old:

$$\theta_{t+1} = f_t (h, \theta_t, I_t), \quad t = 1, \ldots, T. \quad (8)$$

New idea: parental environmental variables affect productivity of investment.

$f_t$ is neoclassical: strictly increasing, strictly concave, and twice continuously differentiable in $I_t$. 
Agent born with initial conditions: $\theta_0$.

This can be influenced by family investment (also has genetic component).

$h$ is parental characteristics (e.g., their IQ, education, etc.).

$\theta_t$ is the vector of capabilities.

The **technology of production of skill** when the child is $t$ years-old:

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New idea: parental environmental variables affect productivity of investment

$f_t$ is neoclassical: strictly increasing, strictly concave, and twice continuously differentiable in $I_t$.

Solve recursively to obtain:

$$\theta_{t+1} = m_t (h, \theta_1, l_1, \ldots, l_t). \quad (9)$$
Dynamic complementarity arises when

\[
\frac{\partial^2 f_t (h, \theta_t, l_t)}{\partial \theta_t \partial l_t} > 0.
\]
Dynamic complementarity arises when
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\frac{\partial^2 f_t(h, \theta_t, l_t)}{\partial \theta_t \partial l'_t} > 0.
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Two distinct ideas:
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1. Higher stocks of capabilities at age \( t \) promote the productivity of investment at that age;
Dynamic complementarity arises when

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Two distinct ideas:

1. Higher stocks of capabilities at age $t$ promote the productivity of investment at that age;
2. Investment today raises the stock of skills in future periods and raises the productivity of future investment.
Self-productivity:

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Self-productivity:

\[ \frac{\partial f_t (h, \theta_t, l_t)}{\partial \theta_t} > 0. \]

This includes own and cross effects.

(Cross complementarity of capabilities)
This technology describes learning in rodents and macaques as documented, respectively, by Meaney (2001), Cameron (2004), and Knudsen (2006).
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- Early parental emotional environments encourage the animals to explore (and learn) more.
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Early parental emotional environments encourage the animals to explore (and learn) more.

This technology also captures the critical and sensitive periods in humans and animals.
Critical and sensitive periods for investment:

1. If \( \frac{\partial f_t(h, \theta_t, I_t)}{\partial I_t} = 0 \) for \( t \neq t^* \), then \( t^* \) is the critical period for that investment.

2. If \( \frac{\partial f_t}{\partial I_t}(\cdot) > \frac{\partial f_t'}{\partial I_t'}(\cdot) \) for \( t \neq t' \), then \( t' \) is a sensitive period, where \( \cdot \) is a common point of evaluation.
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2. If

\[ \frac{\partial f_t(\cdot)}{\partial I_t} > \frac{\partial f_{t'}(\cdot)}{\partial I_{t'}} \quad t \neq t' \]

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Special cases of the technology:
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Ontogenic models:

\[ \theta_{t+1} = f_t(h, \theta_t, l_t) = f_t(h_0, \theta_0), \quad \forall t \geq 0 \]

(initial conditions fully determinative, no investment, no feedback).
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(initial conditions fully determinative, no investment, no feedback).
- Initially-determined trajectories fully determine life cycle evolution ("Types" as in Keane and Wolpin, 1997).
- Dynamic complementarity explains why investment in more adults is more productive than for the less able.
Parental preferences for child outcomes
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- $V^P(V^C)$: the valuation by parents of child value function.
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- Models of Parent-Child Interactions (Akabayashi; Weinberg; Cosconati; Conti et al.)
- Parental altruism.
- Alternative: merit goods: Parents value specific outcomes, not necessarily child utility.
Preferences and the Optimal Life-Cycle Profile of Investments

Assume $T = 2$; stationary environment. (Two periods of childhood)
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Preferences and the Optimal Life-Cycle Profile of Investments

- Assume $T = 2$; stationary environment. (Two periods of childhood)
- $w$: wage rate
- $r$: interest rate
- At the beginning of adulthood, the parents draw the initial level of skill of the child, $\theta_1$, from $J(\theta_1)$, which they can influence through investment.
On reaching adulthood, parents receive bequest $b$. 

\[ c_1 + I_1 + c_2 + I_2 (1 + r) + b' (1 + r)^2 = w + w (1 + r) + b. \]
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State variables for the parent: parental skills, $h$, the parental financial resources, $b$, and the initial skill level of the child, $\theta_1$. 
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$c_1$ and $c_2$ denote the consumption of the household in the first and second period of the life cycle of the child.
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c_1$ and $c_2$ denote the consumption of the household in the first and second period of the life cycle of the child.

The budget constraint is:

$$c_1 + l_1 + \frac{c_2 + l_2}{(1 + r)} + \frac{b'}{(1 + r)^2} = wh + \frac{wh}{(1 + r)} + b.$$  (10)
• $\beta$: discount factor
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- $\delta$: measure of parental altruism toward the child.
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\( \eta(\cdot) \) is the one period utility function.
• $\beta$: discount factor
• $\delta$: measure of parental altruism toward the child.
• $\eta(\cdot)$ is the one period utility function.
• Problem of the parent:

$$V(h, b, \theta_1) = \max \left\{ \eta(c_1) + \beta \eta(c_2) + \beta^2 \delta E \left[ V(h', b', \theta'_1) \right] \right\}.$$  

(11)
A Special Case

- Assume $\theta_1, I_1, I_2$ are scalars.
A Special Case

- Assume $\theta_1$, $l_1$, $l_2$ are scalars.
- The child’s adult stock of skills, $h'$:

$$h' = m_2 (h, \theta_1, l_1, l_2).$$  

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A Special Case

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  \[ h' = m_2 (h, \theta_1, I_1, I_2) \]. \hspace{1cm} (12)

- Conventional specification of technology (12) implicit in one-period models:

  \[ h' = m_2 (h, \theta_1, \gamma I_1 + (1 - \gamma) I_2) \] \hspace{.5cm} \hspace{1cm} (13)

  $\gamma = 1/2$. 

Adult stocks of skills do not depend on how investments are distributed over different periods of childhood.
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Polar opposite:

\[ h' = m_2(h, \theta_1, \min\{l_1, l_2\}) . \] (14)

**Adult stocks of skills critically depend on how investments are distributed over time.** If investments in period one are zero, \(I_1 = 0\), then it does not pay to invest in period two. If late investments are zero, \(I_2 = 0\), it does not pay to invest early.
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Dual Face of Complementarity

- Complementarity has a dual face.

It is essential to invest early to get satisfactory adult outcomes. But it is also essential to invest late to harvest the fruits of the early investment.
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- Complementarity has a dual face.
- It is essential to invest early to get satisfactory adult outcomes.
- But it is also essential to invest late to harvest the fruits of the early investment.
More general technology:

\[ h' = m_2 \left( h, \theta_1, \left[ \gamma (l_1)^\phi + (1 - \gamma) (l_2)^\phi \right]^{1/\phi} \right), \quad (15) \]

for \( \phi \leq 1 \) and \( 0 \leq \gamma \leq 1 \).
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The CES share parameter \( \gamma \) is a *skill multiplier*. 
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It arises from the productivity of early investment not only in directly boosting \( h' \) (through self-productivity) but also in raising the productivity of \( l_2 \) by increasing \( \theta_2 \) through first period investments.
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- Thus $I_1$ directly increases $\theta_2$ which in turn affects the productivity of $I_2$ in forming $h'$. 

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Heckman

The Economics of the Family
More general technology:

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Thus \( l_1 \) directly increases \( \theta_2 \) which in turn affects the productivity of \( l_2 \) in forming \( h' \).

\( \gamma \) captures the net effect of \( l_1 \) on \( h' \) through both self-productivity and direct complementarity.
Elasticity of substitution $1/(1 - \phi)$ is a measure of how easy it is to substitute between $I_1$ and $I_2$. 

$\phi$ represents the degree of complementarity (or substitutability) between early and late investments in producing skills. When $\phi$ is small, low levels of early investment $I_1$ are not easily remediated by later investment $I_2$ in producing human capital. The other face of CES complementarity is that when $\phi$ is small, high early investments should be followed with high late investments if the early investments are to be harvested. In the extreme case when $\phi \to -\infty$, (15) converges to (14).
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In the extreme case when \( \phi \rightarrow -\infty \), (15) converges to (14).
This technology explains — why returns to education are low in the adolescent years for disadvantaged (low $h$, low $l_1$, low $\theta_2$) adolescents but are high in the early years.
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In the one-period model of childhood, inputs at any stage of childhood are perfect substitutes.
This technology explains — why returns to education are low in the adolescent years for disadvantaged (low $h$, low $l_1$, low $\theta_2$) adolescents but are high in the early years.

In the one-period model of childhood, inputs at any stage of childhood are perfect substitutes.

Application of the one period model supports the widely held but empirically unsupported intuition that diminishing returns make investment in less advantaged adolescents more productive.
Optimal Investment Strategies for $\phi = 1$
(perfect substitutes)

- When $\phi = 1$, early and late investments are perfect CES substitutes, the optimal investment strategy is straightforward.

The price of early investment is $\text{\$1.}$

The price of late investment is $\text{\$/}(1 + r)$.

Productivity of early investment: $\gamma$; late investment $(1 - \gamma)$.

Invest early if $\gamma > (1 - \gamma)(1 + r)$.
Optimal Investment Strategies for $\phi = 1$
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General Case

For $-\infty < \phi < 1$, the first-order conditions are necessary and sufficient given concavity of the technology in terms of $l_1$ and $l_2$. 

\[
I_1 I_2 = \gamma (1 - \gamma) (1 + r) \left[ \frac{1}{1 - \phi} \right].
\]
For $-\infty < \phi < 1$, the first-order conditions are necessary and sufficient given concavity of the technology in terms of $l_1$ and $l_2$.

$-\infty < \phi < 1$:

\[ \frac{l_1}{l_2} = \left[ \frac{\gamma}{(1 - \gamma)(1 + r)} \right]^{\frac{1}{1-\phi}}. \]  

(16)
The Ratio of Early to Late Investment in Human Capital As a Function of the Skill Multiplier for Different Values of Complementarity

Assumes $r = 0$)

Source: Cunha et al. (2007, 2009).
Alternative Market Environments

- In a complete-market model, optimal investment levels do not depend on the parental permanent shocks to wages or endowments or the parameters that characterize the utility function $\eta(\cdot)$.
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- Even in this “perfect” credit market setting, parental investments depend on parental skills, $h$, because these characteristics affect the returns to investment.
- (But not other features of the model.)
- This generalizes Becker-Tomes.
- From the point of view of the child, this is a market failure due to the accident of birth.
Constraints on Borrowing Across Generations

Suppose parents cannot borrow against child’s future earnings. (Becker-Tomes, 1986)
### Constraints on Borrowing Across Generations

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Constraints on Borrowing Across Generations

- Suppose parents cannot borrow against child’s future earnings. (Becker-Tomes, 1986)
- A second credit constraint: the parental bequests must be non-negative and parents only have access to of a risk-free bond, and not to contingent claims.
- The problem of the parent is to maximize (11) subject to (10), the technology (15), and the liquidity constraint:

\[ b' \geq 0. \] (17)
If binding, realized investment $\hat{I}_j$ less than optimal $I_j^*$

$\hat{I}_1 \leq I_1^*$ (unconstrained), $\hat{I}_2 \leq I_2^*$ (unconstrained)
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Under liquidity constraints actual investment $\hat{I}_1 < I_2^*$ is lower than the early investment under the perfect credit market model, $I_1^*$, and $\hat{I}_2 < I_2^*$. 
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Under this formulation of market incompleteness, underinvestment in skills starts at early ages and continues throughout the life cycle of the child.
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Under this formulation of market incompleteness, underinvestment in skills starts at early ages and continues throughout the life cycle of the child.

Lower investment in both periods *does not affect ratio of investments* ($I_1/I_2$).
Both early and late investments depend on parental initial wealth $b$ for the families for whom the constraint (17) binds.
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- Both early and late investments depend on parental initial wealth $b$ for the families for whom the constraint (17) binds.
- Children who come from constrained families with lower $b$ will have lower early and late investments.
- Interventions that occur at early stages would exhibit high returns, especially if they are followed up with resources to supplement late investments.
Parents Themselves Face Lifetime Liquidity Constraints

- Cunha and Heckman (2007).
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- Parents are subject to lifetime liquidity constraints and constraints that prevent the parents from borrowing against their own future labor income, which may affect their ability to finance investments in the child’s early years.
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- Cunha and Heckman (2007).
- Parents are subject to lifetime liquidity constraints and constraints that prevent the parents from borrowing against their own future labor income, which may affect their ability to finance investments in the child’s early years.
- Assume that parents’ productivity grows exogenously at rate $\alpha$. 
s: parental savings.
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Parents face a sequence of constraints at each stage of the life cycle of the child:

\[ c_1 + l_1 + \frac{s}{1 + r} = wh + b \]  \hspace{1cm} (18)

\[ c_2 + l_2 + \frac{b'}{1 + r} = w (1 + \alpha) h + s, \]  \hspace{1cm} (19)

\[ s \geq 0 \text{ and } b' \geq 0. \]
The restriction $s \geq 0$ says that parents cannot borrow income from their old age to finance consumption and investment when the child is in the first stage of the life cycle.
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Some parents may be willing to do this, especially when $\alpha$ is high.

In the case when $s \geq 0$ and $b' \geq 0$ bind, and investments are not perfect substitutes, early income matters.
Suppose $\eta(c) = (c^\lambda - 1)/\lambda$:

$$\frac{l_1}{l_2} = \left[\frac{\gamma}{(1 - \gamma)(1 + r)}\right]^{\frac{1}{1 - \phi}} \left[\frac{(wh + b - l_1)}{\beta ((1 + \alpha)wh - l_2)}\right]^{\frac{1 - \lambda}{1 - \phi}} \leq 1.$$
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Now, ratios of investment depend on parental preferences and endowments.
Suppose $\eta(c) = (c^\lambda - 1)/\lambda$:

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\((1 - \lambda)/(1 - \phi) \in [0.8\overline{3}, 1.\overline{3}] \). Family resource influence on investment.
This analysis of credit constrained families joined with a low value of $\phi$ interprets the fact that the timing of family income in the early stages of childhood apparently affects the level of ability and achievement of the children, although there is still some controversy about the empirical importance of this effect.
Estimating and Interpreting the Estimates of the Technology of Skill Formation

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- Cunha and Heckman (2008) and Cunha, Heckman, and Schennach (2010) estimate versions of the technology of skill formation. (Dynamic state space models)
- Can identify the technology under many different credit market structures.
- Econometric Challenges
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- Multiplicity of measured inputs and measured outputs
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- Multiplicity of measured inputs and measured outputs
- Measurement error in inputs and outputs (we only have proxies)
- Endogeneity of Investment and hence stocks of skills
- Omitted inputs
- Need to go beyond the linear technology to capture the notion of substitution between early and late.
- Output as measured by test scores is meaningless.
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A Life Cycle Framework for Organizing Studies and Integrating Evidence

\[ \theta_t = (\theta_C, \theta_N, \theta_H) \] capacities at \( t \)

\[ \theta_{t,h}: \text{parental traits at } t \]

\[ I_t: \text{investment at } t \]

\[ \theta_{t+1} = f_t(\theta_t, I_t, \theta_{t,h}): \text{Technology of Skill Formation} \]
Findings from Nonlinear Model (Cunha et al., 2010)

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  - Self-productivity becomes stronger as children become older, for both cognitive and noncognitive skill formation (i.e., $\frac{\partial \theta_{t+1}}{\partial \theta_t} \uparrow t$).
  - Complementarity between cognitive skills and investment becomes stronger as children become older. The elasticity of substitution for cognition is smaller in second stage production.
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Optimal Investment

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- Noncognitive traits promote the accumulation of cognitive traits (but not vice versa).
- This econometric evidence is consistent with a broad array of evidence from interventions studies on life cycle profile of rates of return.
34% of the variation in educational attainment in the sample is explained by the measures of cognitive and noncognitive capabilities.
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Measured parental investments account for 15% of the variation in educational attainment.

These estimates suggest that the measures of cognitive and noncognitive capabilities are powerful, but not exclusive, determinants of educational attainment and that other factors, besides the measures of family investment that we use, are at work in explaining variation in educational attainment.
Role of Luck

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- 50-60% of the variance in lifetime income determined by factors present at the time college-going decisions are being made (Cunha et al., 2005; Hoffman, 2010; Yaron et al., 2010)
Some Implications for Policy

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Densities of Ratio of Early to Late Investments
Maximizing Aggregate Education Versus Minimizing Aggregate Crime

Ratio Early to Late Investment

Education
Crime
Integrating Family Intervention Studies With Family Influence Studies

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Intro Optimal Investment Est

Integrating Family Intervention Studies With Family Influence Studies

- Beyond treatment effects
- Understanding mechanisms
- Many experiments that target early childhood—some long running (e.g., Perry Preschool)
- Evidence that they are effective (rate of return is 7–10%), and a primary channel of influence is through noncognitive skills — personality (Heckman, Malofeeva, et al., 2008; revised 2011).
Technology of skill formation allows economists to integrate these diverse studies through their effects on $\theta_t$. 
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- Can model interaction of parental investment with governmental investments: components may be perfect substitutes or not.

- Identify different technologies (public and private) that both produce the same $\theta_t$ (may use both)
• $I_t^G$: government investment
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$I_t^P$: private (family) investment

Studies under way doing this (Moon, Pinto, et al. 2011)

Can establish the channels through which government (external) investment promotes capabilities.
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Government technology: $f^G(\theta_t^P, I_t^G, I_t^P, h)$
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Behavioral Genetics

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Epigenetics: Judith Rich Harris, Turkheimer, and beyond
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Experimental interventions that supplement family life show that we can boost $\theta_t$ by interventions (surrogate parenting)
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- But inequality in skills — broadly defined — plays an important role in creating inequality in society.
- Skills are multidimensional.
- They produce inequality in education, wages, health, crime, and determine a host of important outcomes.
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Much recent work shows the importance of the early years in shaping skills.
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• Dynamic of skill formation has a strong biological foundation.
• Redirect and broadens our thinking about policy.
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• Schools matter, but what schools do depends on what parents send them, and how parents support the children in school.
Appendix
Interpreting the Estimates of Cunha, Heckman, and Schennach

- The promise and limitations of the literature
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- To examine the implications of these estimates, analyze a standard social planning problem that can be solved solely from knowledge of the technology of skill formation and without knowledge of parental preferences and parental access to lending markets.
Interpreting the Estimates of Cunha, Heckman, and Schennach

- The promise and limitations of the literature
- To examine the implications of these estimates, analyze a standard social planning problem that can be solved solely from knowledge of the technology of skill formation and without knowledge of parental preferences and parental access to lending markets.
- Determine optimal allocations of investments from a fixed budget to maximize aggregate schooling for a cohort of children.
• Assume that the state has full control over family investment decisions.
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• Do not model parental investment responses to the policy or parent-child interactions: This is a huge open issue, currently being investigated. (Principle — agent problems within the family)
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May understate or overstate the parental response.
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May understate or overstate the parental response.

These simulations produce a measure of the investment that is needed from whatever source to achieve the specified target.
Agent heterogeneity in endowments and parental environments.
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Optimal ratio of $I_1/I_2$ depends on initial conditions.
Even though there is static complementarity in each period

\[ \frac{\partial^2 f_1(\theta_1, l_1, h)}{\partial l_1 \partial \theta_1} > 0, \]

the optimal policy is to invest in the less advantaged in early years.
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- The optimal ratio of early-to-late investment depends on the desired outcome, the endowments of children and the budget.
- Crime is more intensive in noncognitive skill than educational attainment, which depends much more strongly on cognitive skills.
How Does All of This Cause Us to Rethink Education and Human Capital Policies?

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- Education plays an important role, but early life factors create education and play independent roles beyond their effects on education.
- Human capital policy, broadly defined, has important implications for social policy about health, crime, wage inequality, teenage pregnancy.