Notes on Cunha, Heckman, and Schennach (2010)

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INET Summer Workshop
July 14, 2012
This draft, July 13, 2012

- Why is it efficient to invest in least well-off (by endowment) children?
- 2 Children A and B and Two Periods $(1, 2)$

(I)

$$\theta_1^A = f(\theta_0^A, I_0^A) \qquad \theta_1^B = f(\theta_0^B, I_0^B)$$
$$\theta_2^A = f(\theta_1^A, I_1^A) \qquad \theta_2^B = f(\theta_1^B, I_1^B)$$

- Assume $r = 0$.

- We seek to maximize $\theta_2^A + \theta_2^B$ subject to constraints in 1.

$$I_0^A + I_0^B + I_1^A + I_2^B = M$$

- Intuition is that by investing we concavify the production functions.

- Intuition:

Take Simple Case:

$$\theta^A = g(I_A \alpha^A) \tag{1}$$
$$\theta^B = g(I_B \alpha^B) \tag{2}$$

$$g' > 0 \qquad g'' < 0$$

- Allocate I_A and I_B

  $\max \theta^A + \theta^B$

  st. (1), (2) and

$$
\begin{aligned}
I_A + I_B &= M \\
\alpha^A g'(I_A \alpha^A) &= \alpha^B g'(I_B \alpha^B) \\
\frac{\alpha^A}{\alpha^B} &= \frac{g'(I_A \alpha^A)}{g'(I_B \alpha^B)} \\
I_A = I_B & \qquad \text{if } \alpha^A = \alpha^B
\end{aligned}
$$

$$
\begin{aligned}
\text{If } \alpha^A > \alpha^B \Rightarrow & I_A \alpha^A < I_B \alpha^B \\
& \therefore I_B > I_A
\end{aligned}
$$

(II) Cobb-Douglas case

$$g(b) = b^{\phi}$$
$$\phi\alpha^{A}(I_A\alpha^{A})^{\phi-1} = \phi\alpha^{B}(I_B\alpha^{B})^{\phi-1}$$
$$\left(\frac{I_B}{I_A}\right)^{1-\phi} = \left(\frac{I_A}{I_B}\right)^{\phi-1} = \left(\frac{\alpha_B}{\alpha_A}\right)$$

$$
\begin{aligned}
(\phi - 1)\ln(I_A/I_B) &= \phi \ln(\alpha^B/\alpha^A) \\
\ln(I_A/I_B) &= \left(\frac{\phi}{\phi - 1}\right) \ln\left(\frac{\alpha^A}{\alpha^B}\right) \\
& 0 < \phi < 1, \\
& \alpha^A > \alpha^B \\
& \implies \ln(I_A/I_B) < 0 \\
& I_B > I_A \;\text{ if }\; \alpha^A/\alpha_B > 1
\end{aligned}
$$

- Take a second case

$$\theta^A = \alpha^A g(I^A) \qquad g' > 0 \qquad g'' < 0$$
$$\theta^B = \alpha^B g(I^B)$$

$$\max \alpha^A g(I_A) + \alpha^B g(I^B)$$

st.

$$M = I^A + I^B$$
$$\alpha^A g'(I_A) = \alpha^B g'(I^B)$$
$$\frac{\alpha^A}{\alpha_B} = \frac{g'(I^B)}{g'(I^A)}$$
$$\alpha^A > \alpha^B \implies I_B < I^A$$

- Suppose we can choose

$$\alpha^A \text{ and } \alpha^B$$

$$\text{Total Cost: } C(\alpha^A) + C(\alpha^B)$$

- We would always choose to equalize $(\alpha^A) = \alpha^B$ (at equal cost).

(III) Intuition for Cunha, Heckman, and Schennach (2010)

Equality $\Rightarrow$ Efficiency Problem

- Consider a social planner who seeks to maximize the aggregate human capital of society.
- There are two children: A and B.
- They differ in their initial endowments θ_A and θ_B respectively.
- Assume they are biologically determined, outside the control of the social planner.

- Adult human capital of children is h_A and h_B and are produced by investment (X), a scalar, where X_A is investment in A and X_B is investment in B:

$$h_A = g(X_A, \theta_A)$$
$$h_B = g(X_B, \theta_B)$$

where g is increasing in both arguments, $g_{11}(\cdot) < 0$, $g_{22}(\cdot) < 0$, and $g_{12}(\cdot) > 0$.

- Suppose $\theta_A > \theta_B$, if the social planner has a fixed budget for investment, $\bar{X}$, where

$$\bar{X} = X_A + X_B.$$

(i) What is the optimal policy for investment in children?

Is it equalizing or disequalizing in terms of initial conditions θ_A and θ_B?

Characterize the ratio of X_A and X_B.

For specificity start with the special case $h_A = \theta_A g(X_A)$; $h_B = \theta_B g(X_B)$.

(ii) Suppose next that instead of being biologically determined outside the control of the parent, θ is determined by early childhood investment I and early parental background P:

$$\theta = \eta(I, P),$$

where η is increasing in I and P, $\eta_{11}(\cdot) < 0$, $\eta_{22}(\cdot) < 0$ and $\eta_{12}(\cdot)$ may be negative, positive or zero.

Assume $P_A > P_B$.

What is the optimal policy for investment in the first stage of life of the children if the goal is to maximize aggregate human capital?

Assume a constraint $\overline{I} = I_A + I_B$.

Assume also that the policy in part (i) is in effect.

- In answering this question analyze the following examples:
  (a) a Cobb-Douglas case:
  $$\theta = I^{\gamma} P^{\delta}$$
  (b) a Leontief case:
  $$\theta = \min\{I, P\}$$
  (c) and a case with perfect substitutes
  $$\theta = \tau_1 I + \tau_2 P, \quad I > 0, \quad \tau_1, \tau_2 > 0,$$

  Then discuss the general case.

- For each case characterize the optimal policy for I_A and I_B, the ratio I_A/I_B, and the ratio X_A/X_B.
- Under what conditions is the policy (ii) joined with (i) equalizing? Disequalizing?
- At what stage(s) is policy equalizing? Disequalizing?
- Given the intuition for your results.
- In the Cobb-Douglas case for g and η, can the optimal policy be equalizing at any stage?

(iii) Discuss the empirical evidence on the effectiveness of early childhood investments in equalizing adult outcomes.

Intuition: If a social planner could design second stage production functions to optimize aggregate output, he would symmetrize the technology for both people.

First stage investment symmetrizes.