HCEO WORKING PAPER SERIES

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Working Paper

2020-072

10/2020

HUMAN CAPITAL AND
ECONOMIC OPPORTUNITY
GLOBAL WORKING GROUP

The University of Chicago
1126 E. 59th Street Box 107
Chicago IL 60637

www.hceconomics.org

On absolute socioeconomic health inequality comparisons*

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September 2020

Abstract

This paper introduces a new graphical tool: the mean deviation concentration curve. Using a unified approach, we derive the associated dominance conditions that identify robust rankings of absolute socioeconomic health inequality for all indices obeying Bleichrodt and van Doorslaer's (2006) *principle of income-related health transfer.* We also derive dominance conditions that are compatible with other transfer principles available in the literature. In order to make the identification of all robust orderings implementable using survey data, we discuss statistical inference for these dominance tests. To illustrate the empirical relevance of the proposed approach, we compare joint distributions of income and health-related behavior in the United States.

Keywords: mean deviation concentration curves, generalized health concentration curves, generalized health range curves, absolute socioeconomic health inequality, stochastic dominance, inference
JEL Codes: D63, I10

*This research was funded by the Social Sciences and Humanities Research Council of Canada. The authors wish to thank Owen O'Donnell his constructive comments.

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1 Introduction

Measuring socioeconomic health inequality is central for researchers who wish to monitor the evolution of the distribution of health outcomes and health-related behaviors between any two distributions. When doing so, the researcher can adopt an index-based approach, a dominance-based approach or a combination of both. An index-based approach relies on a specific index that aggregates the information in a single cardinal number and allows for a complete ordering of these inequalities. These complete rankings may depend heavily on the specific mathematical form of the index selected. A dominance-based approach to inequalities does not rely on a specific index but on a general class of indices with underlying well-established ethical principles.[1] The use of such a general approach may lead to partial rankings of distributions; nevertheless, these rankings are robust to the specific form of the index.

So far, most of the literature on dominance and health inequality focused on deriving measurement tools and dominance conditions for robust orderings of relative socioeconomic health inequalities and health achievement (see Makdissi and Yazbeck, 2014; Khaled, Makdissi and Yazbeck, 2018). Exceptions are noted for cases where the variable of interest is not ratio-scale as in the work of Allison and Foster (2004) for pure health inequality and the work of Makdissi and Yazbeck (2017) in the context of socioeconomic health inequality. Indeed these papers explore a particular type of dominance for absolute socioeconomic health inequality. Specifically, because of their focus on ordinal data, Allison and Foster (2004) and Makdissi and Yazbeck (2017) develop a dominance-based approach embedded within an index-based approach (embedded dominance approach). In their framework, the dominance-based approach accounts for the uncertainty around the appropriate numerical

[1]These ethical principles are based on a normative consensus established in the literature. See Wagstaff (2002), Bleichrodt and van Doorslaer (2006), Erreygers, Clarke and Van Ourti (2012), Makdissi and Yazbeck (2014), and Khaled, Makdissi and Yazbeck (2018). It is important to note that the well-known concentration index is a rank-dependent index.

scale that one should impose to transform the data and thus compute the index. Nevertheless, the authors do not develop dominance conditions that accounts for the uncertainty around the specific mathematical form of the index itself in the context of absolute inequality. Thus, the literature on the dominance-based approach to socioeconomic health inequalities is still silent as far as robust orderings of absolute socioeconomic health inequalities are concerned.

Motivated by the absence of a dominance-based approach to absolute socioeconomic health inequality, the well-documented selectivity in reporting issues (see among others, King et al. 2012; Kjellsson, Gerdtham, and Petrie, 2015) as well as the importance of reporting both relative and absolute measures of socioeconomic health inequalities,[2] this paper focuses on robust comparisons of absolute socioeconomic health inequalities. Its overarching objective is to fill the gap in the literature on robust rankings of health distributions in the context of absolute socioeconomic health inequalities. In doing so, this paper sheds light on an issue arising from deriving dominance conditions based on the generalized health concentration curves and shows that it is difficult to obtain a proper dominance test for absolute socioeconomic health inequality (for more details see Appendix A).[3] To address this issue, this paper proposes new graphical tools and dominance tests that allow for reporting robust rankings of absolute health inequalities.

This paper contributes to the literature on the measurement of health inequality in two distinct ways. First, it develops a new measurement tool, the mean deviation concentration curve (MDC) and the associated dominance conditions for robust orderings at the second

[2]See among others Wagstaff et al., 1991; Mackenbach and Kunst, 1997; Oliver et al., 2002; Regidor, 2004; Avendano Pabon, 2006; Harper and Lynch, 2007; Kelly et al., 2007; Mackenbach et al., 2008; Regidor et al., 2009; Harper et al., 2010, Erreygers and van Ourti, 2011; King et al. 2012; Kjellsson, Gerdtham, and Petrie, 2015.

[3]In Appendix A we investigate the properties of the canonical absolute socioeconomic health inequality indices (and their implications) when used to produce robust rankings of health distributions. It shows that using the generalized concentration curve yields sufficient dominance conditions (but not necessary ones), which speak to the importance of introducing new graphical tools.

order as well as higher orders. It also contributes to the ongoing debate on the ethical principles that indices should obey by offering a unified dominance approach for absolute measures of socioeconomic health inequality. Whether the *principle of income-related health transfers* alone is appropriate or whether it should be complemented with other ethical principles is still open to debate. In that regard, this paper remains agnostic; nevertheless, it provides evidence that imposing additional ethical principles can lead to robust orderings when absolute socioeconomic health inequality rankings are not possible under the *principle of income-related health transfers* alone.

The remaining of this paper is organized as follows. In Section 2, we establish the measurement framework in which we operate as well as the ethical principles governing these measures' properties. In Section 3, we provide conditions under which robust orderings of joint distributions of health and income can be identified. In Section 4, we discuss the estimation and inference corresponding to the methods developed in Section 3. In Section 5, we provide an empirical illustration using the information on cigarette consumption and overweightedness from the National Health Interview Survey (NHIS) in 1997 and 2014. Finally, in Section 6, we conclude.

2 Measurement framework

The purpose of this section is to present the measurement framework for absolute socioeconomic health inequality indices and provide the necessary measurement background. We will first present the absolute socioeconomic health inequality indices, and explain their relation with achievement indices. We then describe the underlying ethical properties.

2.1 Absolute socioeconomic health inequality indices and health achievement indices

Absolute socioeconomic health inequality indices are functionals of the joint distribution of health, H and income, Y. Let H and Y be 2 random variables that are absolutely continuous with support on the positive half real line and with densities f_H and f_Y respectively.[4] Let $f_{Y,H}$ be the joint density of the 2 random variables and $F_Y(y)$ be the cumulative distribution of income. Let $h(p)$ be the conditional expectation of health, H, with respect to p^{th}-quantile of Y. Formally,

$$h(p) = E[H|Y = F_Y^{-1}(p)] \tag{1}$$

We measure absolute socioeconomic health inequality in a rank dependent framework where ranks are individuals's position in the distribution of socioeconomic statuses. Formally, these indices can be written in the following general form:

$$I_A(h) = \int_0^1 \nu(p)h(p)\mathrm{d}p, \tag{2}$$

where $\nu(p)$ is a social weight function.[5]

If one adopts an Atkinson-Kolm-Sen (AKS) approach,[6] then it is possible to interpret absolute socioeconomic health inequality as the absolute loss in average health that is due to an unequal distribution of health along socioeconomic ranks. Formally,

$$I_A(h) = \mu_h - A(h), \tag{3}$$

where $A(h)$ is health achievement.

Thus, any index of absolute socioeconomic health inequality can be viewed as the cost in health achievement associated with socioeconomic health inequalities. This health achievement can be measured using a health achievement index which accounts simultaneously

[4]In this paper, we assume that this health measure is a ratio-scale variable.

[5]The assumptions made on this social weight function are discussed in details in subsections 2.2, 2.4, 3.2, and 3.3.

[6]See Atkinson (1970), Kolm (1976a and 1976b), and Sen (1973).

for changes in the average health outcome and changes in socioeconomic health inequality. More formally, Wagstaff (2002) shows that it is possible to define health achievement indices $A(h)$ as follows:[7]

$$A(h) = \int_0^1 \omega(p)h(p)\mathrm{d}p, \tag{4}$$

where $\omega(p) \in \Re_+$ is the social weight function. A health achievement index is a weighted average of population's health where social weights are a function of socioeconomic ranks. Using equations (3) and (4), we can rewrite $\omega(p)$ as a function of the social weight function $\nu(p)$ in the following form: $\omega(p) = 1 - \nu(p)$. The mathematical properties of these social weight functions are associated with the indices' ethical principles. In the following subsections, we elaborate on two different ethical principles: the *principle of income-related health transfer* and the *principle of symmetry around the median*. We also discuss the implication of linking inequality indices to health achievement indices.

2.2 Principle of income-related health transfer

The *principle of income-related health transfers* can be expressed in a mathematical form through assumptions on the social weight functions $\nu(p)$ and $\omega(p)$. More specifically, the social weight functions in equations (2) and (4) should satisfy the following assumptions respectively:

A.1 $\nu^{(1)}(p) > 0$,

A.2 $\int_0^1 \nu(p)\mathrm{d}p = 0$,

A'.1 $\omega^{(1)}(p) < 0$,

A'.2 $\int_0^1 \omega(p)\mathrm{d}p = 1$,

[7] While this paper's focus in on absolute inequality indices, the relation between achievement indices and inequality indices will play a key role in this paper if we want to consider higher-order ethical principles.

where $\nu^{(i)}(p) = \frac{\partial^i \nu(p)}{\partial p^i}$ and $\omega^{(i)}(p) = \frac{\partial^i \omega(p)}{\partial p^i}$. The roles of assumption A.1 and A'.1 are embedded in Bleichrodt and van Doorslaer's (2006) *principle of income-related health transfer* where the contribution of an individual health status to socioeconomic health inequality is non-decreasing with socioeconomic status. Keeping everything else constant, this means that if the rich are relatively healthier, then socioeconomic health inequality (health achievement) will be higher (lower). In addition, this principle implies that performing a mean preserving health transfer δ_h from an individual at a higher socioeconomic rank to an individual at a lower socioeconomic rank decreases (increases) socioeconomic health inequality (health achievement). Assumption A.2 guarantees that the weight function $\nu(p)$ sums to zero (i.e., $\int_0^1 \nu(p)\mathrm{d}p = 0$) and assumption A'.2 guarantees that the weight function $\omega(p)$ sums to one.

Assumptions A.1 and A.2 guarantee that inequality measures have the general properties of absolute inequality indices: (i) the absolute inequality indices should be equal to zero ($I_A(h) = 0$) if everyone has the same health status, (ii) the absolute inequality indices should remain unchanged, if everyone's health increases by the same amount. Assumptions A'.1 and A'.2 guarantees that health achievement indices are equal to $\overline{h}$ if everyone's health is at the same level $\overline{h}$.

Having elaborated on the properties of the different relevant indices under the *principle of income-related health transfers*, we will use these properties to define the sets to which they belong. When the social weight function satisfies assumptions A.1 and A.2, I_A (.) is considered to be rank-dependant measures of socioeconomic health inequality. Similarly, $A(.)$ is considered to be a rank-dependant measure of health achievement when the social weight function satisfies assumptions A'.1 and A'2. For the rest of this paper we denote by Λ_A^2 the set of all rank dependent absolute socioeconomic health inequality indices obeying assumptions A.1 and A.2.[8]

2.3 Health achievement-based socioeconomic health inequality indices

If one adopts a health achievement-based approach (i.e., an AKS approach), then the non-negative constraint on the social weight function $\omega(p)$ limits the domain of $\nu(p)$ as follows:

A.3 $\nu(p) \in (-\infty, 1]$

This additional assumption excludes some possible extreme values for the social weight functions under the *principle of income-related health transfer*. More specifically, if one does not impose this restriction on $\nu(p)$, the social weight function for the health achievement index, $\omega(p)$, may take negative values at high socioeconomic ranks. Intuitively, this means that the analyst would be willing to accept a lower average health status provided that those who are in high socioeconomic ranks are in very poor health. So, the restriction on the social weight function ($\omega(p) \in \Re_+$) means that the health achievement index implicitly obeys an additional ethical principle: *monotonicity*. This ethical principle stipulates that, everything else held constant, an increase of the health status at one social position has a non-negative impact on health achievement. While *monotonicity* is not an ethical principle that is associated with socioeconomic health inequality measurement, this principle (as we will see in section 3.2) will play an instrumental role in the identification of higher-order dominance results under *pro-poor transfer sensitivity*.

For the rest of this paper, we denote by $\Lambda^2_{AW} \subset \Lambda^2_A$ the set of health achievement-based rank dependent absolute socioeconomic inequality indices obeying A.1, A.2 as well as the restriction implied by $\omega(p) \in \Re_+$, namely A.3.

[8] A more formal definition of this set and all subsequent sets of indices are given Appendix B.

2.4 Principle of symmetry around the median

In addition to assumptions A.1 and A.2 discussed in the previous section, Erreygers, Clarke and Van Ourti (2012) suggest that there is another ethical property that may be imposed on measures socioeconomic health inequality. More specifically, the authors suggest that researchers may be interested in focusing on indices that pass the upside down test. This test consists of interchanging the health levels of individuals at a certain rank p with their carnival counterpart and then verifying whether inequality indices reflect this change. This means that the health levels of individuals will be swapped such that a person at the 10th percentile of the income distribution will take the health level a person in the 90th percentile, and a person at the 20th percentiles will take the health level of a person at the 80th percentile, and so on. Thus, for a given health distribution $g(p) = h(1-p)$ that is the carnival counterpart for $h(p)$, the upside down test consists in verifying whether an index of inequality $I_A(g)$ is always positive (negative) when $I_A(h)$ is negative (positive). In their paper, Erreygers, Clarke and Van Ourti (2012) show that an index of socioeconomic health inequality passes this test only if its weight function $\nu(p)$ is symmetric around the median of socioeconomic ranks (p = 0.5).[9] This means that in addition to the A.1 and A.2, if one wishes to focus on inequality indices that pass the upside down test, then the following assumption should hold:

A.4 $\nu(1-p) = -\nu(p)$.

It should be noted that, assumption A.4 also implies that $\nu(0.5) = 0$. This additional assumption, when imposed, allows us to define a subset of indices: the set of indices that pass the upside-down test, $\Lambda^2_{A\rho} \subset \Lambda^2_A$.

[9] It should be noted that we can also define a subset of health achievement indices that pass the upside down test and make these symmetric indices compatible with an AKS approach. However, there is no need to restrict achievement indices to obtain robust rankings.

3 Identifying robust orderings of health distributions

The information on absolute socioeconomic health inequalities can be captured via indices or via graphical tools. These graphical tools are also used to derive dominance conditions that identify robust rankings of socioeconomic health inequalities (for more details, see Makdissi and Yazbeck, 2014; and Khaled, Makdissi and Yazbeck, 2018). Intuitively, a dominance condition is a non-intersection condition imposed on the graphical tool that represents a class of indices.

Since the non intersection of health concentration curves identify robust rankings of relative socioeconomic health inequality, one may be tempted to extend this strategy to generalized health concentration curves and absolute socioeconomic health inequality. Unfortunately, this is not feasible because the generalized health concentration curves do not allow for dominance conditions for absolute socioeconomic health inequality (for a formal proof see appendix A). As noted in Makdissi and Yazbeck (2014) and Khaled, Makdissi and Yazbeck (2018), the non intersection of two generalized health concentration curves is a dominance condition for health achievement only (i.e., r.h.s. of equation (3)) and not absolute socioeconomic health inequality (i.e., the second term of the l.h.s of equation (3)).[10]

To address this issue, we introduce a new graphical tool: the mean deviation concentration curve (MDC), and derive the associated dominance conditions required to identify robust rankings of absolute socioeconomic health inequality. We also introduce higher-orders mean deviation concentration curves and their associated dominance conditions that are compatible with *pro-poor transfer sensitivity.* In addition to proposing new graphical tools, we exploit generalized health range curves introduced in Khaled, Makdissi and Yazbeck (2018) in an absolute socioeconomic health inequality framework and develop dominance

[10]This is also true for the Generalized Lorenz curves that are used to identify robust rankings of social welfare and not robust rankings of absolute income inequality (Shorrocks, 1983).

conditions for indices that are compatible with the *symmetry around the median principle* and *pro-extreme ranks transfer sensitivity*.

3.1 Principle of income-related health transfer: Mean Deviation Concentration Curve

In order to identify robust rankings of absolute socioeconomic health inequality, we extend Moyes's (1987) absolute Lorenz curve to a bivariate setting and introduce a new curve; the mean deviation concentration curve.[11] We thus define the following transformation on $h(p)$: $\widetilde{h}(p) = \mu_h - h(p)$ and the resulting mean deviation health concentration as follows:

$$MDC(p) = \int_0^p \widetilde{h}(u)\mathrm{d}u. \tag{5}$$

A mean deviation concentration curve displays the cumulative shortfall in health for the poorest $100p\%$ of the population. It represents p times the cumulative difference between the average health status and the average health status of the poorest $100p\%$. As one can clearly see in Figure 1, the mean deviation concentration curve takes a value of zero at rank $p = 0$ and rank $p = 1$ (that is $MDC(0) = MDC(1) = 0$) by construction. When a population health is (perfectly) equally distributed, the mean deviation concentration curve is a horizontal line and is equal to the horizontal axis. The left panel of Figure 1 displays a mean deviation concentration curve for a case where the health outcome, $h(p)$ is more concentrated among the rich with a positive income-health gradient everywhere. As one moves to higher socioeconomic ranks, deviations from the average health are positive (but decreasing), they reach a point where they are equal to zero, then negative (but increasing). As a result, the MDC curve increases at a decreasing rate, reaches a plateau then decreases at an increasing rate. More specifically, individuals whose ranks correspond with the increasing part of the MDC curve, have health levels below the average health

[11] The existing graphical tools in the health inequality literature do not allow for the identification of such orderings. The interested reader can refer to Appendix A for an explanation.

status, and those whose ranks correspond to the decreasing part of the MDC curve have a health level higher than the average health status. The right panel of Figure 1 represents the opposite case in which the health outcome is concentrated among the poor with a negative income-health gradient everywhere. Also, the mean deviation concentration curve may be in part above and in part below the horizontal axis if the income-health gradient is non monotonic.

In addition to their graphical interpretation, mean deviation concentration curves can be used to identify robust orderings of absolute socioeconomic health inequality. In that respect, the role of the MDC curves is akin to the role of the health concentration curve thus, the associated dominance condition will require a non-intersection condition on the mean deviation concentration curves (instead of the health concentration curves).[12]

Theorem 1 *Let $f^1_{Y,H}$ and $f^2_{Y,H}$ represent two joint densities of income and health. $I_A(h_1(p)) \leq I_A(h_2(p))$ for all $I_A(h(p)) \in \Lambda^2_A$ if and only if*

$$MDC_1(p) \leq MDC_2(p) \text{ for all } p \in [0,1].$$

Theorem 1 presents the dominance conditions for absolute socioeconomic health inequality under the assumption of the *principle of income-related health transfer*. If no robust ranking is obtained with Theorem 1, the analyst may increase aversion to socioeconomic health inequality by imposing higher-order principles. Intuitively, a higher-order principle (in the context of a dominance-based approach) can be considered as equivalent to a higher degree of aversion to socioeconomic health inequality (in the context of an index-based approach). This means that for a given transfer, the magnitude of the social gain associated with a transfer depends on the location of this transfer in the distribution of the socioeconomic status. The appeal in imposing higher-order of aversion to socioeconomic health inequality resides in its capacity to increase the power of ordering by narrowing to a subset of indices

[12]For information regarding the detailed proofs for all dominance Theorems please refer to Appendix C.

instead of relying on the arbitrary choice of a specific index. As pointed in Khaled, Makdissi and Yazbeck (2018), there are two distinct views regarding higher-order principles of aversion to socioeconomic health inequality: *pro-poor health transfer sensitivity principles* and *pro-extreme ranks health transfer sensitivity principles.* In this paper, we adopt a unified approach to dominance without taking any stand on which view is desirable. Thus, we provide higher-order principles that are compatible with both ethical views and leave it up to the analyst to choose which view they would like to adopt.

3.2 Pro-poor transfer sensitivity principles: Health achievement-based inequality indices

If one adopts health achievement-based rank-dependent absolute socioeconomic inequality indices, it is possible to develop higher-order tests for cases where the dominance condition in Theorem 1 does not identify any robust ordering. The higher-order transfer principles associated with a higher-order mean deviation concentration curve are *pro-poor health transfer sensitivity* principles.[13] These transfer principles assume that health transfers become more desirable if they are occurring in the lower part of the distribution of socioeconomic ranks. In the context of this paper, this means that an absolute socioeconomic health inequality index satisfying assumptions A.1, A.2, and A.3, will obey the s-th order *pro-poor health transfer sensitivity* if $(-1)^{i+1}\nu^{(i)}(p) \geq 0$ for all $i = 1$ to $s-1$. It is important to note that we are imposing *pro-poor health transfer sensitivity* and also restricting the indices to be health achievement-based by adding assumption A.3.[14] Let $\Lambda^s_{AW} \subset \Lambda^{s-1}_{AW} \subset \cdots \subset \Lambda^2_{AW} \subset \Lambda^2_A$ denote the set of all achievement-based absolute health inequality indices obeying the s-th order *pro-poor health transfer sensitivity.* One can identify robust rankings of absolute so-

[13]These higher-order conditions are discussed in detail in Makdissi and Yazbeck (2014).

[14]Note that one does not need to impose A.3 at order 2 because $MDC^2(1) = 0$ by definition. Having $\nu^{(s-2)}(1)MDC^s(1) = 0$ or $\omega^{(s-2)}(1)MDC^s(1) = 0$ plays an important role in the proofs in the appendix. Imposing A.3 is not a very strong assumption. It only eliminates social weights function that would assign negative values on some socioeconomic ranks. However, because the technical fact that $MDC(1) = 0$, we don't need this assumption at order 2 to impose as little assumptions as possible.

cioeconomic health inequality with higher-order mean deviation concentration curves which are defined over the $[0,1]$ interval as

$$MDC^s(p) = \int_0^p MDC^{s-1}(u)\mathrm{d}u, \tag{6}$$

where $MDC^2(p) = MDC(p)$.

Theorem 2 *Let $f^1_{Y,H}$ and $f^2_{Y,H}$ represent two joint densities of income and health. $I_A(h_1(p)) \leq I_A(h_2(p))$ for all $I_A(h(p)) \in \Lambda^s_{AW}$, $s \in \{3, 4, \dots\}$, if and only if*

$$MDC^s_1(p) \leq MDC^s_2(p) \text{ for all } p \in [0,1].$$

Intuitively these higher-order tests can be interpreted as an increase aversion to inequality. Each time one increases the order of ethical principles under consideration, this translates into integrating equation (6). Mathematically, this means that equation (6) cumulates the values of the curve of the previous order, adding *de facto* more weight on lower socioeconomic status observations.

3.3 Symmetry around the median and pro-extreme ranks health transfer sensitivity principles

If no robust ranking can be obtained with Theorem 1 nor by imposing *pro-poor transfer sensitivity* (i.e., Theorem 2) or if the researcher is not willing to increase the level of aversion to socioeconomic inequality by imposing *pro-poor transfer sensitivity*, then an alternative path can be followed by imposing the *symmetry around the median principle.*[15] Erreygers, Clarke, and Van Ourti (2012) suggest that in addition to the *principle of income-related health transfer*, it is desirable that a measure of socioeconomic health inequality passes the *upside-down* test. This ethical principle was coined later on as the *symmetry around the median principle.* In their paper, they also discuss a higher-order of aversion to socioeconomic health inequality that is compatible with the *symmetry around the median*

[15] We denote by $\Lambda^2_{A\rho} \subset \Lambda^2_A$ the set of all indices obeying this ethical principle.

principle. These higher-order principles are formalized in Khaled, Makdissi and Yazbeck (2018) and coined as *pro-extreme ranks health transfer sensitivity* approach. These transfer sensitivity principles are compatible with valuing transfers occurring away from the median of socioeconomic ranks more than transfers occurring close to the median. In the context of this paper, this means that an absolute socioeconomic health inequality index satisfying assumptions A.1 A.2, and A.4 will obey the s-th order *pro-extreme ranks health transfer sensitivity* if $(-1)^{i+1}\nu^{(i)}(p) \geq 0$ for all $i = 1, \ldots, s-1$. We thus define the set of indices obeying all *pro-extreme ranks health transfer sensitivity principles* of order $i = 3, \ldots, s$ as $\Lambda^s_{A\rho} \subset \Lambda^{s-1}_{A\rho} \subset \cdots \subset \Lambda^2_{A\rho} \subset \Lambda^2_A$.

It is possible to identify robust rankings of absolute socioeconomic health inequality for indices obeying the *principle of income-related health transfer* and *symmetry around the median* by exploiting Khaled, Makdissi, and Yazbeck's (2018) generalized health range curves. The generalized health range curve, $GR(p)$ represents the cumulative health range at rank p. Let us define the range of health statuses at rank p as $r(p) = h(1-p) - h(p)$, the generalized health range curve $GR(p)$ associated with distribution $f_{Y,H}$ is formally defined over the interval $[0, 0.5]$ as:

$$GR(p) = \int_0^p r(u)\mathrm{d}u. \tag{7}$$

Higher-order generalized health range curves which are defined over the $[0, 0.5]$ interval can then be defined, for $s \in \{3, 4, \ldots\}$ as

$$GR^s(p) = \int_0^p GR^{s-1}(u)\mathrm{d}u, \tag{8}$$

and $GR^2(p) = GR(p)$ for $s = 2$. These generalized health range curves identify robust orderings of absolute socioeconomic health inequality for indices obeying symmetry around the median and *pro-extreme ranks transfer sensitivity* using a dominance based approach. The identification of robust orderings will require a non-intersection condition on the generalized health range curves.

Theorem 3 *Let $f^1_{Y,H}$ and $f^2_{Y,H}$ represent two joint densities of income and health. $I_A(h_1) \leq I_A(h_2)$ for all $I_A(h) \in \Lambda^s_{A\rho}$, $s \in \{2, 3, \dots\}$, if and only if*

$$GR^s_1(p) \leq GR^s_2(p) \text{ for all } p \in [0, 0.5].$$

As in the previous case, the intuition of these higher-order tests can be understood from equation (8). This equation cumulates the values of the curve of the previous order, adding *de facto* more weight on ranges associated with socioeconomic status further away from the median socioeconomic status.

4 Estimation and Inference

In previous sections, we introduced new inequality measurement tools and dominance conditions for robust ordering of absolute health inequality allowing for robust rankings of distributions. However, we did not discuss uncertainty. In this section, we will provide details regarding the estimation and inference approach followed in this paper. It is important to note that while some of the dominance tools are new, the inference methodology is based on theory already available in the literature.

To estimate absolute health concentration curves, we use the following non-parametric estimators.[16] We thus derive the non-parametric estimators $\widehat{MDC}^s(p)$ of $MDC^s(p)$.

$$\widehat{MDC}^s(p) = \frac{1}{N}\sum_{i=1}^{N}\left[\overline{h} - h_i\right]\frac{\left[p - \widehat{F}_Y^{-1}(p)\right]^{s-2}}{(s-2)!}\mathbb{1}(y_i \leqslant \widehat{F}_Y^{-1}(p)). \tag{9}$$

To estimate generalized health range curves, we use the non parametric estimators in Khaled, Makdissi, and Yazbeck (2018) $\widehat{GR}^s(p)$ of $GR^s(p)$.

$$\begin{aligned}\widehat{GR}^s(p) \quad = \quad & \frac{1}{N}\sum_{i=1}^{N} h_i \frac{(\widehat{F}_Y(y_i) - 1 + p)^{s-2}}{(s-2)!}[\mathbb{1}(y_i > \widehat{F}_Y^{-1}(1-p))] \\ & -\frac{1}{N}\sum_{i=1}^{N} h_j \frac{(p - \widehat{F}_Y(y_i))^{s-2}}{(s-2)!}[\mathbb{1}(y_i \leqslant \widehat{F}_Y^{-1}(p))] \end{aligned} \tag{10}$$

[16]The details are in Appendix D.

These estimators are used to produce the figures presented in the empirical section as well as the 95% confidence band around them using a bootstrap procedure.

Depending on the empirical evidence at hand, the researcher may be interested in testing three different null hypotheses. The appropriate test will be determined upon a visual check of the MDCs. For example, if there is an obvious case of intersection, then the third set of hypotheses is appropriate. The first hypothesis test is derived directly from Theorems 1, 2, and 3 and consists of testing a a null of dominance. Assume that we have two samples S_1 and S_2 of sizes n_1 and n_2 drawn from $f^1_{Y,N}$ and $f^2_{Y,H}$. Let $L_i(p)$ be $MDC^s_i(p)$ or $GR^s_i(p)$, for $i = 1$ and 2. We are interested in testing if $I_A(h_1) \leq I_A(h_2)$ for all $I_A(h)$ in a set of indices (Λ^2_A, Λ^s_{AW}, $s \in \{3, 4, \dots\}$, or $\Lambda^s_{A\rho}$, $s \in \{2, 3, \dots\}$). The inference test based on the dominance result of Theorems 1, 2 and 3 consist of testing:

$$\begin{aligned} H_0 &: \ L_2(p) - L_1(p) \geq 0, \forall p \in [0, 1] \\ H_1 &: \ L_2(p) - L_1(p) < 0, \text{ for some } p \end{aligned}$$

The inspection of the above test indicates that to identify robust orders, we test for dominance, i.e., when we reject H_0, we have evidence against that null of dominance of distribution of $f^1_{Y,N}$ over $f^2_{Y,H}$. While one may think that testing the null of non-dominance and establishing a case for dominance would be more intuitive, such a test requires strong evidence against the null. This strong evidence is impossible to obtain over the entire $[0, 1]$ or $[0, 0.5]$ intervals (Davidson and Duclos, 2013). Consequently, we follow the suggestion of Schechtman, Shelef, Yitzhaki, and Zitikis (2008), and perform the above test in both directions, i.e. for $H_0 : \ L_2(p) - L_1(p) \geq 0, \ \forall p \in [0, 1]$ and for $H'_0 : \ L_1(p) - L_2(p) \geq 0, \ \forall p \in [0, 1]$. We interpret that L_1 dominates L_2 if there is no strong evidence against H_0 and there is strong evidence against H'_0.

Let $\tau = \sup_p(L_1(p) - L_2(p))$. It is straightforward to construct a KS type of test statistic

$\widehat{\tau}$ that is a non-parametric estimator of τ as follows:

$$\widehat{\tau} = \sqrt{\frac{n_1 n_2}{n_1 + n_2}} \sup_p \left(\widehat{L}_1(p) - \widehat{L}_2(p) \right) \tag{11}$$

The asymptotic distributions of $\widehat{\tau}$ will be that of functionals of a two-dimensional Gaussian processes. To perform this test, we follow a bootstrap procedure as in Schechtman, Shelef, Yitzhaki, and Zitikis (2008).[17]

There may be two reasons for the rejection of the second order dominance (or any higher order): (i) the two curves are equal or (ii) the two curves intersects. In the first case, imposing higher order ethical principles does not improve the dominance results. In the second case, there may be a higher order that allows for a robust ranking of the two distributions. Let us first consider the case of equality between the two curves. In this case, the test is:

$$\begin{aligned} H_0 &: |L_2(p) - L_1(p)| = 0, \forall p \in [0,1] \\ H_1 &: |L_2(p) - L_1(p)| \neq 0, \text{ for some } p \end{aligned}$$

For this test, we use another statistics that is more akin to a KS test, $\tau' = \sup_p |L_1(p) - L_2(p)|$ with a non-parametric estimator being:

$$\widehat{\tau}' = \sqrt{\frac{n_1 n_2}{n_1 + n_2}} \sup_p \left| \widehat{L}_1(p) - \widehat{L}_2(p) \right| \tag{12}$$

The asymptotic distributions of $\widehat{\tau}'$ will also be that of functionals of a two-dimensional Gaussian processes. The same bootstrapped procedure can be used to perform the test.

Let us now turn our attention to testing for intersecting curves. This allows the analyst to check if it is worth looking for rankings using higher order dominance tests. These higher order rankings are only possible if the two curves are visibly distinct and have a clear point of intersection. In this case, we can restrict our attention on two specific portions of the

[17] Details can be found Appendix E.

$[0,1]$ interval $\mathscr{P}_i \subset [0,1]$ for $i = 1$ and 2 such that $\mathscr{P}_1 \cup \mathscr{P}_2 = [0,1]$ and $\mathscr{P}_1 \cap \mathscr{P}_2 = \emptyset$. Restricting the analysis to subintervals of [0,1] does not complicate the analysis. We simply need to test

$$H_{00} \;:\; L_2(p) - L_1(p) \geq 0, \forall p \in \mathscr{P}_1$$
$$H_{10} \;:\; L_2(p) - L_1(p) < 0, \text{ for some } p \in \mathscr{P}_1$$

and

$$H_{01} \;:\; L_1(p) - L_2(p) \geq 0, \forall p \in \mathscr{P}_2$$
$$H_{11} \;:\; L_1(p) - L_2(p) < 0, \text{ for some } p \in \mathscr{P}_2$$

As for the dominance test case, we follow the recommendation of Schechtman, Shelef, Yitzhaki, and Zitikis (2008) and perform the test in both directions over $\mathscr{P}_1$ and over $\mathscr{P}_2$ using the statistics defined in equation (11).

5 Empirical illustration

To show the empirical applicability of the approaches proposed in this paper, we conduct an empirical illustration using National Health Interview Survey data from the years 1997 and 2014. In this illustration, we show that when there is no robust ranking for all indices obeying the *principle of income-related health transfers*, the researcher can explore two paths. She can focus on health achievement-based indices and impose more structure with the *pro-poor transfer sensitivity principle*, or focus on indices that obey *symmetry around the median principle* and eventually add *pro-extreme rank transfer sensitivity principle*. To guide the analyst through the different options, we provide a hierarchical dominance options tree that summarizes how these ethical principles are added when moving from one theorem to the other (see Figure 2). In addition to the dominance tree, we provide a simple decision guideline (in Table 1) that allows for a rapid interpretation of the empirical results (in the light of the theorems proposed). This guideline synthesizes the dominance results derived in this paper (i.e., Theorems 1, 2, and 3) and is helpful and applicable when interpreting empirical results from this paper or any other empirical application based on the theoretical results proposed in this paper.

5.1 Data

In our illustration, we focus on comparisons of absolute socioeconomic health inequalities using two ill-health variables that have been of great interest in the health economics literature: cigarette consumption (i.e., the number of cigarettes/day) and overweightedness. We follow Bilger, Kruger, and Finkelstein (2016) and use max[0,BMI-25] as a measure of overweightedness. Given that the empirical application is mainly for illustration purposes, we will avoid drawing policy recommendations but will provide some guidance to possible interesting avenues to explore. In order to compare our results with Khaled, Makdissi and Yazbeck's (2018) results on relative socioeconomic health inequality, we use the same NHIS years and variables for the empirical illustration.

The NHIS monitors health outcomes of Americans since 1957. It is a cross-sectional household interview survey representative of American households and non-institutionalized individuals collected via personal household interviews. For comparison purposes, we focus on the adult population in the 1997 and 2014 public-use data for which we have information on income. As a result, sample sizes are 34,776 for overweightedness and 35,667 for cigarette consumption in 1997. For 2014, the sample sizes are 35,197 for overweightedness and 36,363 for cigarette consumption. We use the sample adult file to extract information on health-related behavior and use family income adjusted for family size to infer the socioeconomic rank of individuals.[18] In the set of inequality comparisons presented in this empirical illustration, we focus on temporal comparisons at the national level and complement it

[18] We compute equivalent income by dividing family income by the square root of household size.

with regional comparisons for 2014.

5.2 Comparison of health outcomes and health related behaviors over time

We first start by national comparisons of inequalities over time (i.e., between 1997 and 2014) presented in Table 2 to 5. These comparisons are then complemented with comparisons at the regional level presented in Table 6.

Upon a quick inspection of the mean deviation concentration curves for cigarette consumption (in Figure 3) and their associated p-values (in Table 2), we notice that there is more absolute socioeconomic inequality in cigarette consumption in 2014 than in 1997. The mean deviation concentration curve for cigarette consumption in 1997 is above the corresponding one in 2014, and this is true almost everywhere except for a few intersections on some small intervals. The results of the statistical tests indicate strong evidence against the null $H_0 : MDC^2_{1997} \leq MDC^2_{2014}$ (at 5% significance level) and no strong evidence against the null $H_0 : MDC^2_{1997} \geq MDC^2_{2014}$. This result indicates that the small intersections were not statistically significant and we can assert that $MDC^2_{1997} \geq MDC^2_{2014}$. It is important to keep in mind that the interpretation of the dominance results based on the method developed in section 3 is for outcomes that are considered as healthy behaviour/outcomes. Given that we are dealing with a measure of unhealthy behavior, lower mean deviation concentration curves in 2014 should be interpreted as a higher concentration of this unhealthy behavior among low socioeconomic groups (for details see Table 2).[19] We thus interpret this dominance result as an indication of the presence of more absolute socioeconomic inequality in cigarette consumption in 2014 than in 1997. While this paper focuses on absolute health inequalities, it is important to compare these results to the relative inequality comparisons presented in Khaled, Makdissi, and Yazbeck (2018) because it allows us to appreciate the

[19]It should be highlighted that if we were dealing with healthy behaviors a concentration of this behavior among the lower socioeconomic groups would mean a lower socioeconomic inequality for that behavior. Indeed the interpretation of the direction of inequality depends on the nature of the health related behavior.

importance of using both the absolute dominance approach in addition to the relative approach. Evidence from Figure 3 and the associated p-values in Table 2 indicate that in the case of cigarette consumption, both relative and absolute inequality measures are moving in the same direction, suggesting more socioeconomic inequalities both at the relative and absolute levels in 2014. It is important to note that such concordance between relative and absolute dominance may not always occur.

Another health variable that is interesting to consider in the analysis of socioeconomic health inequality is overweightedness (defined as max[0, BMI - 25]). In this paper, overweightedness is defined as any positive deviation (in BMI units) from the maximal threshold of the healthy weight. Since deviations from the healthy weight threshold are deemed to have a detrimental impact on health, then for a given threshold, Bilger et al.'s (2017) transformation cardinalizes the (ill) health variable such that a value of zero reflects no overweightedness. Looking a Figure 5, we notice that the mean deviation concentration curve in 1997 intersects with the mean deviation concentration curve in 2014. When testing over the $[0, 1]$ interval, we find strong evidence against both the null $H_0 : MDC^2_{1997} \leq MDC^2_{2014}$ and the null $H_0 : MDC^2_{1997} \geq MDC^2_{2014}$, This indicates that the intersections displayed in Figure 5 are statistically significant. The results of the statistical tests presented in Table 3 indicates that for the $[0, 0.1]$ interval, there is no strong evidence against the null $H_0 : MDC^2_{1997} \leq MDC^2_{2014}$ and strong evidence against the null $H_0 : MDC^2_{1997} \geq MDC^2_{2014}$ (at 1% significance level). As for the $(0.1, 1]$ interval, the results of the statistical tests indicate strong evidence against the null $H_0 : MDC^2_{1997} \leq MDC^2_{2014}$ (at 1% significance level) and no strong evidence against the null $H_0 : MDC^2_{1997} \geq MDC^2_{2014}$. The results from the statistical tests indicate that the rankings of absolute socioeconomic inequality in overweightedness are not robust for all absolute inequality indices that obey the *principle of income-related health transfer*,

$I \in \Lambda_A^2$. Consequently, if one considers all indices that obey the *principle of income-related health transfer*, it is impossible to assess whether the inequalities in overweightedness have increased or decreased over time in a robust way.

It is possible to obtain a robust ranking by imposing more assumptions on the set of indices. As mentioned earlier, when no dominance result can be obtained, the analyst can consider the subset of indices those that are achievement-based and obey a higher-order ethical principles such as the *pro-poor transfer sensitivity* principle (i.e., considering, MDC^3). We follow this path. The information provided from the left panel of Figure 6, combined with p-values in Table 4, shows that imposing A.3 and *pro-poor transfer sensitivity* does not help in producing a robust ranking. More specifically, the third-order mean deviation concentration curves for overweightedness in 1997 and 2014, displayed in the left panel of Figure 6, seem to be very close. This is confirmed by the equality test for the two curves that shows no strong evidence against the null of equality (see Table 4). Given that the second-order dominance and the third-order dominance tests do not yield a robust ordering, the analyst will need to consider imposing an additional assumption by focusing on indices that pass the *upside-down test*.

If the researcher is willing to operate under the assumption of *symmetry around the median principle*, then the dominance conditions to be tested are the ones derived in Theorem 3. Thus instead of comparing mean deviation concentration curves, the researcher will have to compare generalized health range curves such as the ones in the right panel of Figure 6. The evidence from the graphical test and the statistical test in Table 5 suggest that there is more absolute socioeconomic health inequality in overweightedness in 2014 than in 1997. More specifically, there is strong evidence against the null hypothesis $H_0 : GR_{1997}^2 \leq GR_{2014}^2$ (at 1% significance level), and we cannot reject $H_0 : GR_{2014}^2 \leq GR_{1997}^2 \; \forall p \in [0,1]$. This means that $I_{2014} \leq I_{1997} \; \forall I \in \Lambda_{A\rho}^2$. As mentioned earlier, since we are comparing ill-health variables, a lower inequality index in 2014 means that the overweightedness is more concentrated among low socioeconomic ranks in 2014. This is thus interpreted as a higher absolute socioeconomic inequality in 2014 than in 1997. As this ordering is obtained at the 1% significance level, we do not explore higher-orders of dominance.

At this point, it may be informative to see whether these results are comparable to previous results obtained for relative measures of socioeconomic inequality. Comparing these results to the findings in Khaled, Makdissi, and Yazbeck (2018), we notice that conclusions are reversed. Thus, a policymaker who looks at a relative index of socioeconomic health inequality will be lead to believe that socioeconomic overweightedness inequality has decreased between 1997 and 2014 whereas the conclusions derived based on absolute indices of socioeconomic health inequality leads to an opposite conclusion.

5.3 Regional comparison of health related behaviors

Turning our attention to regional comparisons, we focus on the year 2014 and compare four regions: the Northeast, the West, the Midwest, and the South.[20] We first consider cigarette consumption. Results suggest that, under the *principle of income-related health transfer*, there are no clear patterns of dominance and thus no complete order. More specifically, results shown in Table 6 indicate that the West has lower socioeconomic inequality than the Midwest and the South at a 1% significance level and lower socioeconomic inequality than the Northeast indices at a 5% significance level for all $I \in \Lambda^2_A$. Imposing additional assumptions such as *pro-poor transfer sensitivity* (i.e., $I \in \Lambda^3_{AW}$) and *symmetry around the median* (i.e., $I \in \Lambda^2_{A\rho}$) increases the significance level of this last comparison to 1% but does not change the conclusions. All other regional comparisons of socioeconomic health inequality in cigarette consumption are not robust for the orders of ethical principles under consideration.

[20] In this empirical illustration, we chose to tests for order 2 to 4.

As for regional comparison for overweightedness, we note that under the *principle of income-related health transfer*, the only (regional) robust ranking occurs when we compare the West to the Northeast. For all $I \in \Lambda_A^2$, the West has less socioeconomic health inequality than the Northeast at a 5% significance level. Focusing on the comparison between the West and South, we note that there are no robust rankings under the *principle of income-related health transfer* (see top left panel of Figure 7 and Table 7). Adding the *symmetry around the median principle* does not change this result (see bottom left panel of Figure 7 and Table 7). However, if we impose *pro-poor transfer sensitivity* (see the top right panel in Figure 7 and Table 7) or *symmetry around the median principle* together with *pro-extreme rank transfer sensitivity* (see the bottom right panel in Figure 7 and Table 7), we note that the West has less socioeconomic inequality in overweightedness than the South at a 5% significance level. All other regional comparisons of socioeconomic inequality in overweightedness are not robust for the orders of ethical principles under consideration.

To conclude, this section provides temporal comparisons of cigarette consumption and overweightedness between 1997 and 2014. It shows a case where both the absolute and relative measures of inequality follow the same patterns. It also shows a case where they diverge. The divergence in the conclusions derived from these two different measures of socioeconomic health inequality is a clear example of why both measures should be reported in a dominance setting. We thus highlight, through empirical evidence, the importance of reporting robust rankings for absolute socioeconomic health inequality. More specifically, we show that comparisons and rankings derived from relative inequality cannot be used to infer comparisons and rankings for absolute inequalities. These results speak to the importance of developing approaches that allow for robust rankings of absolute socioeconomic health inequality. In addition to generally supporting the importance of developing dominance results for absolute socioeconomic health inequality, this empirical illustration provides evidence of the advantage of imposing the *symmetry around the median principle* in cases where the *principle of income-related health transfer* alone or together with *pro-poor transfer sensitivity*, does not allow to derive dominance results. Thus, in this empirical application we show case where a researcher can obtain robust orderings using Theorem 1 and that sometimes neither Theorem 1 nor Theorem 2 can lead to robust orderings. This speaks to the empirical importance and advantage of also using Theorem 3.

6 Conclusion

Previous literature on robust comparisons of socioeconomic health inequalities tackled mainly relative measures of inequality. In this paper, we focus on absolute socioeconomic health inequality comparisons in an attempt to fill a gap in the literature on dominance-based approach to absolute measures of socioeconomic health inequalities. We introduce a new graphical tool, the mean deviation concentration curve, and show how it may be used to identify robust rankings of absolute socioeconomic health inequality for all indices obeying the *principle of income-related health transfer.* We also propose higher-order versions of the mean deviation concentration curve to allow the analyst to increase the power of ordering by restricting his attention to indices obeying *pro-poor transfer sensitivity.*

Furthermore, we show how Khaled, Makdissi, and Yazbeck's (2018) generalized health range curves can be used to increase the power of ordering when the analyst is willing to impose *symmetry around the median* and *pro-extreme rank transfer sensitivity.* They may also be used when robust orderings cannot be obtained by increasing aversion to inequality under *pro-poor transfer sensitivity.* To show the applicability of the theoretical results, we conduct an empirical application in which we investigate temporal and regional comparisons of cigarette consumption and overweightedness and compare them with results obtained from robust comparisons of relative socioeconomic health inequality comparisons in Khaled, Makdissi, and Yazbeck (2018). The results from this empirical illustration are in line with our theoretical results, which speak to the importance of the proposed method and its applicability in empirical studies.

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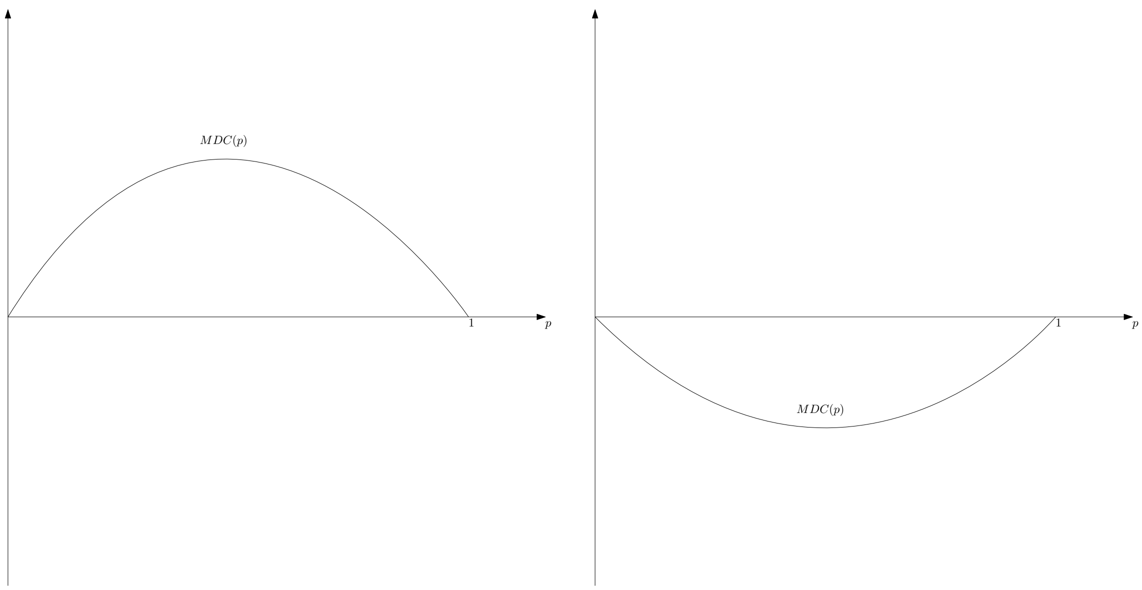
Figure 1: mean deviation concentration curves



$h(p)$ more concentrated among higher socioeconomic classes

$h(p)$ more concentrated among lower socioeconomic classes

Figure 2: Ethical principles and dominance theorems

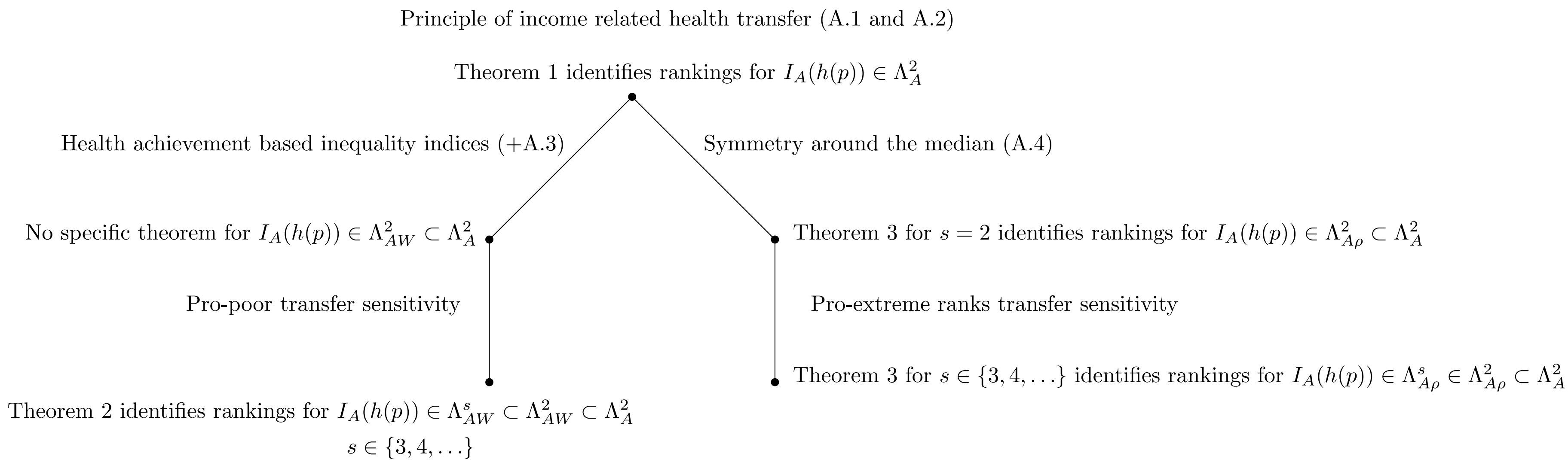


Figure 3: Socioeconomic inequality in cigarettes consumption

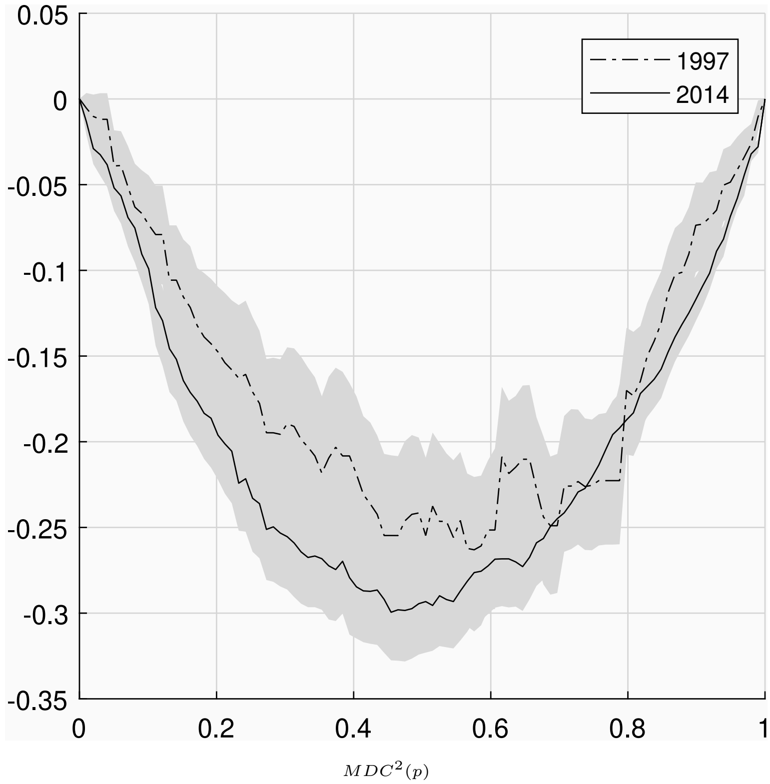


Figure 4: Higher order tests for socioeconomic inequality in cigarettes consumption

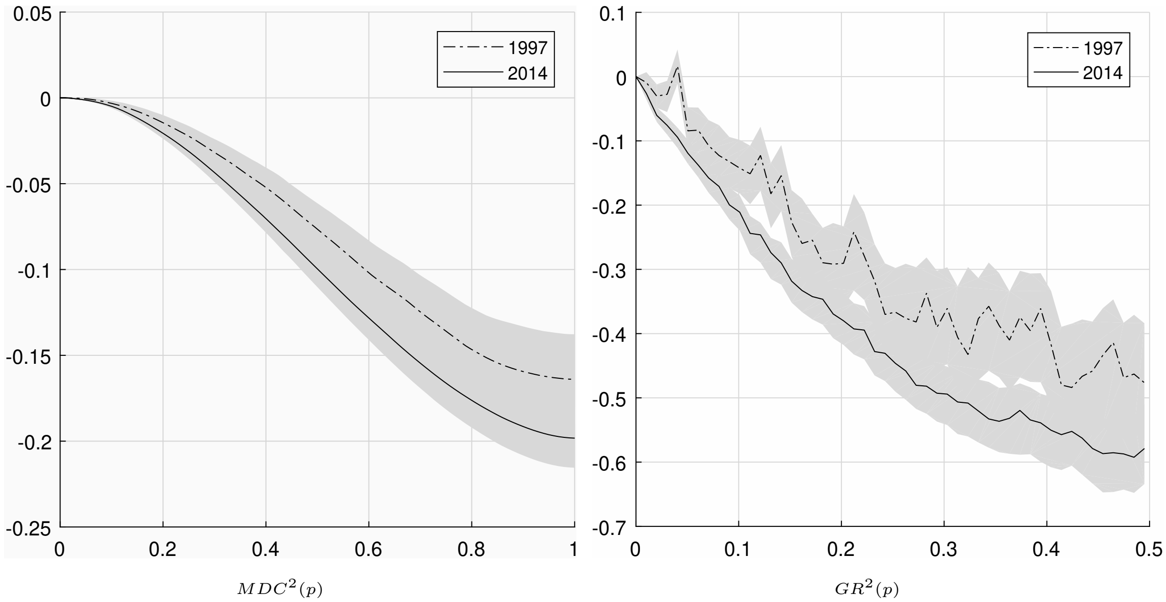


Figure 5: Socioeconomic inequality in obesity/overweight

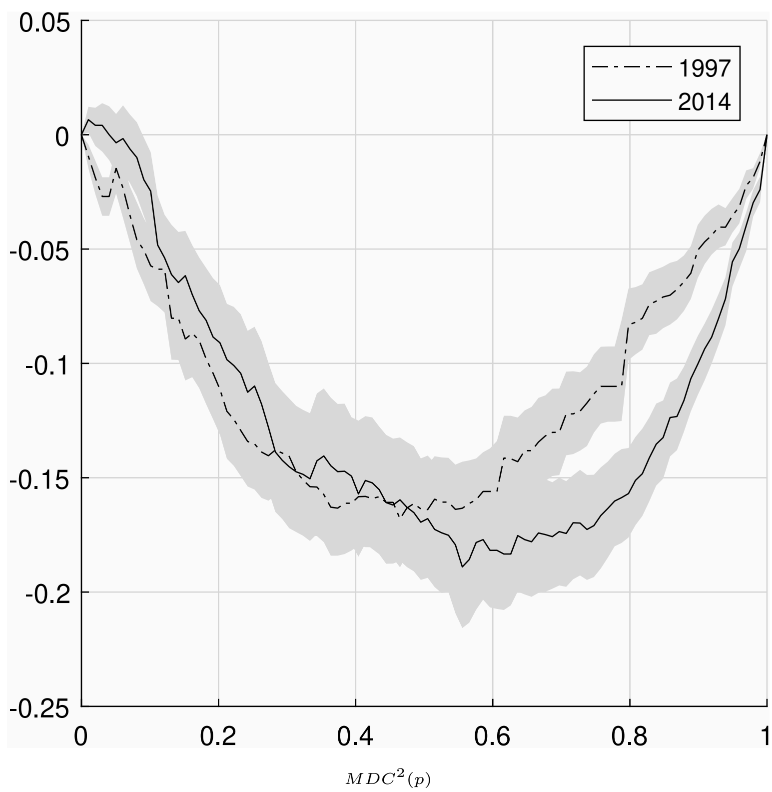


Figure 6: Higher order tests for socioeconomic inequality in obesity/overweight

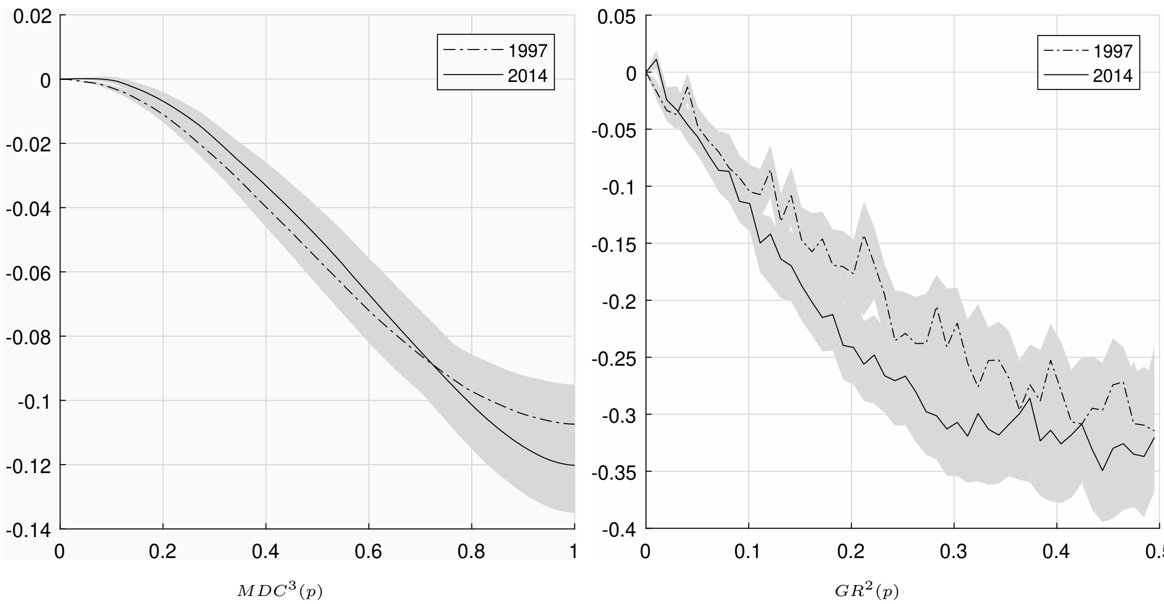


Figure 7: Socioeconomic inequality in BMI, West VS South

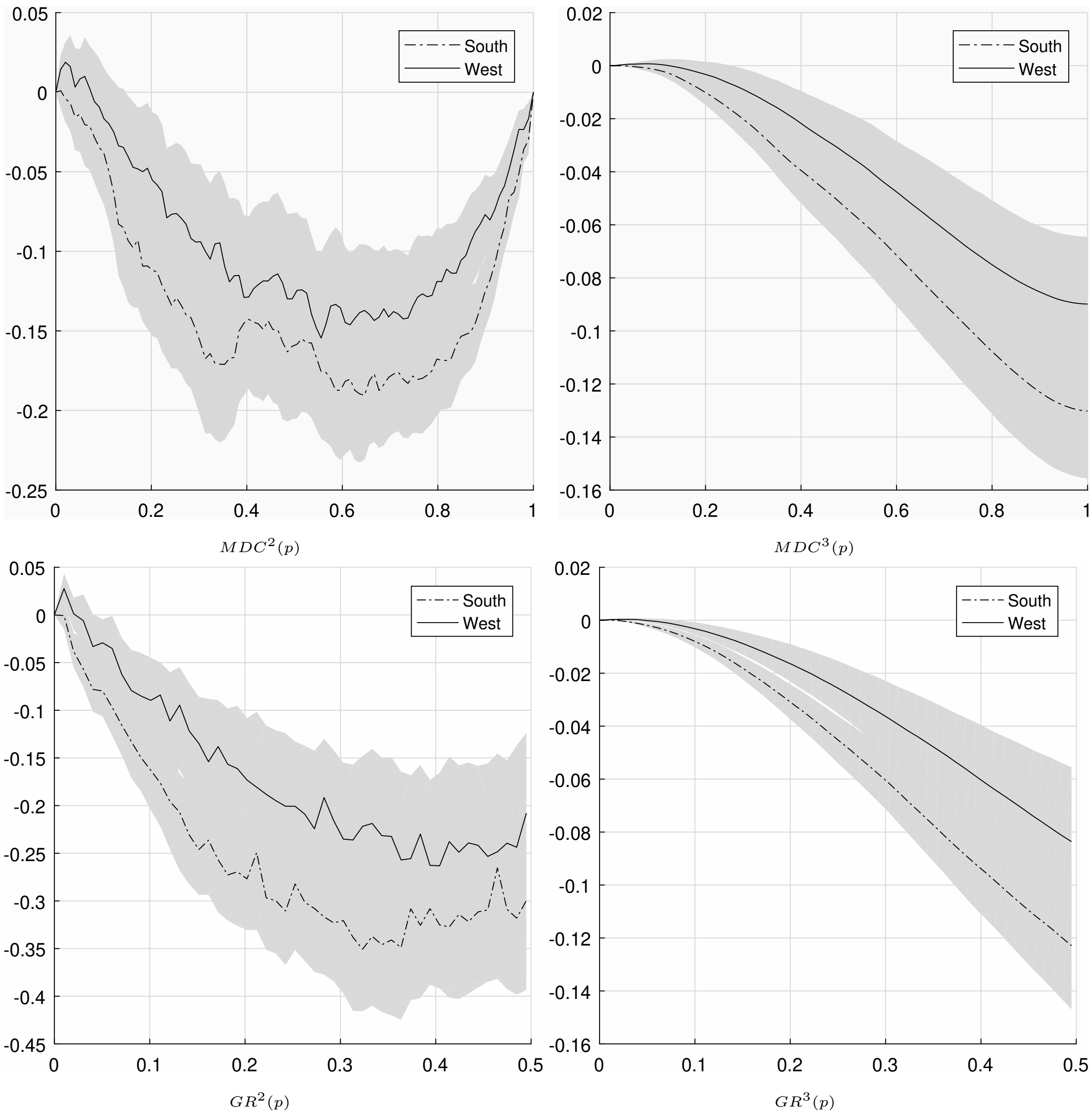


Table 1: Dominance tests practice guide

	Dominance	Index	Interpretation good-health	Interpretation ill-health
Theorem 1	$MDC_1(p) \leq MDC_2(p)$ for all $p \in [0,1]$.	$I_A(h_1) \leq I_A(h_2)$ for all $I_A(h) \in \Lambda_A^2$	Less socioeconomic inequality in 1	More socioeconomic inequality in 1
Theorem 2	$MDC_1^s(p) \leq MDC_2^s(p)$ for all $p \in [0,1]$.	$I_A(h_1) \leq I_A(h_2)$ for all $I_A(h) \in \Lambda_A^s$	Less socioeconomic inequality in 1	More socioeconomic inequality in 1
Theorem 3	$GR_2^s(p) \geq GR_1^s(p)$ for all $p \in [0,0.5]$.	$I_A(h_1) \leq I_A(h_2)$ for all $I_A(h) \in \Lambda_{A\rho}^s$	Less socioeconomic inequality in 1	More socioeconomic inequality in 1

Table 2: Dominance tests for MDC^s and GR^s comparisons for cigarette consumption

	p-value	
	s=2	s=3
$H_0 : MDC^s_{1997}(p) \leqslant MDC^s_{2014}(p), \forall p$ $H_1 : MDC^s_{1997}(p) > MDC^s_{2014}(p)$ for some p	0.0430	0.0170
$H_0 : MDC^s_{2014}(p) \leqslant MDC^s_{1997}(p), \forall p$ $H_1 : MDC^s_{2014}(p) > MDC^s_{1997}(p)$ for some p	0.5065	0.8569
$H_0 : GR^s_{1997}(p) \leqslant GR^s_{2014}(p), \forall p$ $H_1 : GR^s_{1997}(p) > GR^s_{2014}(p)$ for some p	0.0040	0.0010
$H_0 : GR^s_{2014}(p) \leqslant GR^s_{1997}(p), \forall p$ $H_1 : GR^s_{2014}(p) > GR^s_{1997}(p)$ for some p	0.9620	0.8368

Table 3: Dominance tests for MDC^2 comparisons for overweightedness

	p-value	
	$\mathscr{P} := [0, 0.1]$	$\mathscr{P} := [0.1, 1]$
$H_0 : MDC^2_{1997}(p) \leqslant, MDC^2_{2014}(p), \forall p \in \mathscr{P}$ $H_1 : MDC^2_{1997}(p) > MDC^2_{2014}(p)$ for some $p \in \mathscr{P}$	0.9930	0.0010
$H_0 : MDC^2_{2014}(p) \leqslant MDC^2_{1997}(p), \forall p \in \mathscr{P}$ $H_1 : MDC^2_{2014}(p) > MDC^2_{1997}(p)$ for some $p \in \mathscr{P}$	0.0010	0.1712

Table 4: Dominance tests for MDC^3 comparisons for overweightedness

	p-value
$H_0 : MDC^3_{1997}(p) = MDC^3_{2014}(p), \forall p$ $H_1 : MDC^3_{1997}(p) \neq MDC^3_{2014}(p)$ for some p	0.1842

Table 5: Dominance tests for GR^2 comparisons for overweightedness

	p-value
$H_0 : GR^2_{1997}(p) \leqslant GR^2_{2014}(p), \forall p$ $H_1 : GR^2_{1997}(p) > GR^2_{2014}(p)$ for some p	0.0080
$H_0 : GR^2_{2014}(p) \leqslant GR^2_{1997}(p), \forall p$ $H_1 : GR^2_{2014}(p) > GR^2_{1997}(p)$ for some p	0.6456

Table 6: Regional dominance tests: cigarette consumption and overweightedness

Cigarette consumption				
	Northeast	West	Midwest	South
Northeast	-		ND	ND
West	Λ^2_A **, Λ^3_{AW} *** and $\Lambda^2_{A\rho}$ ***	-	Λ^2_A ***	Λ^2_A ***
Midwest	ND		-	ND
South	ND		ND	-
Overweightedness				
	Northeast	West	Midwest	South
Northeast	-		ND	ND
West	Λ^2_A ** and $\Lambda^2_{A\rho}$ ***	-	ND	Λ^3_{AW} **, $\Lambda^3_{A\rho}$ ** and $\Lambda^4_{A\rho}$ ***
Midwest	ND	ND	-	ND
South	ND		ND	-

Significance levels ** 5%; *** 1%

Table 7: Dominance tests for MDC^s and GR^s comparisons for overweightedness between the West and the South

	p-value			
	MDC^s		GR^s	
	s=2	s=3	s=2	s=3
$H_0 : L^s_{1997}(p) \leqslant L^s_{2014}(p), \forall p$ $H_1 : L^s_{1997}(p) > L^s_{2014}(p)$ for some p	0.0801	0.0180	0.9560	0.8448
$H_0 : L^s_{2014}(p) \leqslant L^s_{1997}(p), \forall p$ $H_1 : L^s_{2014}(p) > L^s_{1997}(p)$ for some p	0.9970	0.8709	0.0951	0.0240
$L^s = MDC^s$ or GR^s				

A Health concentration curves and generalized health concentration curves

The generalized health concentration curve $GC(p)$ displays the absolute contribution of the p poorest individuals to average health. If one try to develop a dominance condition based on these curves, the resulting condition would be that $I_A(h_1) \leq I_A(h_2)$ for all $I_A(h) \in \Lambda_A^2$ if

$$GC_1(p) \geq GC_2(p) \text{ for all } p \in [0, 1], \tag{13}$$

and,

$$\mu_{h_2} \geq \mu_{h_1}. \tag{14}$$

Comparing the conditions in equations (13) and (14) to the dominance condition for relative socioeconomic health inequality in Khaled, Makdissi and Yazbeck (2018), we note two important differences. First, there is an additional condition on the average that is more restrictive than it may first appear. This condition implicitly means that the identification of robust rankings of absolute socioeconomic health inequality through generalized health concentration curves is only possible when two distributions have exactly the same average health status. More specifically, given that at rank $p = 1$, $GC(1) = \mu_h$, the inequalities in equations (13) and (14) can only be simultaneously verified if $\mu_{h1} = \mu_{h2}$. Second, the conditions in equations (13) and (14) are sufficient but not necessary conditions. This results indicates that it may be possible to derive a condition with another graphical tool.

Proof of the result in equations (13) and (14) Integrating by parts equation (2) yields

$$I_A(h) = \nu(p)GC(p)|_0^1 - \int_0^1 \nu^{(1)}(p)GC(p)\mathrm{d}p \tag{15}$$

Since by definition $GC(0) = 0$ and $GC(1) = \mu_h$ for all indices $I_A(h) \in \Lambda_A^2$, equation (15) can be rewritten as

$$I_A(h) = \nu(1)\mu_h - \int_0^1 \nu^{(1)}(p)GC(p)\mathrm{d}p \tag{16}$$

From equation (16), we get

$$\Delta I_{A12} = \nu(1)(\mu_{h2} - \mu_{h1}) + \int_0^1 \nu^{(1)}(p)\left[GC_1(p) - GC_2(p)\right]\mathrm{d}p \tag{17}$$

Note that $\nu^{(1)}(p)$ is non negative. This implies that if $GC_1(p) \geq GC_2(p)$ for all $p \in [0,1]$, then $\int_0^1 \nu^{(1)}(p)\left[GC_1(p) - GC_2^s(p)\right]\mathrm{d}p \geq 0$. If in addition, $\mu_{h2} \geq \mu_{h1}$, then $\Delta I_{A12} \geq 0$. This proves for sufficiency of the condition. ■

B Sets of indices

Proofs are based on the following mathematical definition of the set of indices.

Λ_A^2 is the set of all absolute socioeconomic health inequality indices, $I_A(h)$ such that

a) $\nu(p) \in \Re$,

b) $\nu(p)$ is continuous and differentiable almost everywhere over $[0,1]$,

c) $\int_0^1 \nu(p)\mathrm{d}p = 0$,

d) $\nu^{(1)}(p) > 0,\ \forall p \in [0,1]$.

Λ_R^2 is the set of all absolute socioeconomic health inequality indices, $I_R(h)$ such that

a) $\nu(p) \in \Re$,

b) $\nu(p)$ is continuous and differentiable almost everywhere over $[0,1]$,

c) $\int_0^1 \nu(p)\mathrm{d}p = 0$,

d) $\nu^{(1)}(p) > 0,\ \forall p \in [0,1]$.

Λ_{AW}^s is the set of all absolute socioeconomic health inequality indices, $I_A(h) \in \Lambda_A^2$ such that

a) $\nu(p) \in (-\infty, 1]$,

b) $\nu(p)$ is continuous and $s-1$-time differentiable almost everywhere over $[0,1]$,

c) $\nu^{(i)}(1) = 0 \ \forall i \in \{1, 2, \ldots, s-1\}$,

d) $(-1)^{i+1}\nu^{(i)}(p) \geqslant 0 \ \forall p \in [0,1] \ \forall i \in \{1, 2, \ldots, s-1\}$.

$\Lambda^s_{A\rho}$ is the set of all absolute socioeconomic health inequality indices, $I_A(h) \in \Lambda^2_A$ such that

a) $\nu(1-p) = \nu(p) \ \forall p \in [0,1]$,

b) $\nu(p)$ is continuous and $s-1$-time differentiable almost everywhere over $[0,1]$,

c) $\nu^{(i)}(0.5) = 0 \ \forall i \in \{1, 2, \ldots, s-1\}$,

d) $(-1)^{i+1}\nu^{(i)}(p) \geqslant 0 \ \forall p \in [0,0.5] \ \forall i \in \{1, 2, \ldots, s-1\}$.

C Proofs of dominance theorems

Proof of Theorem 1. First note that for $I_A(h) \in \Lambda^2_A$, equation (2) can be rewritten as

$$I_A(h) = \int_0^1 \nu(p)h(p)\mathrm{d}p = \int_0^1 (1-\nu(p))\widetilde{h}(p)\mathrm{d}p \tag{18}$$

Integrating by parts equation (18), we get

$$I_A(h) = (1-\nu(p))MDC^2(p)\big|_0^1 + \int_0^1 \nu^{(1)}(p)MDC^2(p)\mathrm{d}p \tag{19}$$

Since by definition $MDC^2(0) = MDC^2(1) = 0$, $I_A(h) \in \Lambda^2_A$, the first term on the right hand side of the equation is nil. This yields to

$$I_A(h) = \int_0^1 \nu^{(1)}(p)MDC^2(p)\mathrm{d}p. \tag{20}$$

Let $\Delta I_{A12} = I_A(h_2) - I_A(h_1)$. From equation (20), we get

$$\Delta I_{A12} = \int_0^1 \nu^{(1)}(p)\left[MDC^s_2(p) - MDC^s_1(p)\right]\mathrm{d}p. \tag{21}$$

Note that $\nu^{(1)}(p)$ is non negative. This implies that if $MDC_2^2(p) \geq MDC_1^2(p)$ for all $p \in [0,1]$, then $\Delta I_{A12} \geq 0$. This proves for sufficiency of the condition.

Having provided a sufficiency condition let us now prove for the necessity of the condition. Consider now the set of indices $I_A(h) \in \Lambda_A^2$ for which $\nu(p)$ takes the following form:

$$\nu(p) = \begin{cases} -\theta & 0 \leq p_c \\ [p_c + \varepsilon - p] & p_c \leq p \leq p_c + \varepsilon \\ \varepsilon - \theta & p \geq p_c + \varepsilon \end{cases}, \tag{22}$$

where $p_c \in [0,1]$ and θ is chosen so that $\int_0^1 \nu(p)\mathrm{d}p = 0$. Since $\nu(p)$ is differentiable almost everywhere, it satisfies the conditions in the definition of Λ_A^2. Differentiating equation (22) yields

$$\nu^{(1)}(p) = \begin{cases} 0 & 0 \leq p_c \\ 1 & p_c \leq p \leq p_c + \varepsilon \\ 0 & p \geq p_c + \varepsilon \end{cases} \tag{23}$$

Imagine now that $MDC_2^2(p) < MDC_1^2(p)$ on an interval $[p_c, p_c + \varepsilon]$ for ε that can be arbitrarily close to 0. For any $\nu(p)$ obeying the relation in equation (23), the expression in equation (21) is negative. Hence it cannot be that $MDC_2^s(p) < MDC_1^s(p)$ for $p \in [p_c, p_c+\varepsilon]$. This proves the necessity of the condition. ■

Proof of Theorem 2. First note that for $I_A(h) \in \Lambda_{AW}^s$, equation (2) can be rewritten as

$$I_A(h) = \int_0^1 (1 - \omega(p))h(p)\mathrm{d}p = \int_0^1 \omega(p)\widetilde{h}(p)\mathrm{d}p \tag{24}$$

Integrating by parts equation (24), we get

$$I_A(h) = \omega(p)MDC^2(p)\big|_0^1 - \int_0^1 \omega^{(1)}(p)MDC^2(p)\mathrm{d}p \tag{25}$$

Since by definition $MDC^2(0) = MDC^2(1) = 0$, $I_A(h) \in \Lambda_A^2$, the first term on the right hand side of the equation is nil. This yields to

$$I_A(h) = -\int_0^1 \omega^{(1)}(p)MDC^2(p)\mathrm{d}p \tag{26}$$

Now assume that for $s-1$, we have

$$I_A(h) = (-1)^{s-2} \int_0^1 \omega^{(s-2)}(p) MDC^{s-1}(p) \mathrm{d}p. \tag{27}$$

Integrating by parts equation (27) yields

$$I_A(h) = (-1)^{s-2} \left\{ \omega^{(s-2)}(p) MDC^s(p) \Big|_0^1 - \int_0^1 \omega^{(s-1)} MDC^s(p) \mathrm{d}p \right\}. \tag{28}$$

Since by definition $MDC^s(0) = 0$ and $\omega^{(s-2)}(1) = 0$ (because $\nu^{(s-2)}(1) = 0$) for all indices $I_A(h) \in \Lambda^s_{AW}$, the first term in the braces on the right hand side of the equation is nil. This yield

$$I_A(h) = (-1)^{s-1} \int_0^1 \omega^{(s-1)}(p) MDC^s(p) \mathrm{d}p. \tag{29}$$

Given that equations (27) and (29) both conform to the relation depicted in equation (26), it follows that equation (29) holds for all $s \in \{2, 3, \dots\}$. Let $\Delta I_{A12} = I_A(h_2) - I_A(h_1)$. From equation (29), we get

$$\Delta I_{A12} = (-1)^{s-1} \int_0^1 \omega^{(s-1)}(p) \left[MDC^s_2(p) - MDC^s_1(p) \right] \mathrm{d}p. \tag{30}$$

Note that $(-1)^{s-1}\omega^{(s-1)}(p)$ is non negative. This implies that if $MDC^s_2(p) \geq MDC^s_1(p)$ for all $p \in [0, 1]$, then $\Delta I_{A12} \geq 0$. This proves for sufficiency of the condition.

Having provided a sufficiency condition let us now prove for the necessity of the condition. Consider now the set of indices $I_A(h) \in \Lambda^s_{AW}$ for which $\omega^{(s-2)}(p)$ takes the following form:

$$\omega^{(s-2)}(p) = \begin{cases} (-1)^{s-2}\varepsilon & 0 \leq p_c \\ (-1)^{s-2}\left[p_c + \varepsilon - p\right] & p_c \leq p \leq p_c + \varepsilon \\ 0 & p \geq p_c + \varepsilon \end{cases} \tag{31}$$

where $p_c \in [0, 1]$. Since $\omega(p)$ is differentiable almost everywhere, it satisfies the conditions in the definition of Λ^s_{AW}. Differentiating equation (31) yields

$$\omega^{(s-1)}(p) = \begin{cases} 0 & 0 \leq p_c \\ (-1)^{s-1} & p_c \leq p \leq p_c + \varepsilon \\ 0 & p \geq p_c + \varepsilon \end{cases} \tag{32}$$

Imagine now that $MDC^s(p) < MDC_1^s(p)$ on an interval $[p_c, p_c + \varepsilon]$ for ε that can be arbitrarily close to 0. For any $\omega(p)$ obeying the relation in equation (32), the expression in equation (30) is negative. Hence it cannot be that $MDC_2^s(p) < MDC_1^s(p)$ for $p \in [p_c, p_c+\varepsilon]$. This proves the necessity of the condition. ■

Proof of Theorem 3. First note that for $I_A(h) \in \Lambda^s_{A\rho}$, equation (2) can be rewritten as

$$I_A(h) = -\int_0^{0.5} \nu(p) r(p) \mathrm{d}p \tag{33}$$

Integrating by parts equation (33), we get

$$I_A(h) = -\nu(p) GR^2(p)\Big|_0^{0.5} + \int_0^{0.5} \nu^{(1)}(p) GR^2(p) \mathrm{d}p. \tag{34}$$

Since by definition $GR^2(0) = 0$ and $\nu(0.5) = 0$ for all indices $I_A(h) \in \Lambda^s_{A\rho}$, the first term on the right hand side of the equation is nil. This yields to

$$I_A(h) = \int_0^{0.5} \nu^{(1)}(p) GR^2(p) \mathrm{d}p. \tag{35}$$

Now assume that for $s-1$, we have

$$I_A(h) = (-1)^{s-1} \int_0^{0.5} \nu^{(s-2)}(p) GR^{s-1}(p) \mathrm{d}p. \tag{36}$$

Integrating by parts equation (36) yields

$$I_A(h) = (-1)^{s-1} \left\{ \nu^{(s-2)}(p) GR^s(p)\Big|_0^{0.5} - \int_0^{0.5} \nu^{(s-1)} GR^s(p) \mathrm{d}p \right\}. \tag{37}$$

Since by definition $GR^s(0) = 0$ and $\nu^{(s-2)}(0.5) = 0$ for all indices $I_A(h) \in \Lambda^s_{A\rho}$, the first term in the braces on the right hand side of the equation is nil. This yield

$$I_A(h) = (-1)^s \int_0^{0.5} \nu^{(s-1)}(p) GR^s(p) \mathrm{d}p. \tag{38}$$

Given that equations (35) and (38) both conform to the relation depicted in equation (36), it follows that equation (38) holds for all $s \in \{2, 3, \dots\}$. Let $\Delta I_{A12} = I_A(h_2) - I_A(h_1)$.

From equation (38), we get

$$\Delta I_{A12} = (-1)^s \int_0^{0.5} \nu^{(s-1)}(p) \left[GR_2^s(p) - GR_1^s(p)\right] \mathrm{d}p. \tag{39}$$

Note that $(-1)^s \nu^{(s-1)}(p)$ is non negative. This implies that if $GR_2^s(p) \geq GR_1^s(p)$ for all $p \in [0, 0.5]$, then $\Delta I_{A12} \geq 0$. This proves for sufficiency of the condition.

Having provided a sufficiency condition let us now prove for the necessity of the condition. Consider now the set of indices $I_A(h) \in \Lambda^s_{A\rho}$ for which $\nu^{(s-2)}(p)$ takes the following form:

$$\nu^{(s-2)}(p) = \begin{cases} (-1)^{s-1}\varepsilon & 0 \leq p_c \\ (-1)^{s-1}\left[p_c + \varepsilon - p\right] & p_c \leq p \leq p_c + \varepsilon \\ 0 & p \geq p_c + \varepsilon \end{cases} \tag{40}$$

where $p_c \in [0, 0.5]$. Since $\nu(p)$ is differentiable almost everywhere, it satisfies the conditions in the definition of $\Lambda^s_{A\rho}$. Differentiating equation (40) yields

$$\nu^{(s-1)}(p) = \begin{cases} 0 & 0 \leq p_c \\ (-1)^s & p_c \leq p \leq p_c + \varepsilon \\ 0 & p \geq p_c + \varepsilon \end{cases} \tag{41}$$

Imagine now that $GR_2^s(p) < GR_1^s(p)$ on an interval $[p_c, p_c + \varepsilon]$ for ε that can be arbitrarily close to 0. For any $\nu(p)$ obeying the relation in equation (40), the expression in equation (39) is negative. Hence it cannot be that $GR_2^s(p) < GR_1^s(p)$ for $p \in [p_c, p_c + \varepsilon]$. This proves the necessity of the condition. ■

D Estimator for $MDC^s(p)$

Computation of integrals containing indicator variables involving inverse of $\widehat{F}_Y^1$. Even though $\widehat{F}_Y$ is a step function, the following standard result holds: $y_i \leqslant \widehat{F}_Y^{-1}(p)$ if and only if $\widehat{F}_Y(y_i) \leqslant p$. In what follows, We will check the formula for the estimator by induction.

First compute

$$
\begin{aligned}
I_3(p) &= \int_0^p \mathbb{1}(y_i \leqslant \widehat{F}_Y^{-1}(u))\mathrm{d}u && (42)\\
&= \int_0^p \mathbb{1}(\widehat{F}_Y(y_i) \leqslant u)\mathrm{d}u && (43)\\
&= (p - \widehat{F}_Y(y_i))\mathbb{1}(\widehat{F}_Y(y_i) \leqslant p) && (44)\\
&= \frac{(p - \widehat{F}_Y(y_i))^{3-2}}{(3-2)!}\mathbb{1}(\widehat{F}_Y(y_i) \leqslant p). && (45)
\end{aligned}
$$

Then recursively compute for $s > 3$

$$
\begin{aligned}
I_s(p) &= \int_0^p I_{s-1}(u)\mathrm{d}u && (46)\\
&= \int_0^p \frac{(u - \widehat{F}_Y(y_i))^{s-3}}{(s-3)!}\mathbb{1}(\widehat{F}_Y(y_i) \leqslant u)\mathrm{d}u && (47)\\
&= \frac{(p - \widehat{F}_Y(y_i))^{s-2}}{(s-2)!}\mathbb{1}(\widehat{F}_Y(y_i) \leqslant p). && (48)
\end{aligned}
$$

■

Estimator for $MDC^s(p)$. The fact that $MDC(p)$ can be written as

$$MDC(p) = \int_0^\infty \int_0^\infty (\mu_h - h)\mathbb{1}((y \leq F_Y^{-1}(p))f_{H,Y}(h,y)\mathrm{d}h\mathrm{d}y, \quad (49)$$

results into the simple estimator for $MDC(p)$ from a sample (h_i, y_i) for $i = 1, \ldots, N$ to be given by:

$$\widehat{MDC}(p) = \frac{1}{N}\sum_{i=1}^{N}\left[\overline{h} - h_i\right]\mathbb{1}(y_i \leqslant \widehat{F}_Y^{-1}(p)), \quad (50)$$

where $\overline{h} = N^{-1}\sum_{i=1}^N h_i$. From equations (48) and (49), we can also get the simple estimator for $MDC^s(p)$

$$\widehat{MDC}^s(p) = \frac{1}{N}\sum_{i=1}^{N}\left[\overline{h} - h_i\right]\frac{\left[p - \widehat{F}_Y^{-1}(p)\right]^{s-2}}{(s-2)!}\mathbb{1}(y_i \leqslant \widehat{F}_Y^{-1}(p)). \quad (51)$$

■

E Bootstrap algorithm

The bootstrap algorithm is constructed as follows. Assume that we have an i.i.d. sample of size n_0 from the random variable corresponding to first theoretical curve L_0 and and and i.i.d. sample of size n_1 from the random variable corresponding to the second theoretical curve L_1. Denote those samples by $\mathcal{S}_0$ and $\mathcal{S}_1$ respectively. Let $\mathcal{S}$ be the combined sample. Let $\widehat{L}_0$ and $\widehat{L}_1$ be the nonparametric estimators of L_0 and L_1 respectively, constructed from those two samples and let $\widehat{L}_{01}(p) = \widehat{L}_0(p) - \widehat{L}_1(p)$. Let

$$\widehat{\tau} = \sqrt{\frac{n_0 n_1}{n_0 + n_1}} \sup_p \widehat{L}_{01}(p)$$

1. Repeat for $b = 1, \ldots, B$

   (a) Draw a sample of size n_0 from $\mathcal{S}$. Compute the nonparametric estimator $\widehat{L}_{0b}$.

   (b) Draw a sample of size n_1 from $\mathcal{S}$. Compute the nonparametric estimator $\widehat{L}_{1b}$.

   (c) Compute $\widehat{L}_{01b}(p) = \widehat{L}_{0b}(p) - \widehat{L}_{1b}(p)$.

   (d) Compute[21] $\widehat{\tau}_b = \sqrt{\frac{n_0 n_1}{n_0+n_1}} \sup_p \left(\widehat{L}_{01b}(p) - \widehat{L}_{01}(p)\right)$.

2. Using the sample $\widehat{\tau}_1, \ldots, \widehat{\tau}_B$, compute the bootstrap p-value

$$\frac{1}{B} \sum_{b=1}^{B} \mathbb{1}(\widehat{\tau}_b > \widehat{\tau}).$$

[21] In our empirical application we use a recenterred statistics as proposed by Schechtman, Shelef, Yitzhaki and Zitikis (2008). One could have also used the statistics $\widehat{\tau}_b = \sqrt{\frac{n_0 n_1}{n_0+n_1}} \sup_p \widehat{L}_{01b}(p)$