Notes on Cunha, Heckman, and Schennach (2010)

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I had some notes originally in June 2011, then in March 2012.

Stefano had notes of his own.

It gave intuition for the results in Cunha, Heckman, and Schennach (2010).

Why is it efficient to invest in least well-off (by endowment) children?

2 Children A and B and Two Periods $(1, 2)$

(I)

$$\theta_1^A = f(\theta_0^A, I_0^A) \qquad \theta_1^B = f(\theta_0^B, I_0^B)$$
$$\theta_2^A = f(\theta_1^A, I_1^A) \qquad \theta_2^B = f(\theta_1^B, I_1^B)$$

Assume $r = 0$.

We seek to maximize $\theta_2^A + \theta_2^B$ subject to constraints in 1.

$$I_0^A + I_0^B + I_1^A + I_2^B = M$$

Intuition is that by investing we concavify the production functions.

Intuition:

Take Simple Case:

$$\theta^A = g(I_A \alpha^A) \tag{1}$$

$$\theta^B = g(I_B \alpha^B) \tag{2}$$

$$g' > 0 \qquad g'' < 0$$

Allocate I_A and I_B

$\max \theta^A + \theta^B$

st. (1), (2) and

$$\begin{aligned} I_A + I_B &= M \\ \alpha^A g'(I_A\alpha^A) &= \alpha^B g'(I_B\alpha^B) \\ \frac{\alpha^A}{\alpha^B} &= \frac{g'(I_A\alpha^A)}{g'(I_B\alpha^B)} \\ I_A &= I_B \qquad \text{if } \alpha^A = \alpha^B \end{aligned}$$

$$\text{If } \alpha^A > \alpha^B \Rightarrow I_A\alpha^A < I_B\alpha^B$$

$$\therefore I_B > I_A$$

(II) Cobb-Douglas case

$$\begin{aligned} g(b) &= b^\phi \\ \phi\alpha^A(I_A\alpha^A)^{\phi-1} &= \phi\alpha^B(I_B\alpha^B)^{\phi-1} \\ \left(\frac{I_B}{I_A}\right)^{1-\phi} &= \left(\frac{I_A}{I_B}\right)^{\phi-1} = \left(\frac{\alpha_B}{\alpha_A}\right) \end{aligned}$$

$$
\begin{aligned}
(\phi - 1)\ln(I_A/I_B) &= \phi \ln(\alpha^B/\alpha^A) \\
\ln(I_A/I_B) &= \left(\frac{\phi}{\phi - 1}\right) \ln\left(\frac{\alpha^A}{\alpha^B}\right) \\
& 0 < \phi < 1, \\
& \alpha^A > \alpha^B \\
& \implies \ln(I_A/I_B) < 0 \\
& I_B > I_A \ \text{ if } \ \alpha^A/\alpha_B > 1
\end{aligned}
$$

Take a second case

$$
\begin{aligned}
\theta^A &= \alpha^A g(I^A) \qquad g' > 0 \qquad g'' < 0 \\
\theta^B &= \alpha^B g(I^B)
\end{aligned}
$$

$$
\max \alpha^A g(I_A) + \alpha^B g(I^B)
$$

st.

$$
\begin{aligned}
M &= I^A + I^B \\
\alpha^A g'(I_A) &= \alpha^B g'(I^B) \\
\frac{\alpha^A}{\alpha_B} &= \frac{g'(I^B)}{g'(I^A)} \\
\alpha^A > \alpha^B &\implies I_B < I^A
\end{aligned}
$$

Suppose we can choose

$$\alpha^A \text{ and } \alpha^B$$

$$\text{Total Cost: } C(\alpha^A) + C(\alpha^B)$$

We would always choose to equalize $(\alpha^A) = \alpha^B$ (at equal cost).

(III) Intuition for Cunha, Heckman, and Schennach (2010)

Equality $\Rightarrow$ Efficiency Problem

Consider a social planner who seeks to maximize the aggregate human capital of society. There are two children: A and B. They differ in their initial endowments θ_A and θ_B respectively. Assume they are biologically determined, outside the control of the social planner. Adult human capital of children is h_A and h_B and are produced by investment (X), a scalar, where X_A is investment in A and X_B is investment in B:

$$h_A = g(X_A, \theta_A)$$

$$h_B = g(X_B, \theta_B)$$

where g is increasing in both arguments, $g_{11}(\cdot) < 0$, $g_{22}(\cdot) < 0$, and $g_{12}(\cdot) > 0$. Suppose $\theta_A > \theta_B$, if the social planner has a fixed budget for investment, $\bar{X}$, where

$$\bar{X} = X_A + X_B.$$

(i) What is the optimal policy for investment in children? Is it equalizing or disequalizing in terms of initial conditions θ_A and θ_B? Characterize the ratio of X_A and X_B. For specificity start with the special case $h_A = \theta_A g(X_A)$; $h_B = \theta_B g(X_B)$.

(ii) Suppose next that instead of being biologically determined outside the control of the parent, θ is determined by early childhood investment I and early parental background P:

$$\theta = \eta(I, P),$$

where η is increasing in I and P, $\eta_{11}(\cdot) < 0$, $\eta_{22}(\cdot) < 0$ and $\eta_{12}(\cdot)$ may be negative, positive or zero. Assume $P_A > P_B$. What is the optimal policy for investment in the first stage of life of the children if the goal is to maximize aggregate human capital? Assume a constraint $\bar{I} = I_A + I_B$. Assume also that the policy in part (i) is in effect. In answering this question analyze the following examples:

(a) a Cobb-Douglas case:

$$\theta = I^{\gamma} P^{\delta}$$

(b) a Leontief case:

$$\theta = \min \{I, P\}$$

(c) and a case with perfect substitutes

$$\theta = \tau_1 I + \tau_2 P, \quad I > 0, \quad \tau_1, \tau_2 > 0,$$

Then discuss the general case. For each case characterize the optimal policy for I_A and I_B, the ratio I_A/I_B, and the ratio X_A/X_B. Under what conditions is the policy (ii) joined with (i) equalizing? Disequalizing? At what stage(s) is policy equalizing? Disequalizing? Given the intuition for your results. In the Cobb-Douglas case for g and η, can the optimal policy be equalizing at any stage?

(iii) Discuss the empirical evidence on the effectiveness of early childhood investments in equalizing adult outcomes.

Intuition: If a social planner could design second stage production functions to optimize aggregate output, he would symmetrize the technology for both people. First stage investment symmetrizes.